6-4

Parallel Lines and Proportional Parts

What You'll Learn

- Use proportional parts of triangles.
- Divide a segment into parts.

Vocabulary

- midsegment

How do city planners use geometry?

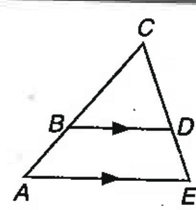
Street maps frequently have parallel and perpendicular lines. In Chicago, because of Lake Michigan, Lake Shore Drive runs at an angle between Oak Street and Ontario Street. City planners need to take this angle into account when determining dimensions of available land along Lake Shore Drive.

PROPORTIONAL PARTS OF TRIANGLES Nonparallel transversals that intersect parallel lines can be extended to form similar triangles. So the sides of the triangles are proportional.

Theorem 6.4

Triangle Proportionality Theorem If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

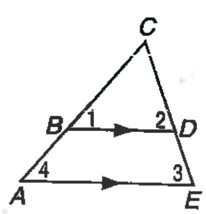
Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.



Proof Theorem 6.4

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$



Paragraph Proof:

Since $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. Then, by AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$. Substituting for CA and CE in the ratio, we get the following proportion.

$$\frac{BA + CB}{CB} = \frac{DE + CD}{CD}$$

$$\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD}$$ Rewrite as a sum.

$$\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1$$ $\frac{CB}{CB} = 1$ and $\frac{CD}{CD} = 1$

$$\frac{BA}{CB} = \frac{DE}{CD}$$ Subtract 1 from each side.

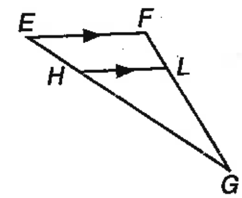
Study Tip

Overlapping Triangles

Trace two copies of $\triangle ACE$. Cut along $\overline{BD}$ to form $\triangle BCD$. Now $\triangle ACE$ and $\triangle BCD$ are no longer overlapping. Place the triangles side-by-side to compare corresponding angles and sides.

Example 1 Find the Length of a Side

In $\triangle EFG$, $\overline{HL} \parallel \overline{EF}$, $EH = 9$, $HG = 21$, and $FL = 6$. Find LG.



From the Triangle Proportionality Theorem, $\frac{EH}{HG} = \frac{FL}{LG}$.

Substitute the known measures.

$\frac{9}{21} = \frac{6}{LG}$

$9(LG) = (21)6$ Cross products

$9(LG) = 126$ Multiply.

$LG = 14$ Divide each side by 9.

Study Tip

Using Fractions You can also rewrite $\frac{9}{21}$ as $\frac{3}{7}$. Then use your knowledge of fractions to find the missing denominator.

$$\frac{3}{7} = \frac{6}{?} \quad (\times 2)$$

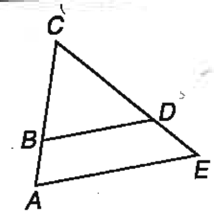
The correct denominator is 14.

Proportional parts of a triangle can also be used to prove the converse of Theorem 6.4.

Theorem 6.5

Converse of the Triangle Proportionality Theorem If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

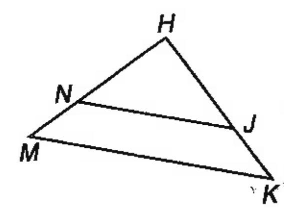
Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.



You will prove Theorem 6.5 in Exercise 38.

Example 2 Determine Parallel Lines

In $\triangle HKM$, $HM = 15$, $HN = 10$, and $\overline{HJ}$ is twice the length of $\overline{JK}$. Determine whether $\overline{NJ} \parallel \overline{MK}$. Explain.



$HM = HN + NM$ Segment Addition Postulate

$15 = 10 + NM$ $HM = 15$, $HN = 10$

$5 = NM$ Subtract 10 from each side.

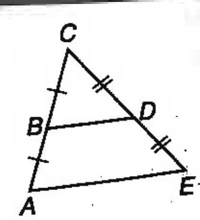
In order to show $\overline{NJ} \parallel \overline{MK}$, we must show that $\frac{HN}{NM} = \frac{HJ}{JK}$. $HN = 10$ and $NM = HM - HN$ or 5. So $\frac{HN}{NM} = \frac{10}{5}$ or 2. Let $JK = x$. Then $HJ = 2x$. So, $\frac{HJ}{JK} = \frac{2x}{x}$ or 2. Thus, $\frac{HN}{NM} = \frac{HJ}{JK} = 2$. Since the sides have proportional lengths, $\overline{NJ} \parallel \overline{MK}$.

A **midsegment** of a triangle is a segment whose endpoints are the midpoints of two sides of the triangle.

Theorem 6.6

Triangle Midsegment Theorem A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

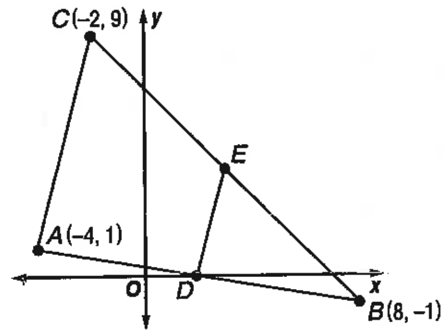
Example: If B and D are midpoints of $\overline{AC}$ and $\overline{EC}$ respectively, $\overline{BD} \parallel \overline{AE}$ and $BD = \frac{1}{2}AE$.



You will prove Theorem 6.6 in Exercise 39.

Example 3 Midsegment of a Triangle

Triangle ABC has vertices $A(-4, 1)$, $B(8, -1)$, and $C(-2, 9)$. $\overline{DE}$ is a midsegment of $\triangle ABC$.



a. Find the coordinates of D and E.

Use the Midpoint Formula to find the midpoints of $\overline{AB}$ and $\overline{CB}$.

$$D\left(\frac{-4+8}{2}, \frac{1+(-1)}{2}\right) = D(2, 0)$$

$$E\left(\frac{-2+8}{2}, \frac{9+(-1)}{2}\right) = E(3, 4)$$

b. Verify that $\overline{AC}$ is parallel to $\overline{DE}$.

If the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

slope of $\overline{AC} = \frac{9-1}{-2-(-4)}$ or 4

slope of $\overline{DE} = \frac{4-0}{3-2}$ or 4

Because the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

c. Verify that $DE = \frac{1}{2}AC$.

First, use the Distance Formula to find AC and DE.

$$AC = \sqrt{[-2-(-4)]^2 + (9-1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$DE = \sqrt{(3-2)^2 + (4-0)^2} = \sqrt{1+16} = \sqrt{17}$$

$$\frac{DE}{AC} = \frac{\sqrt{17}}{\sqrt{68}} = \sqrt{\frac{1}{4}} \text{ or } \frac{1}{2}$$

If $\frac{DE}{AC} = \frac{1}{2}$, then $DE = \frac{1}{2}AC$.

Study Tip

Look Back

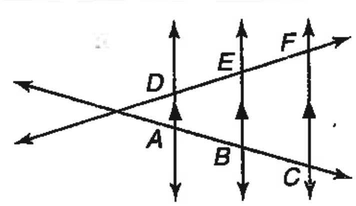
To review the **Distance and Midpoint Formulas,** see Lesson 1-3.

DIVIDE SEGMENTS PROPORTIONALLY We have seen that parallel lines cut the sides of a triangle into proportional parts. Three or more parallel lines also separate transversals into proportional parts. If the ratio is 1, they separate the transversals into congruent parts.

Corollaries

6.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example: If $\overleftrightarrow{DA} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$, $\frac{AC}{DF} = \frac{BC}{EF}$, and $\frac{AC}{BC} = \frac{DF}{EF}$.



6.2 If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

Study Tip

Three Parallel Lines

Corollary 6.1 is a special case of Theorem 6.4. In some drawings, the transversals are not shown to intersect. But, if extended, they will intersect and therefore, form triangles with each parallel line and the transversals.

Example 4 Proportional Segments

MAPS Refer to the map at the beginning of the lesson. The streets from Oak Street to Ontario Street are all parallel to each other. The distance from Oak Street to Ontario along Michigan Avenue is about 3800 feet. The distance between the same two streets along Lake Shore Drive is about 4430 feet. If the distance from Delaware Place to Walton Street along Michigan Avenue is about 411 feet, what is the distance between those streets along Lake Shore Drive?

Make a sketch of the streets in the problem. Notice that the streets form the bottom portion of a triangle that is cut by parallel lines. So you can use the Triangle Proportionality Theorem.

$$\frac{\text{Michigan Ave. Delaware to Walton}}{\text{Oak to Ontario}} = \frac{\text{Lake Shore Drive Delaware to Walton}}{\text{Oak to Ontario}}$$ Triangle Proportionality Theorem

$$\frac{411}{3800} = \frac{x}{4430}$$ Substitution

$$3800 \cdot x = 411(4430)$$ Cross products

$$3800x = 1{,}820{,}730$$ Multiply.

$$x = 479$$ Divide each side by 3800.

The distance from Delaware Place to Oak Street along Lake Shore Drive is about 479 feet.

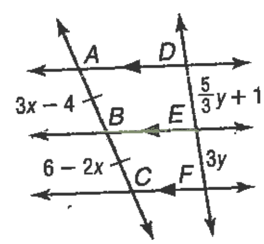
More About. . .

Maps Modern map-making techniques use images taken from space to produce accurate representations on paper. In February 2000, the crew of the space shuttle *Endeavor* collected a trillion radar images of 46 million square miles of Earth.

Source: www.nima.mil

Example 5 Congruent Segments

Find x and y.



To find x:

$AB = BC$	Given
$3x - 4 = 6 - 2x$	Substitution
$5x - 4 = 6$	Add $2x$ to each side.
$5x = 10$	Add 4 to each side.
$x = 2$	Divide each side by 5.

To find y:

$\overline{DE} \cong \overline{EF}$	Parallel lines that cut off congruent segments on one transversal cut off congruent segments on every transversal.
$DE \cong EF$	Definition of congruent segments
$3y = \frac{5}{3}y + 1$	Substitution
$9y = 5y + 3$	Multiply each side by 3 to eliminate the denominator.
$4y = 3$	Subtract $5y$ from each side.
$y = \frac{3}{4}$	Divide each side by 4.

Study Tip

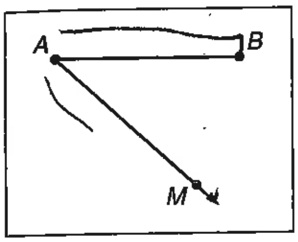
Locus The locus of points in a plane equidistant from two parallel lines is a line that lies between the lines and is parallel to them. In Example 5, $\overleftrightarrow{BE}$ is the locus of points in the plane equidistant from $\overleftrightarrow{AD}$ and $\overleftrightarrow{CF}$.

It is possible to separate a segment into two congruent parts by constructing the perpendicular bisector of a segment. However, a segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and the similarity theorems from this lesson.

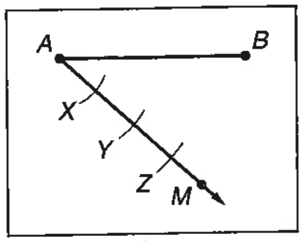
Construction

Trisect a Segment

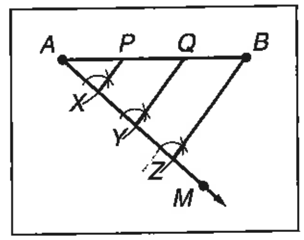
1. Draw $\overline{AB}$ to be trisected. Then draw $\overrightarrow{AM}$.



2. With the compass at A, mark off an arc that intersects $\overrightarrow{AM}$ at X. Use the same compass setting to construct $\overline{XY}$ and $\overline{YZ}$ congruent to $\overline{AX}$.



3. Draw $\overline{ZB}$. Then construct lines through Y and X that are parallel to $\overline{ZB}$. Label the intersection points on $\overline{AB}$ as P and Q.



Conclusion: Because parallel lines cut off congruent segments on transversals, $\overline{AP} \cong \overline{PQ} \cong \overline{QB}$.

Check for Understanding

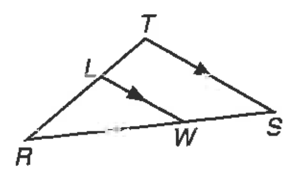
Concept Check

1. **Explain** how you would know if a line that intersects two sides of a triangle is parallel to the third side.
2. **OPEN ENDED** Draw two segments that are intersected by three lines so that the parts are proportional. Then draw a counterexample.
3. **Compare and contrast** Corollary 6.1 and Corollary 6.2.

Guided Practice

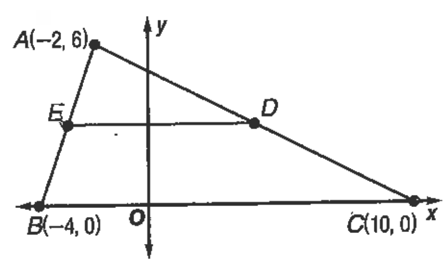
For Exercises 4 and 5, refer to $\triangle RST$.

4. If $RL = 5$, $RT = 9$, and $WS = 6$, find RW.
5. If $TR = 8$, $LR = 3$, and $RW = 6$, find WS.

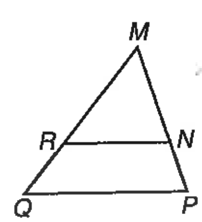


COORDINATE GEOMETRY For Exercises 6–8, use the following information.
Triangle ABC has vertices $A(-2, 6)$, $B(-4, 0)$, and $C(10, 0)$. $\overline{DE}$ is a midsegment.

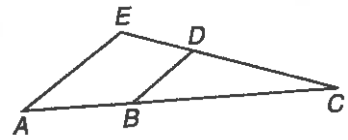
6. Find the coordinates of D and E.
7. Verify that $\overline{DE}$ is parallel to $\overline{BC}$.
8. Verify that $DE = \frac{1}{2}BC$.



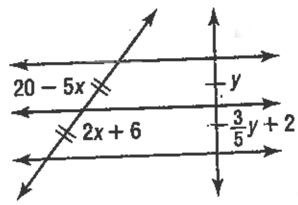
9. In $\triangle MQP$, $MP = 25$, $MN = 9$, $MR = 4.5$, and $MQ = 12.5$. Determine whether $\overline{RN} \parallel \overline{QP}$. Justify your answer.



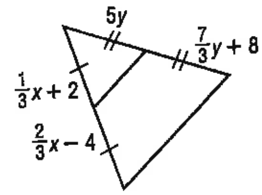
10. In $\triangle ACE$, $ED = 8$, $DC = 20$, $BC = 25$, and $AB = 12$. Determine whether $\overline{DB} \parallel \overline{AE}$.



11. Find x and y.

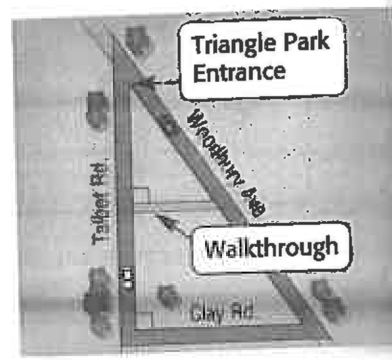


12. Find x and y.



Application

13. **MAPS** The distance along Talbot Road from the Triangle Park entrance to the Walkthrough is 880 yards. The distance along Talbot Road from the Walkthrough to Clay Road is 1408 yards. The distance along Woodbury Avenue from the Walkthrough to Clay Road is 1760 yards. If the Walkthrough is parallel to Clay Road, find the distance from the entrance to the Walkthrough along Woodbury.



Practice and Apply

Homework Help

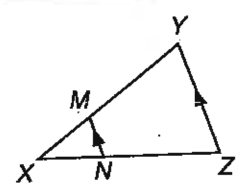
For Exercises	See Examples
14–19	1
20–26	2
27, 28	3
35–37, 43	4
33, 34	5

Extra Practice
See page 765.

For Exercises 14 and 15, refer to $\triangle XYZ$.

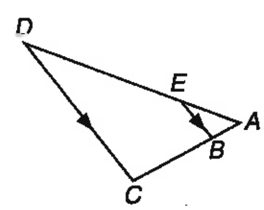
14. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY.

15. If $XN = t - 2$, $NZ = t + 1$, $XM = 2$, and $XY = 10$, solve for t.

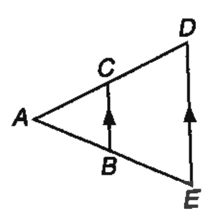


16. If $DB = 24$, $AE = 3$, and $EC = 18$, find AD.

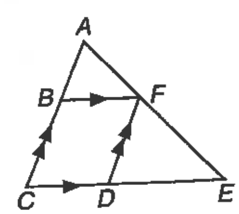
17. Find x and ED if $AE = 3$, $AB = 2$, $BC = 6$, and $ED = 2x - 3$.



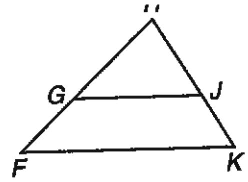
18. Find x, AC, and CD if $AC = x - 3$, $BE = 20$, $AB = 16$, and $CD = x + 5$.



19. Find BC, FE, CD, and DE if $AB = 6$, $AF = 8$, $BC = x$, $CD = y$, $DE = 2y - 3$, and $FE = x + \frac{10}{3}$.

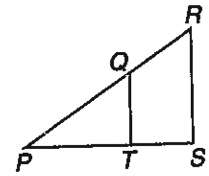


Find x so that $\overline{GJ} \parallel FK$.



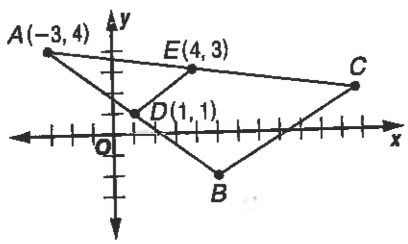
20. $GF = 12, HG = 6, HJ = 8, JK = x - 4$
21. $HJ = x - 5, JK = 15, FG = 18, HG = x - 4$
22. $GH = x + 3.5, HJ = x - 8.5, FH = 21, HK = 7$

Determine whether $\overline{QT} \parallel \overline{RS}$. Justify your answer.

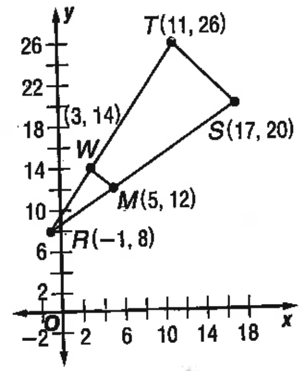


23. $PR = 30, PQ = 9, PT = 12$, and $PS = 18$
24. $QR = 22, RP = 65$, and SP is 3 times TS.
25. $TS = 8.6, PS = 12.9$, and PQ is half RQ.
26. $PQ = 34.88, RQ = 18.32, PS = 33.25$, and $TS = 11.45$

27. Find the length of $\overline{BC}$ if $\overline{BC} \parallel \overline{DE}$ and $\overline{DE}$ is a midsegment of $\triangle ABC$.

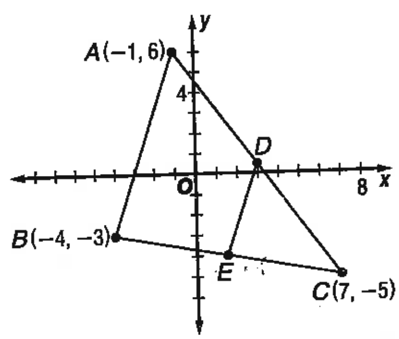


28. Show that $\overline{WM} \parallel \overline{TS}$ and determine whether $\overline{WM}$ is a midsegment.



COORDINATE GEOMETRY For Exercises 29 and 30, use the following information.

Triangle ABC has vertices $A(-1, 6), B(-4, -3)$, and $C(7, -5)$. $\overline{DE}$ is a midsegment.



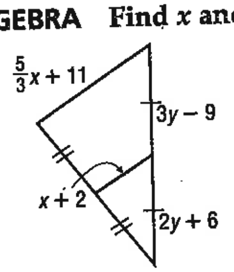
29. Verify that $\overline{DE}$ is parallel to $\overline{AB}$.
30. Verify that $DE = \frac{1}{2}AB$.

31. **COORDINATE GEOMETRY** Given $A(2, 12)$ and $B(5, 0)$, find the coordinates of P such that P separates $\overline{AB}$ into two parts with a ratio of 2 to 1.

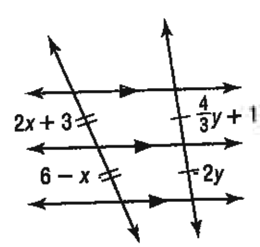
32. **COORDINATE GEOMETRY** In $\triangle LMN$, $\overline{PR}$ divides $\overline{NL}$ and $\overline{MN}$ proportionally. If the vertices are $N(8, 20), P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of L and M.

ALGEBRA Find x and y.

33.

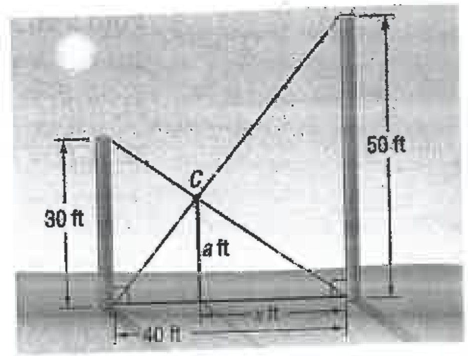


34.



CONSTRUCTION **For Exercises 35–37, use the following information and drawing.**

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



35. Find x, the distance from C to the taller pole.

36. How high above the ground is the coupling?

37. How far down the wire from the smaller pole is the coupling?

PROOF **Write a two-column proof of each theorem.**

38. Theorem 6.5

39. Theorem 6.6

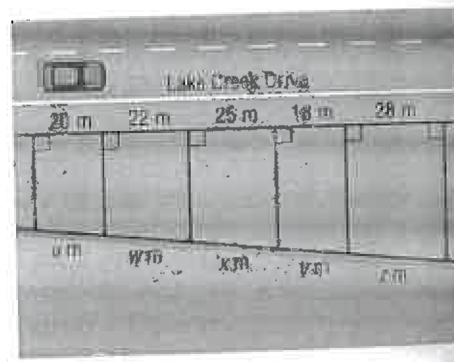
CONSTRUCTION **Construct each segment as directed.**

40. a segment 8 centimeters long, separated into three congruent segments

41. a segment separated into four congruent segments

42. a segment separated into two segments in which their lengths have a ratio of 1 to 4

43. **REAL ESTATE** In Lake Creek, the lots on which houses are to be built are laid out as shown. What is the lake frontage for each of the five lots if the total frontage is 135.6 meters?



44. **CRITICAL THINKING** Copy the figure that accompanies Corollary 6.1 on page 309. Draw $\overline{DC}$. Let G be the intersection point of $\overline{DC}$ and $\overline{BE}$. Using that segment, explain how you could prove $\frac{AB}{BC} = \frac{DE}{EF}$.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

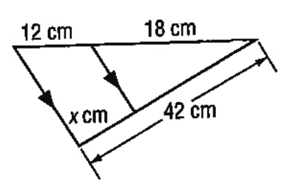
How do city planners use geometry?

Include the following in your answer:

- why maps are important to city planners, and
- what geometry facts a city planner needs to know to explain why the block between Chestnut and Pearson is longer on Lake Shore Drive than on Michigan Avenue.

Standardized Test Practice

46. Find x.



Ⓐ 16 Ⓑ 16.8
Ⓒ 24 Ⓓ 28.4

47. **GRID IN** The average of a and b is 18, and the ratio of a to b is 5 to 4. What is the value of $a - b$?

Extending the Lesson

48. **MIDPOINTS IN POLYGONS** **Draw any quadrilateral *ABCD* on a coordinate plane. Points *E, F, G,* and *H* are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively.**

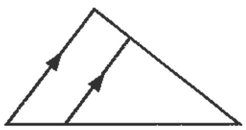
a. Connect the midpoints to form quadrilateral *EFGH*. Describe what you know about the sides of quadrilateral *EFGH*.

b. Will the same reasoning work with five-sided polygons? Explain why or why not.

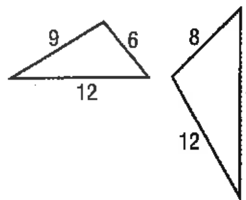
tain Your Skills

lixed Review **Determine whether each pair of triangles is similar. Justify your answer.** *(Lesson 6-3)*

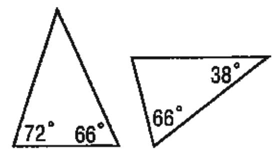
49.



50.

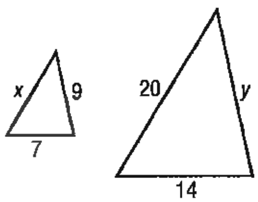


51.

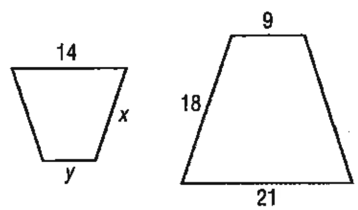


Each pair of polygons is similar. Find *x* and *y*. *(Lesson 6-2)*

52.



53.



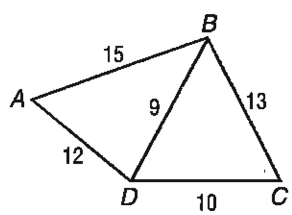
Determine the relationship between the measures of the given angles. *(Lesson 5-2)*

54. $\angle ADB$, $\angle ABD$

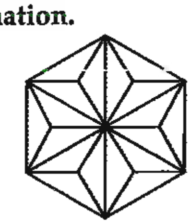
55. $\angle ABD$, $\angle BAD$

56. $\angle BCD$, $\angle CDB$

57. $\angle CBD$, $\angle BCD$



ARCHITECTURE **For Exercises 58 and 59, use the following information.** The geodesic dome was developed by Buckminster Fuller in the 1940s as an energy-efficient building. The figure at the right shows the basic structure of one geodesic dome. *(Lesson 4-1)*



58. How many equilateral triangles are in the figure?

59. How many obtuse triangles are in the figure?

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. *(Lesson 2-3)*

60. You do not have a fever, and you are sick.

61. You have a fever, and you are not sick.

62. You do not have a fever, and you are not sick.

63. You have a fever, and you are sick.

g Ready for Next Lesson **PREREQUISITE SKILL** **Write all the pairs of corresponding parts for each pair of congruent triangles.** *(To review **corresponding congruent parts**, see Lesson 4-3.)*

64. $\triangle ABC \cong \triangle DEF$ 65. $\triangle RST \cong \triangle XYZ$ 66. $\triangle PQR \cong \triangle KLM$

6-5 Parts of Similar Triangles

What You'll Learn

- Recognize and use proportional relationships of corresponding perimeters of similar triangles.
- Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

How is geometry related to photography?

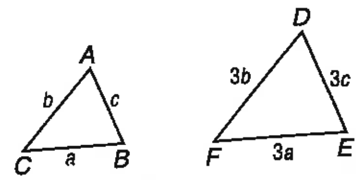
The camera lens was 6.16 meters from this Dale Chihuly glass sculpture when this photograph was taken. The image on the film is 35 millimeters tall. Similar triangles enable us to find the height of the actual sculpture.

More About. . .

Dale Chihuly

Dale Chihuly (1941–), born in Tacoma, Washington, is widely recognized as one of the greatest glass artists in the world. His sculptures are made of hundreds of pieces of hand-blown glass that are assembled to resemble patterns in nature.

PERIMETERS Triangle ABC is similar to $\triangle DEF$ with a scale factor of 1:3. You can use variables and the scale factor to compare their perimeters. Let the measures of the sides of $\triangle ABC$ be a, b, and c. The measures of the corresponding sides of $\triangle DEF$ would be $3a$, $3b$, and $3c$.



$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{a+b+c}{3a+3b+3c}$$
$$= \frac{1(a+b+c)}{3(a+b+c)} \text{ or } \frac{1}{3}$$

The perimeters are in the same proportion as the side measures of the two similar figures. This suggests Theorem 6.7, the Proportional Perimeters Theorem.

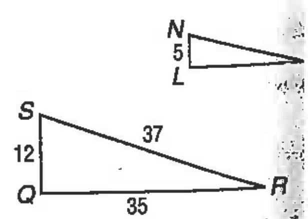
Theorem 6.7

Proportional Perimeters Theorem If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

You will prove Theorem 6.7 in Exercise 8.

Example 1 Perimeters of Similar Triangles

If $\triangle LMN \sim \triangle QRS$, $QR = 35$, $RS = 37$, $SQ = 12$, and $NL = 5$, find the perimeter of $\triangle LMN$.



Let x represent the perimeter of $\triangle LMN$. The perimeter of $\triangle QRS = 35 + 37 + 12$ or 84.

$\frac{NL}{SQ} = \frac{\text{perimeter of } \triangle LMN}{\text{perimeter of } \triangle QRS}$	Proportional Perimeter Theorem
$\frac{5}{12} = \frac{x}{84}$	Substitution
$12x = 420$	Cross products
$x = 35$	Divide each side by 12.

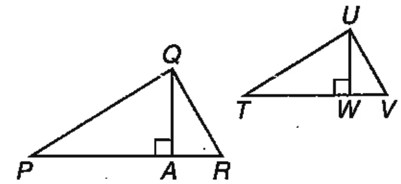
The perimeter of $\triangle LMN$ is 35 units.

SPECIAL SEGMENTS OF SIMILAR TRIANGLES Think about a triangle drawn on a piece of paper being placed on a copy machine and either enlarged or reduced. The copy is similar to the original triangle. Now suppose you drew in special segments of a triangle, such as the altitudes, medians, or angle bisectors, on the original. When you enlarge or reduce that original triangle, all of those segments are enlarged or reduced at the same rate. This conjecture is formally stated in Theorems 6.8, 6.9, and 6.10.

Theorems — *Special Segments of Similar Triangles*

6.8 If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

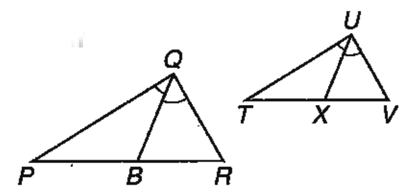
Abbreviation: ~ $\triangle$s *have corr. altitudes proportional to the corr. sides.*



$$\frac{QA}{UW} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

6.9 If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

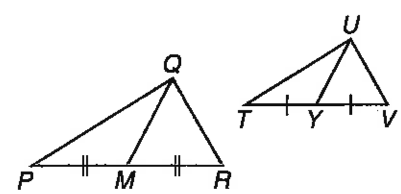
Abbreviation: ~ $\triangle$s *have corr. $\angle$ bisectors proportional to the corr. sides.*



$$\frac{QB}{UX} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

6.10 If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Abbreviation: ~ $\triangle$s *have corr. medians proportional to the corr. sides.*



$$\frac{QM}{UY} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

You will prove Theorems 6.8 and 6.10 in Exercises 30 and 31, respectively.

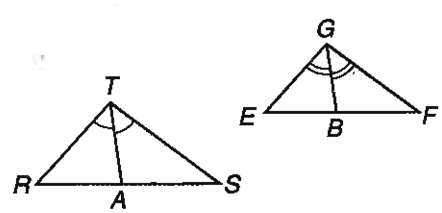
Example 2 Write a Proof

Write a paragraph proof of Theorem 6.9.

Since the corresponding angles to be bisected are chosen at random, we need not prove this for every pair of bisectors.

Given: $\triangle RTS \sim \triangle EGF$
$\overline{TA}$ and $\overline{GB}$ are angle bisectors.

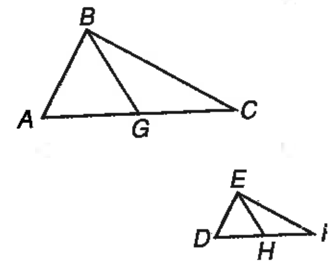
Prove: $\frac{TA}{GB} = \frac{RT}{EG}$



Paragraph Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\angle RTA \cong \angle EGB$ and $\triangle RTA \sim \triangle EGB$ by AA Similarity. Thus, $\frac{TA}{GB} = \frac{RT}{EG}$.

Example 3 Medians of Similar Triangles

In the figure, $\triangle ABC \sim \triangle DEF$. $\overline{BG}$ is a median of $\triangle ABC$, and $\overline{EH}$ is a median of $\triangle DEF$. Find EH if $BC = 30$, $BG = 15$, and $EF = 15$.



Let x represent EH.

$\frac{BG}{EH} = \frac{BC}{EF}$ **Write a proportion.**

$\frac{15}{x} = \frac{30}{15}$ **$BG = 15$, $EH = x$, $BC = 30$, and $EF = 15$**

$30x = 225$ **Cross products**

$x = 7.5$ **Divide each side by 30.**

Thus, $EH = 7.5$.

More About. . .

Photography

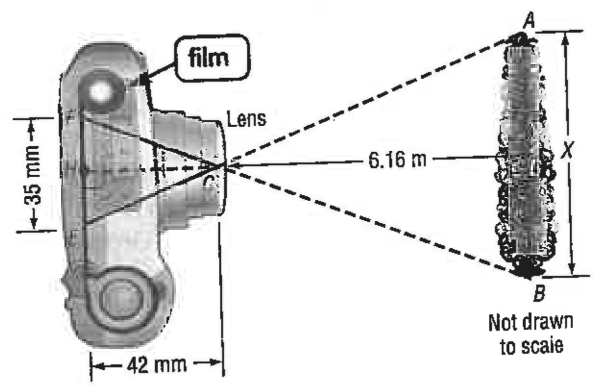
The first consumer-oriented digital cameras were produced for sale in 1994 with a 640 × 480 pixel resolution. In 2002, a 3.3-megapixel camera could take a picture with 2048 × 1536 pixel resolution, which is a sharper picture than most computer monitors can display.

Source: www.howstuffworks.com

The theorems about the relationships of special segments in similar triangles be used to solve real-life problems.

Example 4 Solve Problems with Similar Triangles

PHOTOGRAPHY **Refer to the application at the beginning of the lesson. Th drawing below illustrates the position of the camera and the distance from lens of the camera to the film. Find the height of the sculpture.**



$\triangle ABC$ and $\triangle EFC$ are similar. The distance from the lens to the film in the cam is $CH = 42$ mm. $\overline{CG}$ and $\overline{CH}$ are altitudes of $\triangle ABC$ and $\triangle EFC$, respectively. I two triangles are similar, then the measures of the corresponding altitudes a proportional to the measures of the corresponding sides. This leads to the proportion $\frac{AB}{EF} = \frac{GC}{HC}$.

$\frac{AB}{EF} = \frac{GC}{HC}$ **Write the proportion.**

$\frac{x \text{ m}}{35 \text{ mm}} = \frac{6.16 \text{ m}}{42 \text{ mm}}$ **$AB = x$ m, $EF = 35$ m, $GC = 6.16$ m, $HC = 42$ mm**

$x \cdot 42 = 35(6.16)$ **Cross products**

$42x = 215.6$ **Simplify.**

$x \approx 5.13$ **Divide each side by 42.**

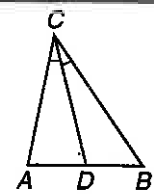
The sculpture is about 5.13 meters tall.

An angle bisector also divides the side of the triangle opposite the angle proportionally.

Theorem 6.11

Angle Bisector Theorem An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

Example: $\frac{AD}{DB} = \frac{AC}{BC}$ ← segments with vertex A ← segments with vertex B

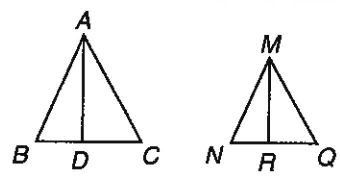


You will prove this theorem in Exercise 32.

k for Understanding

ncept Check

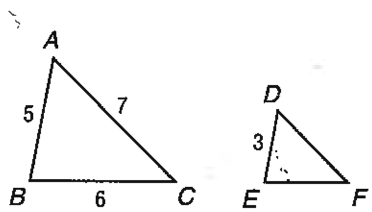
1. **Explain** what must be true about $\triangle ABC$ and $\triangle MNQ$ before you can conclude that $\frac{AD}{MR} = \frac{BA}{NM}$.



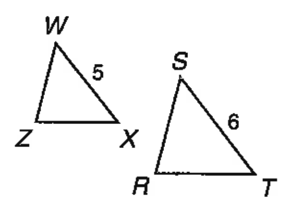
2. **OPEN ENDED** The perimeter of one triangle is 24 centimeters, and the perimeter of a second triangle is 36 centimeters. If the length of one side of the smaller triangle is 6, find possible lengths of the other sides of the triangles so that they are similar.

ed Practice **Find the perimeter of the given triangle.**

3. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and $DE = 3$

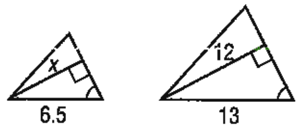


4. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\triangle SRT = 15$

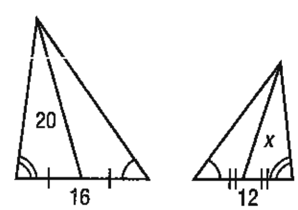


Find x.

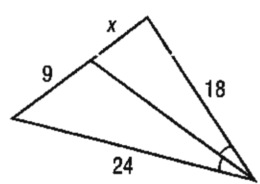
5.



6.



7.

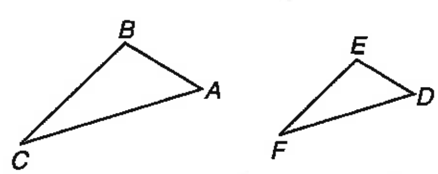


8. **PROOF** Write a paragraph proof of Theorem 6.7.

Given: $\triangle ABC \sim \triangle DEF$

$\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



pplication

9. **PHOTOGRAPHY** The distance from the film to the lens in a camera is 10 centimeters. The film image is 5 centimeters high. Tamika is 165 centimeters tall. How far should she be from the camera in order for the photographer to take a full-length picture?

Practice and Apply

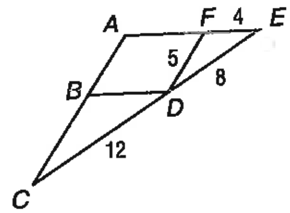
Homework Help

For Exercises	See Examples
10–15	1
16, 17, 28	4
18–27	3
30–37	2

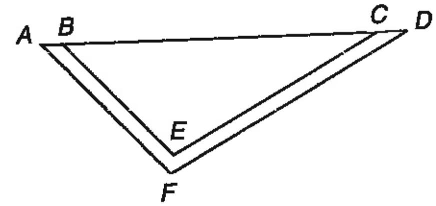
Extra Practice
See page 766.

Find the perimeter of the given triangle.

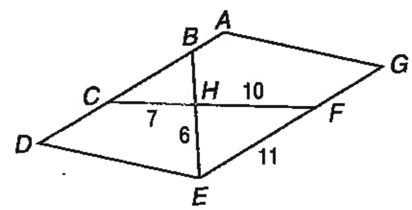
10. $\triangle BCD$, if $\triangle BCD \sim \triangle FDE$, $CD = 12$, $FD = 5$, $FE = 4$, and $DE = 8$



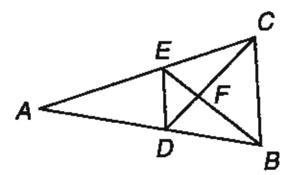
11. $\triangle ADF$, if $\triangle ADF \sim \triangle BCE$, $BC = 24$, $EB = 12$, $CE = 18$, and $DF = 21$



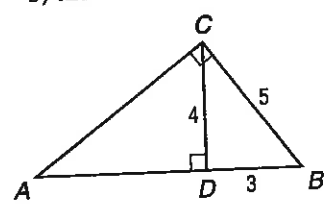
12. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$, $ADEG$ is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$



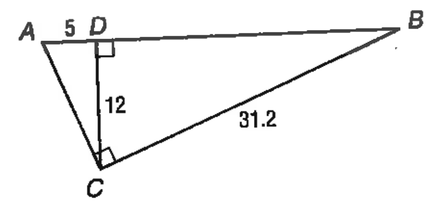
13. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$, perimeter of $\triangle CBF = 27$, $DF = 6$, and $FC = 8$



14. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $CD = 4$, $DB = 3$, and $CB = 5$

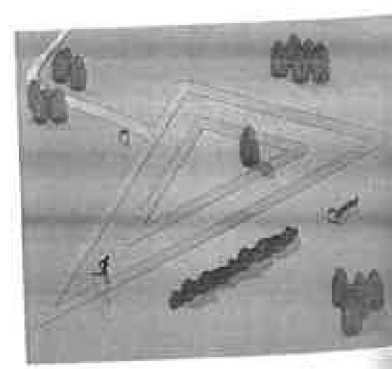


15. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $AD = 5$, $CD = 12$, and $BC = 31.2$

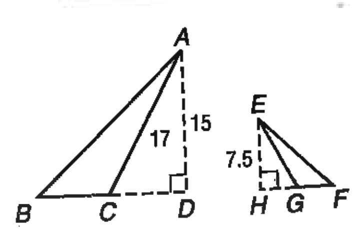


16. **DESIGN** Rosario wants to enlarge the dimensions of an 18-centimeter by 24-centimeter picture by 30%. She plans to line the inside edge of the frame with blue cord. The store only had 110 centimeters of blue cord in stock. Will this be enough to fit on the inside edge of the frame? Explain.

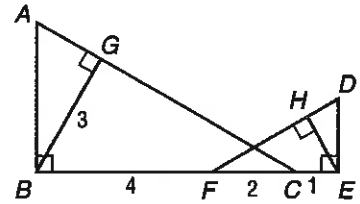
17. **PHYSICAL FITNESS** A park has two similar triangular jogging paths as shown. The dimensions of the inner path are 300 meters, 350 meters, and 550 meters. The shortest side of the outer path is 600 meters. Will a jogger on the inner path run half as far as one on the outer path? Explain.



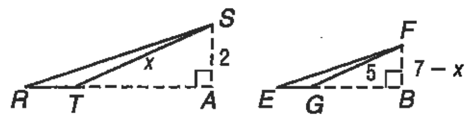
18. Find EG if $\triangle ACB \sim \triangle EGF$, $\overline{AD}$ is an altitude of $\triangle ACB$, $\overline{EH}$ is an altitude of $\triangle EGF$, $AC = 17$, $AD = 15$, and $EH = 7.5$.



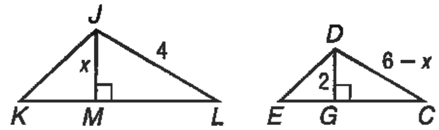
19. Find EH if $\triangle ABC \sim \triangle DEF$, $\overline{BG}$ is an altitude of $\triangle ABC$, $\overline{EH}$ is an altitude of $\triangle DEF$, $BG = 3$, $BF = 4$, $FC = 2$, and $CE = 1$.



20. Find FB if $\overline{SA}$ and $\overline{FB}$ are altitudes and $\triangle RST \sim \triangle EFG$.

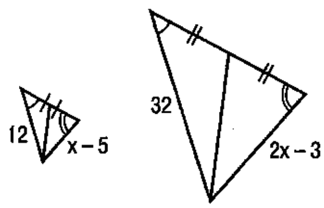


21. Find DC if $\overline{DG}$ and $\overline{JM}$ are altitudes and $\triangle KJL \sim \triangle EDC$.



Find *x*.

22.



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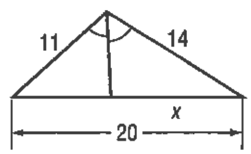

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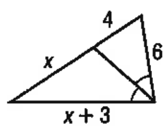

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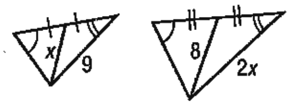
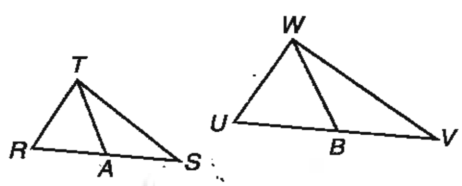
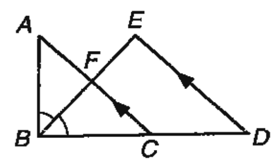


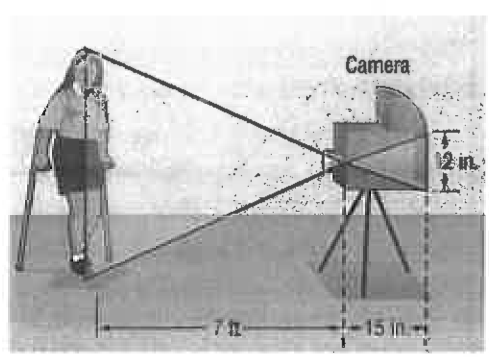
26. Find UB if $\triangle RST \sim \triangle UVW$, $\overline{TA}$ and $\overline{WB}$ are medians, $TA = 8$, $RA = 3$, $WB = 3x - 6$, and $UB = x + 2$.



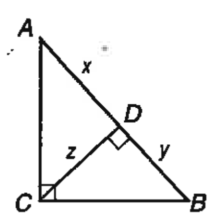
27. Find CF and BD if $\overline{BF}$ bisects $\angle ABC$ and $\overline{AC} \parallel \overline{ED}$, $BA = 6$, $BC = 7.5$, $AC = 9$, and $DE = 9$.



28. **PHOTOGRAPHY** One of the first cameras invented was called a *camera obscura.* Light entered an opening in the front, and an image was reflected in the back of the camera, upside down, forming similar triangles. If the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long, how tall is the person being photographed?



29. **CRITICAL THINKING** $\overline{CD}$ is an altitude to the hypotenuse $\overline{AB}$. Make a conjecture about x, y, and z. Justify your reasoning.



PROOF **Write the indicated type of proof.**

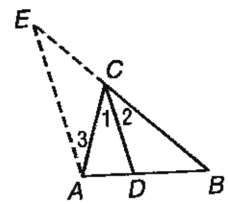
30. a paragraph proof of Theorem 6.8

31. a two-column proof of Theorem 6.10

32. a two-column proof of the Angle Bisector Theorem (Theorem 6.11)

Given: $\overline{CD}$ bisects $\angle ACB$
By construction, $\overline{AE} \parallel \overline{CD}$.

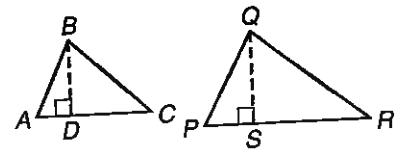
Prove: $\frac{AD}{AC} = \frac{BD}{BC}$



33. a paragraph proof

Given: $\triangle ABC \sim \triangle PQR$
$\overline{BD}$ is an altitude of $\triangle ABC$.
$\overline{QS}$ is an altitude of $\triangle PQR$.

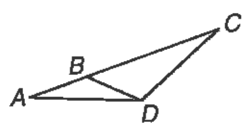
Prove: $\frac{QP}{BA} = \frac{QS}{BD}$



34. a flow proof

Given: $\angle C \cong \angle BDA$

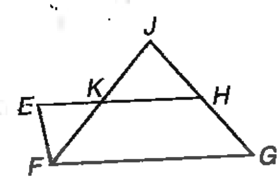
Prove: $\frac{AC}{DA} = \frac{AD}{BA}$



35. a two-column proof

Given: $\overline{JF}$ bisects $\angle EFG$.
$\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

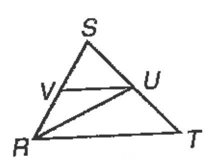
Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



36. a two-column proof

Given: $\overline{RU}$ bisects $\angle SRT$;
$\overline{VU} \parallel \overline{RT}$.

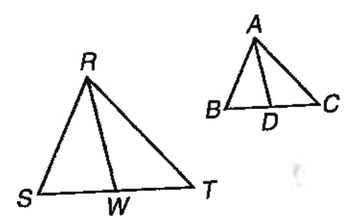
Prove: $\frac{SV}{VR} = \frac{SR}{RT}$



37. a flow proof

Given: $\triangle RST \sim \triangle ABC$; W and D are midpoints of $\overline{TS}$ and $\overline{CB}$.

Prove: $\triangle RWS \sim \triangle ADB$



38. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is geometry related to photography?

Include the following in your answer:

- a sketch of how a camera works showing the image and the film, and
- why the two isosceles triangles are similar.

Standardized Test Practice

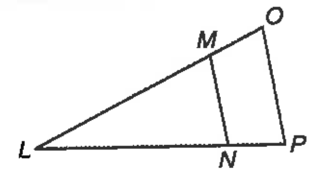
39. GRID IN Triangle ABC is similar to $\triangle DEF$. If $AC = 10.5$, $AB = 6.5$, and $DE = 8$, find DF.

40. ALGEBRA The sum of three numbers is 180. Two of the numbers are the same, and each of them is one-third of the greatest number. What is the least number?

Ⓐ 30 Ⓑ 36 Ⓒ 45 Ⓓ 72

Maintain Your Skills

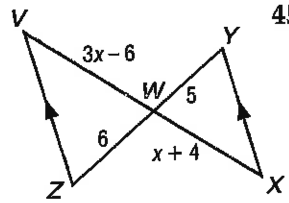
Mixed Review Determine whether $\overline{MN} \parallel \overline{OP}$. Justify your answer. *(Lesson 6-4)*



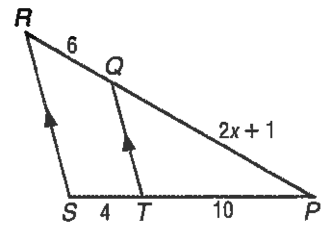
41. $LM = 7, LN = 9, LO = 14, LP = 16$
42. $LM = 6, MN = 4, LO = 9, OP = 6$
43. $LN = 12, NP = 4, LM = 15, MO = 5$

Identify the similar triangles. Find x and the measure(s) of the indicated side(s). *(Lesson 6-3)*

44. VW and WX



45. PQ



Write an equation in slope-intercept form for the line that satisfies the given conditions. *(Lesson 3-4)*

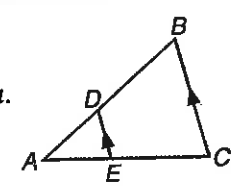
46. x-intercept is 3, y-intercept is -3
47. $m = 2$, contains $(-1, -1)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Name the next two numbers in each pattern.
*(To review **patterns**, see Lesson 2-1.)*

48. 5, 12, 19, 26, 33, …
49. 10, 20, 40, 80, 160, …
50. 0, 5, 4, 9, 8, 13, …

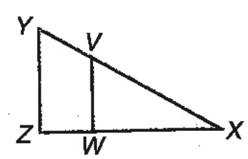
Practice Quiz 2 — *Lessons 6-4 and 6-5*

Refer to $\triangle ABC$. *(Lesson 6-4)*



1. If $AD = 8$, $AE = 12$, and $EC = 18$, find AB.
2. If $AE = m - 2$, $EC = m + 4$, $AD = 4$, and $AB = 20$, find m.

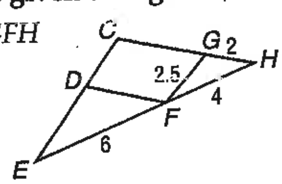
Determine whether $\overline{YZ} \parallel \overline{VW}$. Justify your answer. *(Lesson 6-4)*



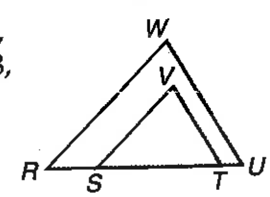
3. $XY = 30$, $XV = 9$, $XW = 12$, and $XZ = 18$
4. $XV = 34.88$, $VY = 18.32$, $XZ = 33.25$, and $WZ = 11.45$

Find the perimeter of the given triangle. *(Lesson 6-5)*

5. $\triangle DEF$ if $\triangle DEF \sim \triangle GFH$

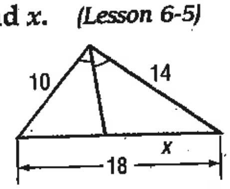


6. $\triangle RUW$ if $\triangle RUW \sim \triangle STV$, $ST = 24$, $VS = 12$, $VT = 18$, and $UW = 21$

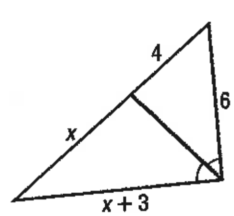


Find x. *(Lesson 6-5)*

7.



8.



9.

10. **LANDSCAPING** Paulo is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. He wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. *(Lesson 6-5)*

Geometry Activity

A Preview of Lesson 6-6

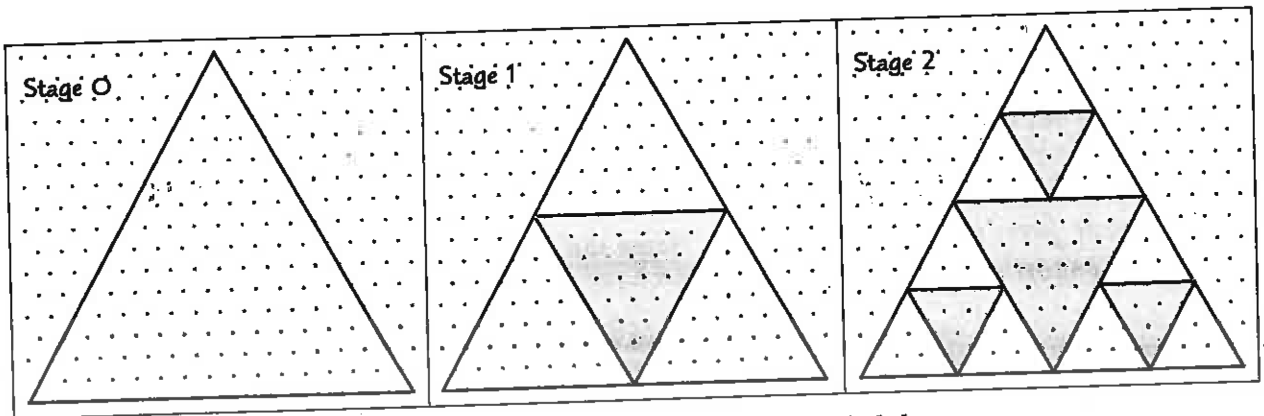
Sierpinski Triangle

Collect Data

Stage 0 On isometric dot paper, draw an equilateral triangle in which each side is 16 units long.

Stage 1 Connect the midpoints of each side to form another triangle. Shade the center triangle.

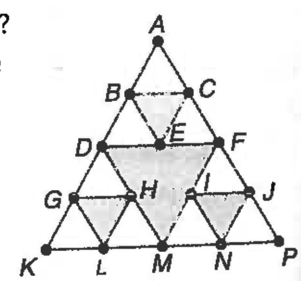
Stage 2 Repeat the process using the three nonshaded triangles. Connect the midpoints of each side to form other triangles.



If you repeat this process indefinitely, the pattern that results is called the **Sierpinski Triangle.** Since this figure is created by repeating the same procedure over and over again, it is an example of a geometric shape called a *fractal*.

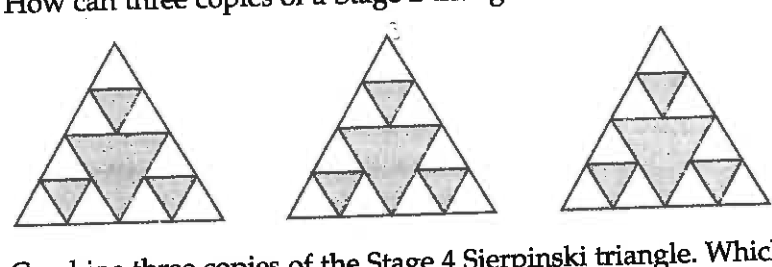
Analyze the Data

1. Continue the process through Stage 4. How many nonshaded triangles do you have at Stage 4?
2. What is the perimeter of a nonshaded triangle in Stage 0 through Stage 4?
3. If you continue the process indefinitely, describe what will happen to the perimeter of each nonshaded triangle.
4. Study $\triangle DFM$ in Stage 2 of the Sierpenski Triangle shown at the right. Is this an equilateral triangle? Are $\triangle BCE$, $\triangle GHL$, or $\triangle IJN$ equilateral?
5. Is $\triangle BCE \sim \triangle DFM$? Explain your answer.
6. How many Stage 1 Sierpinski triangles are there in Stage 2?



Make a Conjecture

7. How can three copies of a Stage 2 triangle be combined to form a Stage 3 triangle?



8. Combine three copies of the Stage 4 Sierpinski triangle. Which stage of the Sierpinski Triangle is this?
9. How many copies of the Stage 4 triangle would you need to make a Stage 6 triangle?

6-6 Fractals and Self-Similarity

What You'll Learn

- Recognize and describe characteristics of fractals.
- Identify nongeometric iteration.

How is mathematics found in nature?

Patterns can be found in many objects in nature, including broccoli. If you take a piece of broccoli off the stalk, the small piece resembles the whole. This pattern of repeated shapes at different scales is part of fractal geometry.

Vocabulary
- iteration
- fractal
- self-similar

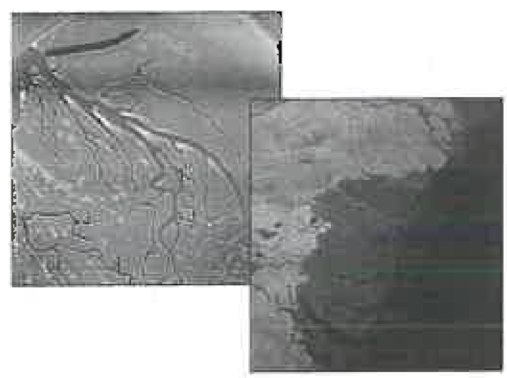
CHARACTERISTICS OF FRACTALS

Benoit Mandelbrot, a mathematician, coined the term *fractal* to describe things in nature that are irregular in shape, such as clouds, coastlines, or the growth of a tree. The patterns found in nature are analyzed and then recreated on a computer, where they can be studied more closely. These patterns are created using a process called **iteration**. Iteration is a process of repeating the same procedure over and over again. A **fractal** is a geometric figure that is created using iteration. The pattern's structure appears to go on infinitely.

WebQuest

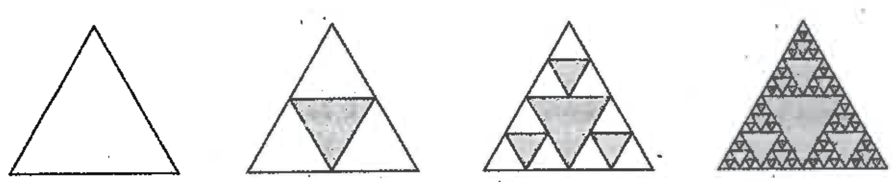
By creating a Sierpinski Triangle, you can find a pattern in the area and perimeter of this well-known fractal. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

Compare the pictures of a human circulatory system and the mouth of the Ganges in Bangladesh. Notice how the branches of the tributaries have the same pattern as the branching of the blood vessels.



One characteristic of fractals is that they are **self similar**. That is, the smaller and smaller details of a shape have the same geometric characteristics as the original form.

The Sierpinski Triangle is a fractal that is self-similar. Stage 1 is formed by drawing the midsegments of an equilateral triangle and shading in the triangle formed by them. Stage 2 repeats the process in the unshaded triangles. This process can continue indefinitely with each part still being similar to the original.

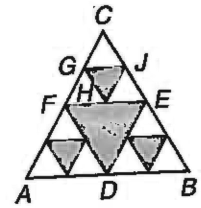


The Sierpinski Triangle is said to be *strictly self-similar*, which means that any of its parts, no matter where they are located or what size is selected, contain a figure that is similar to the whole.

Example 1 Self-Similarity

Prove that a triangle formed in Stage 2 of a Sierpinski triangle is similar to the triangle in Stage 0.

The argument will be the same for any triangle in Stage 2, so we will use only $\triangle CGJ$ from Stage 2.



Given: $\triangle ABC$ is equilateral.
D, E, F, G, J, and H are midpoints of $\overline{AB}, \overline{BC}, \overline{CA}, \overline{FC}, \overline{CE}$, and $\overline{FE}$, respectively.

Prove: $\triangle CGJ \sim \triangle CAB$

Statements	Reasons
1. $\triangle ABC$ is equilateral; D, E, F are midpoints of $\overline{AB}, \overline{BC}, \overline{CA}$; G, J, and H are midpoints of $\overline{FC}$, $\overline{CE}$, $\overline{FE}$.	1. Given
2. $\overline{FE}$ is a midsegment of $\triangle CAB$; $\overline{GJ}$ is a midsegment of $\triangle CFE$.	2. Definition of a Triangle Midsegment
3. $\overline{FE} \parallel \overline{AB}$; $\overline{GJ} \parallel \overline{FE}$	3. Triangle Midsegment Theorem
4. $\overline{GJ} \parallel \overline{AB}$	4. Two segments parallel to the same segment are parallel.
5. $\angle CGJ \cong \angle CAB$	5. Corresponding $\angle$s Postulate
6. $\angle C \cong \angle C$	6. Reflexive Property
7. $\triangle CGJ \sim \triangle CAB$	7. AA Similarity

Thus, using the same reasoning, every triangle in Stage 2 is similar to the original triangle in Stage 0.

Study Tip

Look Back
To review **midsegment**, see Lesson 6-4.

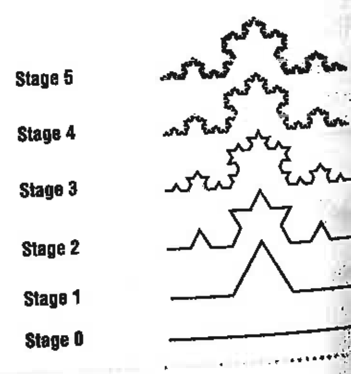
You can generate many other fractal images using an iterative process.

Example 2 Create a Fractal

Draw a segment and trisect it. Create a fractal by replacing the middle third of the segment with two segments of the same length as the removed segment.

After the first geometric iteration, repeat the process on each of the four segments in Stage 1. Continue to repeat the process.

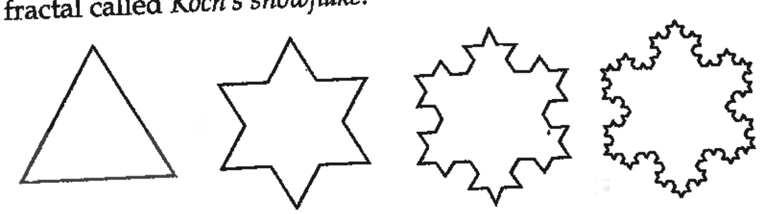
This fractal image is called a *Koch curve*.



Study Tip

Common Misconceptions
Not all repeated patterns are self-similar. The Koch curve is one such example when applied to a triangle.

If the first stage is an equilateral triangle, instead of a segment, this iteration will produce a fractal called *Koch's snowflake*.



NONGEOMETRIC ITERATION An iterative process does not always include manipulation of geometric shapes. Iterative processes can be translated into formulas or algebraic equations. These are called *recursive formulas.*

Study Tip

Recursion on the Graphing Calculator

To do recursion on a graphing calculator, store 2 as the value for X and press [ENTER]. Then enter $X^2 \to X$. Press [ENTER] for each iteration.

Example 3 Evaluate a Recursive Formula

Find the value of x^2, where x initially equals 2. Then use that value as the next x in the expression. Repeat the process four times and describe your observations.

The iterative process is to square the value repeatedly. Begin with $x = 2$. The value of x^2 becomes the next value for x.

x	2	4	16	256	65,536
x^2	4	16	256	65,536	4,294,967,296

The values grow greater with each iteration, approaching infinity.

Example 4 Find a Recursive Formula

PASCAL'S TRIANGLE ***Pascal's Triangle*** **is a numerical pattern in which each row begins and ends with 1 and all other terms in the row are the sum of the two numbers above it.**

a. Find a formula in terms of the row number for the sum of the values in any row in the Pascal's triangle.

To find the sum of the values in the tenth row, we can investigate a simpler problem. What is the sum of values in the first four rows of the triangle?

Row	Pascal's Triangle	Sum	Pattern
1	1	1	$2^0 = 2^{1-1}$
2	1 1	2	$2^1 = 2^{2-1}$
3	1 2 1	4	$2^2 = 2^{3-1}$
4	1 3 3 1	8	$2^3 = 2^{4-1}$
5	1 4 6 4 1	16	$2^4 = 2^{5-1}$

It appears that the sum of any row is a power of 2. The formula is 2 to a power that is one less than the row number: $A_n = 2^{n-1}$.

b. What is the sum of the values in the tenth row of Pascal's triangle?

The sum of the values in the tenth row will be 2^{10-1} or 512.

Example 5 Solve a Problem Using Iteration

BANKING Felisa has \$2500 in a money market account that earns 3.2% interest. If the interest is compounded annually, find the balance of her account after 3 years.

First, write an equation to find the balance after one year.

current balance + (current balance × interest rate) = new balance

$$2500 + (2500 \cdot 0.032) = 2580$$

$$2580 + (2580 \cdot 0.032) = 2662.56$$

$$2662.56 + (2662.56 \cdot 0.032) = 2747.76$$

After 3 years, Felisa will have \$2747.76 in her account.

www.geometryonline.com/extra_examples

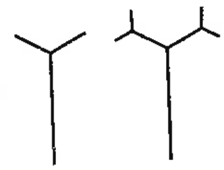
Check for Understanding

Concept Check

1. **Describe** a *fractal* in your own words. Include characteristics of fractals in your answer.
2. **Explain** why computers provide an efficient way to generate fractals.
3. **OPEN ENDED** Find an example of fractal geometry in nature, excluding those mentioned in the lesson.

Guided Practice

For Exercises 4–6, use the following information.
A *fractal tree* can be drawn by making two new branches from the endpoint of each original branch, each one-third as long as the previous branch.



Stage 1 Stage 2

4. Draw Stages 3 and 4 of a fractal tree. How many total branches do you have in Stages 1 through 4? (Do not count the stems.)
5. Find a pattern to predict the number of branches at each stage.
6. Is a fractal tree strictly self-similar? Explain.

For Exercises 7–9, use a calculator.

7. Find the square root of 2. Then find the square root of the result.
8. Find the square root of the result in Exercise 7. What would be the result after 100 repeats of taking the square root?
9. Determine whether this is an iterative process. Explain.

Application

10. **BANKING** Jamir has $4000 in a savings account. The annual percent interest rate is 1.1%. Find the amount of money Jamir will have after the interest is compounded four times.

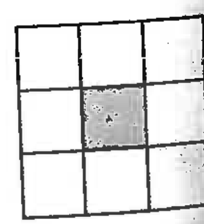
Practice and Apply

Homework Help

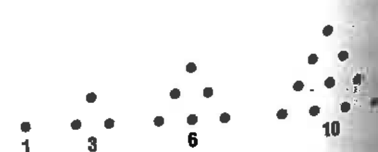
For Exercises	See Examples
11–15, 21–23	1, 2
14–20, 25, 28	4
24–29	2
30–37	3
38–40	5

Extra Practice
See page 766.

For Exercises 11–13, Stage 1 of a fractal is drawn on grid paper so that each side of the large square is 27 units long. The trisection points of the sides are connected to make 9 smaller squares with the middle square shaded. The shaded square is known as a *hole*.



11. Copy Stage 1 on your paper. Then draw Stage 2 by repeating the Stage 1 process in each of the outer eight squares. How many holes are in this stage?
12. Draw Stage 3 by repeating the Stage 1 process in each unshaded square of Stage 2. How many holes are in Stage 3?
13. If you continue the process indefinitely, will the figure you obtain be strictly self-similar? Explain.
14. Count the number of dots in each arrangement. These numbers are called *triangular numbers*. The second triangular number is 3 because there are three dots in the array. How many dots will be in the seventh triangular number?



For Exercises 15–20, refer to Pascal's triangle on page 327. Look at the third diagonal from either side, starting at the top of the triangle.**

15. Describe the pattern.
16. Explain how Pascal's triangle relates to the triangular numbers.
17. Generate eight rows of Pascal's triangle. Replace each of the even numbers with a 0 and each of the odd numbers with a 1. Color each 1 and leave the 0s uncolored. Describe the picture.
18. Generate eight rows of Pascal's triangle. Divide each entry by 3. If the remainder is 1 or 2, shade the number cell black. If the remainder is 0, leave the cell unshaded. Describe the pattern that emerges.
19. Find the sum of the first 25 numbers in the outside diagonal of Pascal's triangle.
20. Find the sum of the first 50 numbers in the second diagonal.

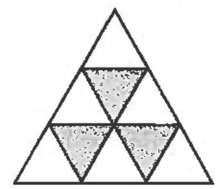
More About...

Blaise Pascal

Pascal (1623–1662) did not discover Pascal's triangle, but it was named after him in honor of his work in 1653 called *Treatise on the Arithmetical Triangle*. The patterns in the triangle are also used in probability.

Source: *Great Moments in Mathematics/After 1650*

The three shaded interior triangles shown were made by trisecting the three sides of an equilateral triangle and connecting the points.



21. Prove that one of the nonshaded triangles is similar to the original triangle.
22. Repeat the iteration once more.
23. Is the new figure strictly self-similar?
24. How many nonshaded triangles are in Stages 1 and 2?

Refer to the Koch Curve on page 326.

25. What is a formula for the number of segments in terms of the stage number? Use your formula to predict the number of segments in Stage 8 of a Koch curve.
26. If the length of the original segment is 1 unit, how long will the segments be in each of the first four stages? What will happen to the length of each segment as the number of stages continues to increase?

Refer to the Koch Snowflake on page 326. At Stage 1, the length of each side is 1 unit.

27. What is the perimeter at each of the first four stages of a Koch snowflake?
28. What is a formula for the perimeter in terms of the stage number? Describe the perimeter as the number of stages continues to increase.
29. Write a paragraph proof to show that the triangles generated on the sides of a Koch Snowflake in Stage 1, are similar to the original triangle.

Find the value of each expression. Then use that value as the next x in the expression. Repeat the process four times, and describe your observations.

30. $\sqrt{x}$, where x initially equals 12
31. $\frac{1}{x}$, where x initially equals 5
32. $x^{\frac{1}{3}}$, where x initially equals 0.3
33. 2^x, where x initially equals 0

Find the first three iterates of each expression.

34. $2x + 1$, x initially equals 1
35. $x - 5$, where x initially equals 5
36. $x^2 - 1$, x initially equals 2
37. $3(2 - x)$, where x initially equals 4

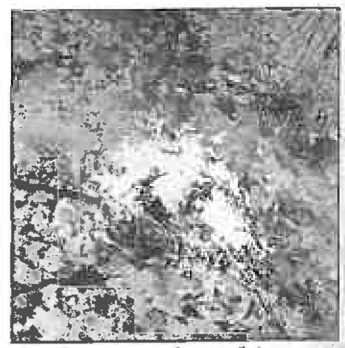
38. **BANKING** Raini has a credit card balance of $1250 and a monthly interest rate of 1.5%. If he makes payments of $100 each month, what will the balance be after 3 months?

WEATHER **For Exercises 39 and 40, use the following information.**
There are so many factors that affect the weather that it is difficult for meteorologists to make accurate long term predictions. Edward N. Lorenz called this dependence *the Butterfly Effect* and posed the question "Can the flap of a butterfly's wings in Brazil cause a tornado in Texas?"

39. Use a calculator to find the first ten iterates of $4x(1 - x)$ when x initially equals 0.200 and when the initial value is 0.201. Did the change in initial value affect the tenth value?

40. Why do you think this is called the Butterfly Effect?

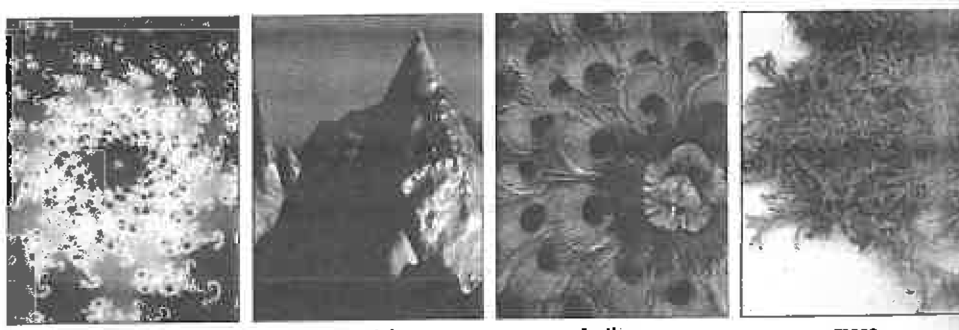
41. **ART** Describe how artist Jean-Paul Agosti used iteration and self-similarity in his painting *Jardin du Creuset*.



42. **NATURE** Some of these pictures are of real objects and others are fractal images of objects.

a. Compare the pictures and identify those you think are of real objects.

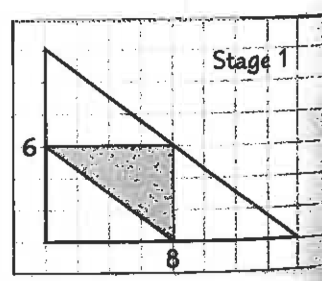
b. Describe the characteristics of fractals shown in the images.



flower | mountain | feathers | moss

43. **RESEARCH** Use the Internet or other sources to find the names and pictures of the other fractals Waclaw Sierpinski developed.

44. **CRITICAL THINKING** Draw a right triangle on grid paper with 6 and 8 units for the lengths of the perpendicular sides. Shade the triangle formed by the three midsegments. Repeat the process for each unshaded triangle. Find the perimeter of the shaded triangle in Stage 1. What is the total perimeter of all the shaded triangles in Stage 2?



45. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is mathematics related to nature?

Include the following in your answer:

- explain why broccoli is an example of fractal geometry, and
- how scientists can use fractal geometry to better understand nature.

Standardized Test Practice

46. **GRID IN** A triangle has side lengths of 3 inches, 6 inches, and 8 inches. A similar triangle is 24 inches on one side. Find the maximum perimeter, in inches, of the second triangle.

47. **ALGEBRA** A repair technician charges $80 for the first thirty minutes of each house call plus $2 for each additional minute. The repair technician charged a total of $170 for a job. How many minutes did the repair technician work?

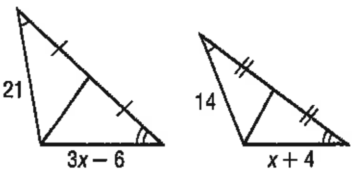
Ⓐ 45 min Ⓑ 55 min Ⓒ 75 min Ⓓ 85 min

Maintain Your Skills

Mixed Review

Find x. *(Lesson 6-5)*

48.



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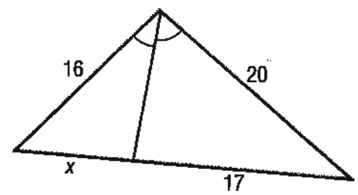

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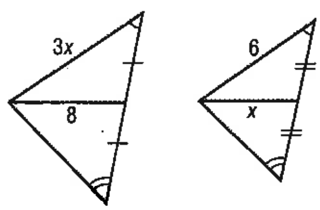

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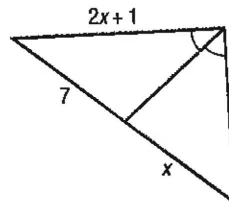
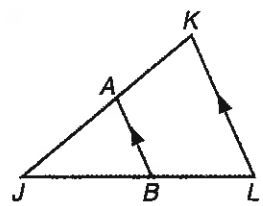


For Exercises 52–54, refer to $\triangle JKL$. *(Lesson 6-4)*

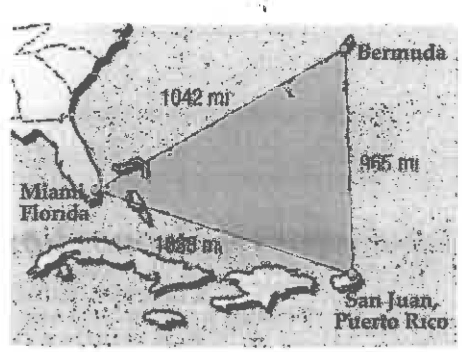


52. If $JL = 27$, $BL = 9$, and $JK = 18$, find JA.

53. If $AB = 8$, $KL = 10$, and $JB = 13$, find JL.

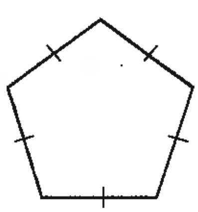
54. If $JA = 25$, $AK = 10$, and $BL = 14$, find JB.

55. **FOLKLORE** The Bermuda Triangle is an imaginary region located off the southeastern Atlantic coast of the United States. It is the subject of many stories about unexplained losses of ships, small boats, and aircraft. Use the vertex locations to name the angles in order from least measure to greatest measure. *(Lesson 5-4)*

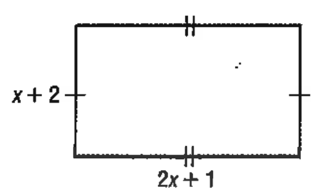


Find the length of each side of the polygon for the given perimeter. *(Lesson 1-6)*

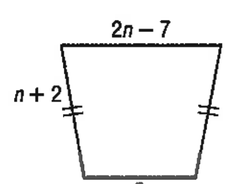
56. $P = 60$ centimeters



57. $P = 54$ feet



58. $P = 57$ units



Chapter 6 Study Guide and Review

Vocabulary and Concept Check

cross products (p. 283)
extremes (p. 283)
fractal (p. 325)
iteration (p. 325)
means (p. 283)
midsegment (p. 308)
proportion (p. 283)
ratio (p. 282)
scale factor (p. 290)
self-similar (p. 325)
similar polygons (p. 289)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **State whether each sentence is *true* or *false*. If false, replace the underlined expression to make a true sentence.**

1. A midsegment of a triangle is a segment whose endpoints are the <u>midpoints of two sides</u> of the triangle.
2. Two polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are <u>congruent</u>.
3. If two angles of one triangle are congruent to two angles of another triangle, then the triangles are <u>similar</u>.
4. If two triangles are similar, then the perimeters are proportional to the measures of the corresponding <u>angles</u>.
5. A fractal is a geometric figure that is created using <u>*recursive formulas*</u>.
6. A midsegment of a triangle is parallel to one side of the triangle, and its length is <u>twice</u> the length of that side.
7. For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if <u>$ad = bc$</u>.
8. If two triangles are similar, then the measures of the corresponding angle bisectors of the triangle are proportional to the <u>measures of the corresponding sides</u>.
9. If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is <u>equal to one-half the length of</u> the third side.

Lesson-by-Lesson Review

6-1 Proportions

See pages 282–287.

Concept Summary

- A ratio is a comparison of two quantities.
- A proportion is an equation stating that two ratios are equal.

Example **Solve $\frac{z}{40} = \frac{5}{8}$.**

$\frac{z}{40} = \frac{5}{8}$	Original proportion
$z \cdot 8 = 40(5)$	Cross products
$8z = 200$	Multiply.
$z = 25$	Divide each side by 8.

Exercises **Solve each proportion.** *See Example 3 on page 284.*

10. $\frac{3}{4} = \frac{x}{12}$ **11.** $\frac{7}{3} = \frac{28}{z}$ **12.** $\frac{x+2}{5} = \frac{14}{10}$

13. $\frac{3}{7} = \frac{7}{y-3}$ **14.** $\frac{4-x}{3+x} = \frac{16}{25}$ **15.** $\frac{x-12}{6} = \frac{x+7}{-4}$

16. BASEBALL A player's slugging percentage is the ratio of the number of total bases from hits to the number of total at-bats. The ratio is converted to a decimal (rounded to three places) by dividing. If Alex Rodriguez of the Texas Rangers has 263 total bases in 416 at-bats, what is his slugging percentage?

17. A 108-inch-long board is cut into two pieces that have lengths in the ratio 2:7. How long is each new piece?

6-2 Similar Polygons

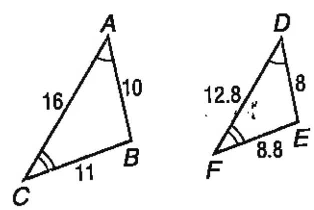
See pages 289–297.

Concept Summary

- In similar polygons, corresponding angles are congruent, and corresponding sides are in proportion.
- The ratio of two corresponding sides in two similar polygons is the scale factor.

Example

Determine whether the pair of triangles is similar. Justify your answer.



$\angle A \cong \angle D$ and $\angle C \cong \angle F$, so by the Third Angle Theorem, $\angle B \cong \angle E$. All of the corresponding angles are congruent.

Now, check to see if corresponding sides are in proportion.

$\frac{AB}{DE} = \frac{10}{8}$ $= \frac{5}{4}$ or 1.25

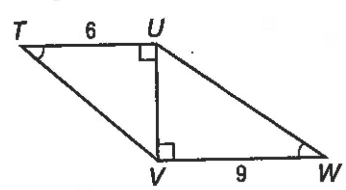
$\frac{BC}{EF} = \frac{11}{8.8}$ $= \frac{5}{4}$ or 1.25

$\frac{CA}{FD} = \frac{16}{12.8}$ $= \frac{5}{4}$ or 1.25

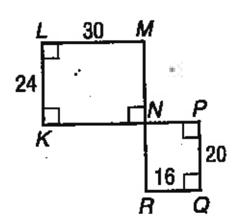
The corresponding angles are congruent, and the ratios of the measures of the corresponding sides are equal, so $\triangle ABC \sim \triangle DEF$.

Exercises **Determine whether each pair of figures is similar. Justify your answer.**
See Example 1 on page 290.

18.

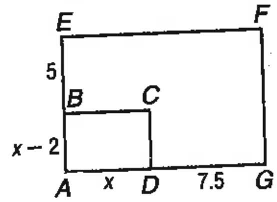


19.

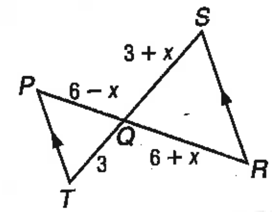


Each pair of polygons is similar. Write a similarity statement, and find x, the measures of the indicated sides, and the scale factor. *See Example 3 on page 291.*

20. $\overline{AB}$ and $\overline{AG}$



21. $\overline{PQ}$ and $\overline{QS}$



6-3 Similar Triangles

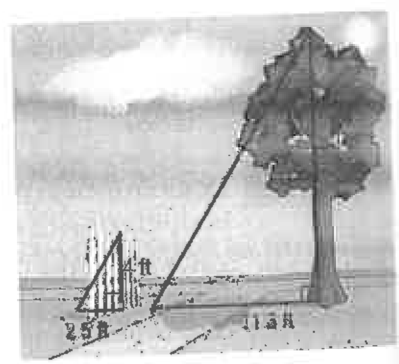
See pages 298–306.

Concept Summary

- AA, SSS, and SAS Similarity can all be used to prove triangles similar.
- Similarity of triangles is reflexive, symmetric, and transitive.

Example

INDIRECT MEASUREMENT **Alonso wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 P.M., he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree?**



Since the triangles formed are similar, a proportion can be written. Let x be the height of the tree.

$$\frac{\text{height of the tree}}{\text{height of the fence}} = \frac{\text{tree shadow length}}{\text{fence shadow length}}$$

$$\frac{x}{4} = \frac{11.3}{2.5} \quad \text{Substitution}$$

$$x \cdot 2.5 = 4(11.3) \quad \text{Cross products}$$

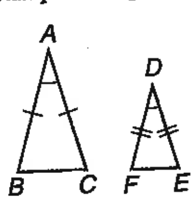
$$2.5x = 45.2 \quad \text{Simplify.}$$

$$x = 18.08 \quad \text{Divide each side by 2.5.}$$

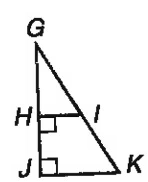
The height of the tree is 18.08 feet.

Exercises **Determine whether each pair of triangles is similar. Justify your answer.** *See Example 1 on page 299.*

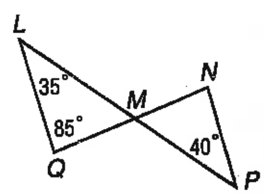
22.



23.

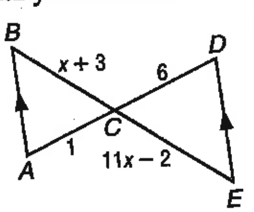


24.

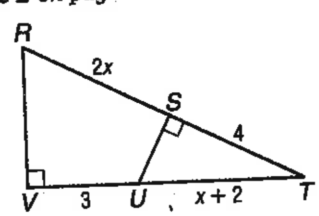


Identify the similar triangles. Find x. *See Example 2 on page 300.*

25.



26.



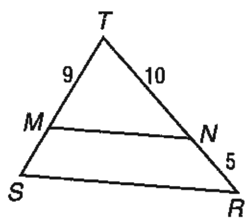
6-4 Parallel Lines and Proportional Parts

See pages 307–315.

Concept Summary

- A segment that intersects two sides of a triangle and is parallel to the third side divides the two intersected sides in proportion.
- If two lines divide two segments in proportion, then the lines are parallel.

Example **In $\triangle TRS$, $TS = 12$. Determine whether $\overline{MN} \parallel \overline{SR}$.**

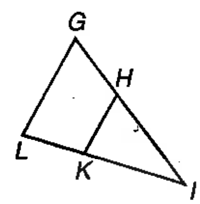


If $TS = 12$, then $MS = 12 - 9$ or 3. Compare the segment lengths to determine if the lines are parallel.

$\frac{TM}{MS} = \frac{9}{3} = 3$ $\frac{TN}{NR} = \frac{10}{5} = 2$

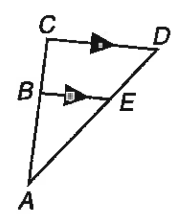
Because $\frac{TM}{MS} \neq \frac{TN}{NR}$, $\overline{MN} \not\parallel \overline{SR}$.

Exercises **Determine whether $\overline{GL} \parallel \overline{HK}$. Justify your answer.**
See Example 2 on page 308.



27. $IH = 21, HG = 14, LK = 9, KI = 15$
28. $GH = 10, HI = 35, IK = 28, IL = 36$
29. $GH = 11, HI = 22$, and IL is three times the length of $\overline{KL}$.
30. $LK = 6, KI = 18$, and IG is three times the length of $\overline{HI}$.

Refer to the figure at the right. *See Example 1 on page 308.*



31. Find ED if $AB = 6, BC = 4$, and $AE = 9$.
32. Find AE if $AB = 12, AC = 16$, and $ED = 5$.
33. Find CD if $AE = 8, ED = 4$, and $BE = 6$.
34. Find BC if $BE = 24, CD = 32$, and $AB = 33$.

6-5 Parts of Similar Triangles

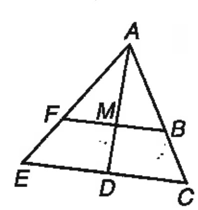
See pages 316–323.

Concept Summary

- Similar triangles have perimeters proportional to the corresponding sides.
- Corresponding angle bisectors, medians, and altitudes of similar triangles have lengths in the same ratio as corresponding sides.

Example **If $\overline{FB} \parallel \overline{EC}$, $\overline{AD}$ is an angle bisector of $\angle A$, $BF = 6$, $CE = 10$, and $AD = 5$, find AM.**

By AA Similarity using $\angle AFE \cong \angle ABF$ and $\angle A \cong \angle A$, $\triangle ABF \sim \triangle ACE$.



$\frac{AM}{AD} = \frac{BF}{CE}$ ~$\triangle$s have angle bisectors in the same proportion as the corresponding sides.

$\frac{x}{5} = \frac{6}{10}$ $AD = 5, AF = 6, FE = 4, AM = x$

$10x = 30$ Cross products

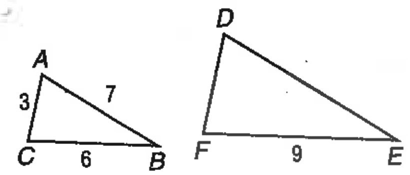
$x = 3$ Divide each side by 10.

Thus, $AM = 3$.

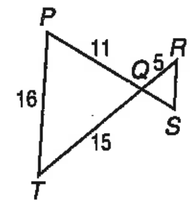
• Extra Practice, see pages 764–766.
• Mixed Problem Solving, see page 787.

Exercises **Find the perimeter of the given triangle.** *See Example 1 on page 316.*

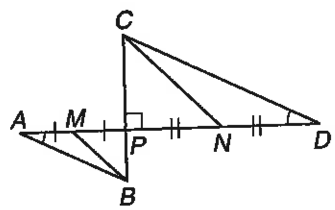
35. $\triangle DEF$ if $\triangle DEF \sim \triangle ABC$



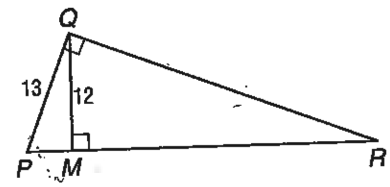
36. $\triangle QRS$ if $\triangle QRS \sim \triangle QTP$



37. $\triangle CPD$ if the perimeter of $\triangle BPA$ is 12, $BM = \sqrt{13}$, and $CN = 3\sqrt{13}$



38. $\triangle PQR$, if $\triangle PQM \sim \triangle PRQ$



6-6 Fractals and Self-Similarity

See pages 325–331.

Concept Summary

- Iteration is the creation of a sequence by repetition of the same operation.
- A fractal is a geometric figure created by iteration.
- An iterative process involving algebraic equations is a recursive formula.

Example **Find the value of $\frac{x}{2} + 4$, where x initially equals −8. Use that value as the next x in the expression. Repeat the process five times and describe your observations.**

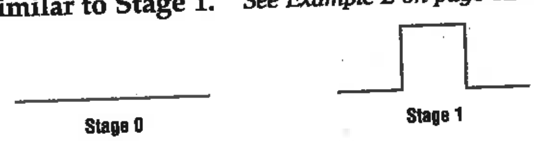
Make a table to organize each iteration.

Iteration	1	2	3	4	5	6
x	−8	0	4	6	7	7.5
$\frac{x}{2} + 4$	0	4	6	7	7.5	7.75

The x values appear to get closer to the number 8 with each iteration.

Exercises **Draw Stage 2 of the fractal shown below. Determine whether Stage 2 is similar to Stage 1.** *See Example 2 on page 326.*

39.



Find the first three iterates of each expression. *See Example 3 on page 327.*

40. $x^3 - 4$, x initially equals 2

41. $3x + 4$, x initially equals −4

42. $\frac{1}{x}$, x initially equals 10

43. $\frac{x}{10} - 9$, x initially equals 30