Chapter 6

Proportions and Similarity

What You'll Learn

- **Lessons 6-1, 6-2, and 6-3** Identify similar polygons, and use ratios and proportions to solve problems.
- **Lessons 6-4 and 6-5** Recognize and use proportional parts, corresponding perimeters, altitudes, angle bisectors, and medians of similar triangles to solve problems.
- **Lesson 6-6** Identify the characteristics of fractals and nongeometric iteration.

Key Vocabulary

- **proportion** (p. 283)
- **cross products** (p. 283)
- **similar polygons** (p. 289)
- **scale factor** (p. 290)
- **midsegment** (p. 308)

Why It's Important

Similar figures are used to represent various real-world situations involving a scale factor for the corresponding parts. For example, photography uses similar triangles to calculate distances from the lens to the object and to the image size. *You will use similar triangles to solve problems about photography in Lesson 6-5.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lesson 6-1, 6-3, and 6-4 — Solve Rational Equations

Solve each equation. *(For review, see pages 737 and 738.)*

1. $\frac{2}{3}y - 4 = 6$ **2.** $\frac{5}{6} = \frac{x-4}{12}$ **3.** $\frac{4}{3} = \frac{y+2}{y-1}$ **4.** $\frac{2y}{4} = \frac{32}{y}$

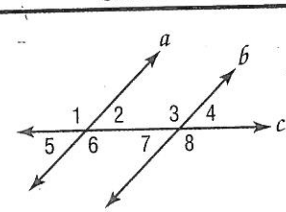
For Lesson 6-2 — Slopes of Lines

Find the slope of the line given the coordinates of two points on the line.
(For review, see Lesson 3-3.)

5. $(3, 5)$ and $(0, -1)$ **6.** $(-6, -3)$ and $(2, -3)$ **7.** $(-3, 4)$ and $(2, -2)$

For Lesson 6-5 — Show Lines Parallel

Given the following information, determine whether $a \parallel b$. State the postulate or theorem that justifies your answer.
(For review, see Lesson 3-5.)



8. $\angle 1 \cong \angle 8$

9. $\angle 3 \cong \angle 6$

10. $\angle 5 \cong \angle 3$

For Lesson 6-6 — Evaluate Expressions

Evaluate each expression for $n = 1, 2, 3,$ and 4. *(For review, see page 736.)*

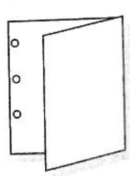
11. 2^n **12.** $n^2 - 2$ **13.** $3^n - 2$

FOLDABLES™ Study Organizer

Proportions and Similarity Make this Foldable to help you organize your notes. Begin with one sheet of 11" by 17" paper.

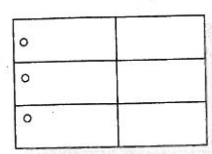
Step 1 Punch and Fold

Fold widthwise. Leave space to punch holes so it can be placed in your binder.



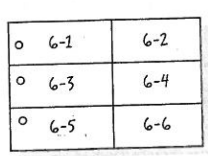
Step 2 Divide

Open the flap and draw lines to divide the inside into six equal parts.



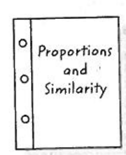
Step 3 Label

Label each part using the lesson numbers.



Step 4 Label

Put the name of the chapter on the front flap.



Reading and Writing As you read and study the chapter, use the Foldable to write down questions you have about the concepts in each lesson. Leave room to record the answers to your questions.

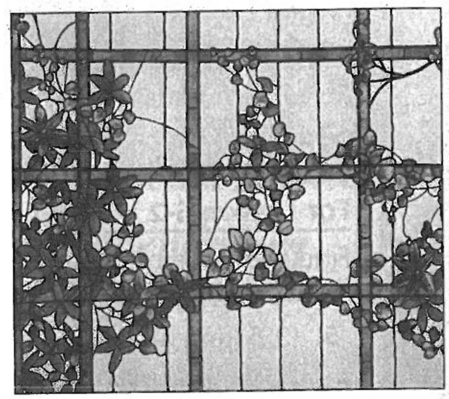
6-1 Proportions

What You'll Learn

- Write ratios.
- Use properties of proportions.

How do artists use ratios?

Stained-glass artist Louis Comfort Tiffany used geometric shapes in his designs. In a portion of *Clematis Skylight* shown at the right, rectangular shapes are used as the background for the flowers and vines. Tiffany also used ratio and proportion in the design of this piece.



Vocabulary

- ratio
- proportion
- cross products
- extremes
- means

WRITE RATIOS A **ratio** is a comparison of two quantities. The ratio of a to b can be expressed as $\frac{a}{b}$, where b is not zero. This ratio can also be written as $a:b$.

Example 1 Write a Ratio

SOCCER The U.S. Census Bureau surveyed 8218 schools nationally about their girls' soccer programs. They found that 270,273 girls participated in a high school soccer program in the 1999–2000 school year. Find the ratio of girl soccer players per school to the nearest tenth.

Divide the number of girl soccer players by the number of schools.

$$\frac{\text{number of girl soccer players}}{\text{number of schools}} = \frac{270{,}273}{8{,}218} \text{ or about } 32.9$$

A ratio in which the denominator is 1 is called a *unit ratio.*

The ratio for this survey was 32.9 girl soccer players for each school.

Extended ratios can be used to compare three or more numbers. The expression $a:b:c$ means that the ratio of the first two numbers is $a:b$, the ratio of the last two numbers is $b:c$, and the ratio of the first and last numbers is $a:c$.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

Example 2 Extended Ratios in Triangles

Multiple-Choice Test Item

In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.

Ⓐ 10 in. Ⓑ 60 in. Ⓒ 90 in. Ⓓ 100 in.

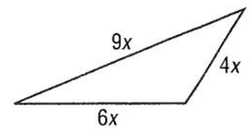
Read the Test Item

You are asked to apply the ratio to the three sides of the triangle and the perimeter to find the longest side.

Solve the Test Item

Recall that equivalent fractions can be found by multiplying the numerator and the denominator by the same number. So, $2:3 = \frac{2}{3} \cdot \frac{x}{x}$ or $\frac{2x}{3x}$. Thus, we can rewrite $4:6:9$ as $4x:6x:9x$ and use those measures for the sides of the triangle. Write an equation to represent the perimeter of the triangle as the sum of the measures of its sides.

$4x + 6x + 9x = 190$	Perimeter
$19x = 190$	Combine like terms.
$x = 10$	Divide each side by 19.



Use this value of x to find the measures of the sides of the triangle.

$4x = 4(10)$ or 40 inches

$6x = 6(10)$ or 60 inches

$9x = 9(10)$ or 90 inches

The longest side is 90 inches. The answer is C.

CHECK Add the lengths of the sides to make sure that the perimeter is 190.

$40 + 60 + 90 = 190$ ✓

Test-Taking Tip

Extended ratio problems require a variable to be the common factor among the terms of the ratio. This will enable you to write an equation to solve the problem.

USE PROPERTIES OF PROPORTIONS

An equation stating that two ratios are equal is called a **proportion**. Equivalent fractions set equal to each other form a proportion. Since $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions, $\frac{2}{3} = \frac{6}{9}$ is a proportion.

Every proportion has two **cross products**. The cross products in $\frac{2}{3} = \frac{6}{9}$ are 2 times 9 and 3 times 6. The **extremes** of the proportion are 2 and 9. The **means** are 3 and 6.

extremes → $\frac{2}{3} = \frac{6}{9}$ ← means

cross product of extremes → $2(9) = 3(6)$ ← cross product of means

$18 = 18$

The product of the means equals the product of the extremes, so the cross products are equal. Consider the general case.

$\frac{a}{b} = \frac{c}{d}$	$b \neq 0, d \neq 0$
$(bd)\frac{a}{b} = (bd)\frac{c}{d}$	Multiply each side by the common denominator, bd.
$da = bc$	Simplify.
$ad = bc$	Commutative Property

Study Tip

Reading Mathematics

When a proportion is written using colons, it is read using the word *to* for the colon. For example, 2:3 is read *2 to 3*. The means are the inside numbers, and the extremes are the outside numbers.

$\overbrace{2:\underbrace{3 = 6}_{\text{means}}:9}^{\text{extremes}}$

Key Concept — *Property of Proportions*

- **Words** For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
- **Examples** $\frac{4}{5} = \frac{12}{15}$ if and only if $4 \cdot 15 = 5 \cdot 12$.

To *solve a proportion* means to find the value of the variable that makes the proportion true.

Example 3 Solve Proportions by Using Cross Products

Solve each proportion.

a. $\frac{3}{5} = \frac{x}{75}$

$\frac{3}{5} = \frac{x}{75}$	Original proportion
$3(75) = 5x$	Cross products
$225 = 5x$	Multiply.
$45 = x$	Divide each side by 5.

b. $\frac{3x-5}{4} = \frac{-13}{2}$

$\frac{3x-5}{4} = \frac{-13}{2}$	Original proportion
$(3x-5)2 = 4(-13)$	Cross products
$6x - 10 = -52$	Simplify.
$6x = -42$	Add 10 to each side.
$x = -7$	Divide each side by 6.

Proportions can be used to solve problems involving two objects that are said to be *in proportion*. This means that if you write ratios comparing the measures of all parts of one object with the measures of comparable parts of the other object, a true proportion would always exist.

Study Tip

Common Misconception
The proportion shown in Example 4 is not the only correct proportion. There are many equivalent proportions, such as:
$\frac{a}{b} = \frac{c}{d}$, $\frac{a}{c} = \frac{b}{d}$,
$\frac{b}{a} = \frac{d}{c}$, and $\frac{c}{a} = \frac{d}{b}$.
All of these have identical cross products.

Example 4 Solve Problems Using Proportions

AVIATION A twinjet airplane has a length of 78 meters and a wingspan of 90 meters. A toy model is made in proportion to the real airplane. If the wingspan of the toy is 36 centimeters, find the length of the toy.

Because the toy airplane and the real plane are in proportion, you can write a proportion to show the relationship between their measures. Since both ratios compare meters to centimeters, you need not convert all the lengths to the same unit of measure.

$$\frac{\text{plane's length (m)}}{\text{model's length (cm)}} = \frac{\text{plane's wingspan (m)}}{\text{model's wingspan (cm)}}$$

$\frac{78}{x} = \frac{90}{36}$	Substitution
$(78)(36) = x \cdot 90$	Cross products
$2808 = 90x$	Multiply.
$31.2 = x$	Divide each side by 90.

The length of the model would be 31.2 centimeters.

Check for Understanding

Concept Check

1. **Explain** how you would solve $\frac{28}{48} = \frac{21}{x}$.
2. **OPEN ENDED** Write two possible proportions having the extremes 5 and 8.
3. **FIND THE ERROR** Madeline and Suki are solving $\frac{15}{x} = \frac{3}{4}$.

Madeline	Suki
$\frac{15}{x} = \frac{3}{4}$	$\frac{15}{x} = \frac{3}{4}$
$45 = 4x$	$60 = 3x$
$11.25 = x$	$20 = x$

Who is correct? Explain your reasoning.

Guided Practice

4. **HOCKEY** A hockey player scored 9 goals in 12 games. Find the ratio of goals to games.

5. **SCULPTURE** A replica of *The Thinker* is 10 inches tall. A statue of *The Thinker*, located in front of Grawemeyer Hall on the Belnap Campus of the University of Louisville in Kentucky, is 10 feet tall. What is the ratio of the replica to the statue in Louisville?

Solve each proportion.

6. $\frac{x}{5} = \frac{11}{35}$ 7. $\frac{2.3}{4} = \frac{x}{3.7}$ 8. $\frac{x-2}{2} = \frac{4}{5}$

9. The ratio of the measures of three sides of a triangle is 9:8:7, and its perimeter is 144 units. Find the measure of each side of the triangle.

10. The ratio of the measures of three angles of a triangle 5:7:8. Find the measure of each angle of the triangle.

Standardized Test Practice

A B C D

11. **GRID IN** The scale on a map indicates that 1.5 centimeters represent 200 miles. If the distance on the map between Norfolk, Virginia, and Atlanta, Georgia, measures 2.4 centimeters, how many miles apart are the cities?

Practice and Apply

Homework Help

For Exercises	See Examples
12–17, 23, 25	1
18–22	2
26, 27	4
28–35	3

Extra Practice
See page 764.

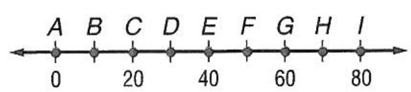
12. **BASEBALL** A designated hitter made 8 hits in 10 games. Find the ratio of hits to games.

13. **SCHOOL** There are 76 boys in a sophomore class of 165 students. Find the ratio of boys to girls.

14. **CURRENCY** In a recent month, 208 South African rands were equivalent to 18 United States dollars. Find the ratio of rands to dollars.

15. **EDUCATION** In the 2000–2001 school year, Arizona State University had 44,125 students and 1747 full-time faculty members. What was the ratio of the students to each teacher rounded to the nearest tenth?

16. Use the number line at the right to determine the ratio of *AC* to *BH*.



17. A cable that is 42 feet long is divided into lengths in the ratio of 3:4. What are the two lengths into which the cable is divided?

Find the measures of the angles of each triangle.

18. The ratio of the measures of the three angles is 2:5:3.
19. The ratio of the measures of the three angles is 6:9:10.

Find the measures of the sides of each triangle.

20. The ratio of the measures of three sides of a triangle is 8:7:5. Its perimeter is 240 feet.
21. The ratio of the measures of the sides of a triangle is 3:4:5. Its perimeter is 72 inches.
22. The ratio of the measures of three sides of a triangle are $\frac{1}{2}:\frac{1}{3}:\frac{1}{5}$, and its perimeter is 6.2 centimeters. Find the measure of each side of the triangle.

More About. . .

Entertainment

In the model, Lincoln is 8 inches tall. In the theater, Lincoln is 6 feet 4 inches tall (his actual adult height).

Source: ©Disney Enterprises, Inc.

LITERATURE **For Exercises 23 and 24, use the following information.**
Throughout Lewis Carroll's book, *Alice's Adventures in Wonderland,* Alice's size changes. Her normal height is about 50 inches tall. She comes across a door, about 15 inches high, that leads to a garden. Alice's height changes to 10 inches so she can visit the garden.

23. Find the ratio of the height of the door to Alice's height in Wonderland.

24. How tall would the door have been in Alice's normal world?

25. **ENTERTAINMENT** Before actual construction of the Great Moments with Mr. Lincoln exhibit, Walt Disney and his design company built models that were in proportion to the displays they planned to build. What is the ratio of the height of the model of Mr. Lincoln compared to his actual height?

ICE CREAM **For Exercises 26 and 27, use the following information.**
There were approximately 255,082,000 people in the United States in a recent year. According to figures from the United States Census, they consumed about 4,183,344,800 pounds of ice cream that year.

26. If there were 276,000 people in the city of Raleigh, North Carolina, about how much ice cream might they have been expected to consume?

27. Find the approximate consumption of ice cream per person.

Online Research **Data Update** Use the Internet or other resource to find the population of your community. Determine how much ice cream you could expect to be consumed each year in your community. Visit www.geometryonline.com/data_update to learn more.

ALGEBRA **Solve each proportion.**

28. $\frac{3}{8} = \frac{x}{5}$ **29.** $\frac{a}{5.18} = \frac{1}{4}$ **30.** $\frac{3x}{23} = \frac{48}{92}$ **31.** $\frac{13}{49} = \frac{26}{7x}$

32. $\frac{2x - 13}{28} = \frac{-4}{7}$ **33.** $\frac{4x + 3}{12} = \frac{5}{4}$ **34.** $\frac{b + 1}{b - 1} = \frac{5}{6}$ **35.** $\frac{3x - 1}{2} = \frac{-2}{x + 2}$

PHOTOGRAPHY **For Exercises 36 and 37, use the following information.**
José reduced a photograph that is 21.3 centimeters by 27.5 centimeters so that it would fit in a 10-centimeter by 10-centimeter area.

36. Find the maximum dimensions of the reduced photograph.

37. What percent of the original length is the length of the reduced photograph?

38. **CRITICAL THINKING** The ratios of the lengths of the sides of three polygons are given below. Make a conjecture about identifying each type of polygon.

a. 2:2:3 **b.** 3:3:3:3 **c.** 4:5:4:5

39. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do artists use ratios?

Include the following in your answer:

- four rectangles from the photo that appear to be in proportion, and
- an estimate in inches of the ratio of the width of the skylight to the length of the skylight given that the dimensions of the rectangle in the bottom left corner are approximately 3.5 inches by 5.5 inches.

Standardized Test Practice

40. **SHORT RESPONSE** In a golden rectangle, the ratio of the length of the rectangle to its width is approximately 1.618:1. Suppose a golden rectangle has a length of 12 centimeters. What is its width to the nearest tenth?

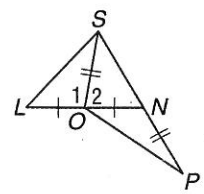
41. **ALGEBRA** A breakfast cereal contains wheat, rice, and oats in the ratio 3:1:2. If the manufacturer makes a mixture using 120 pounds of oats, how many pounds of wheat will be used?

Ⓐ 60 lb Ⓑ 80 lb Ⓒ 120 lb Ⓓ 180 lb

Maintain Your Skills

Mixed Review

In the figure, $\overline{SO}$ is a median of $\triangle SLN$, $\overline{OS} \cong \overline{NP}$, $m\angle 1 = 3x - 50$, and $m\angle 2 = x + 30$. Determine whether each statement is *always*, *sometimes*, or *never* true. *(Lesson 5-5)*



42. $LS > SN$

43. $SN < OP$

44. $x = 45$

Find the range for the measure of the third side of a triangle given the measures of two sides. *(Lesson 5-4)*

45. 16 and 31 46. 26 and 40 47. 11 and 23

48. **COORDINATE GEOMETRY** Given $\triangle STU$ with vertices $S(0, 5)$, $T(0, 0)$, and $U(-2, 0)$ and $\triangle XYZ$ with vertices $X(4, 8)$, $Y(4, 3)$, and $Z(6, 3)$, show that $\triangle STU \cong \triangle XYZ$. *(Lesson 4-4)*

Graph the line that satisfies each condition. *(Lesson 3-3)*

49. $m = \frac{3}{5}$ and contains $P(-3, -4)$

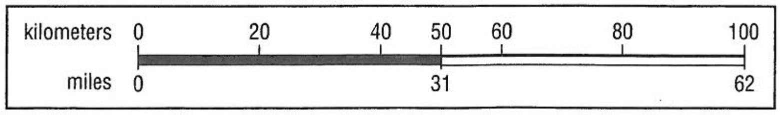
50. contains $A(5, 3)$ and $B(-1, 8)$

51. parallel to $\overleftrightarrow{JK}$ with $J(-1, 5)$ and $K(4, 3)$ and contains $E(2, 2)$

52. contains $S(8, 1)$ and is perpendicular to $\overleftrightarrow{QR}$ with $Q(6, 2)$ and $R(-4, -6)$

C Rao

53. **MAPS** On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



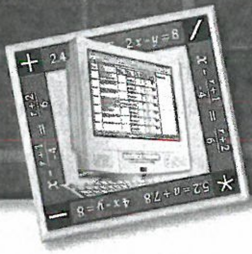
Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. *(Lesson 2-7)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the distance between each pair of points to the nearest tenth. *(To review the **Distance Formula**, see Lesson 1-3.)*

54. $A(12, 3)$, $B(-8, 3)$ 55. $C(0, 0)$, $D(5, 12)$

56. $E\left(\frac{4}{5}, -1\right)$, $F\left(2, \frac{-1}{2}\right)$ 57. $G\left(3, \frac{3}{7}\right)$, $H\left(4, -\frac{2}{7}\right)$



Spreadsheet Investigation

A Follow-Up of Lesson 6-1

Fibonacci Sequence and Ratios

The Fibonacci sequence is a set of numbers that begins with 1 as its first and second terms. Each successive term is the sum of the two numbers before it. This sequence continues on indefinitely.

term	1	2	3	4	5	6	7
Fibonacci number	1	1	2	3	5	8	13
			↑	↑	↑	↑	↑
			$1+1$	$1+2$	$2+3$	$3+5$	$5+8$

Example

Use a spreadsheet to create twenty terms of the Fibonacci sequence. Then compare each term with its preceding term.

Step 1 Enter the column headings in rows 1 and 2.

Step 2 Enter 1 into cell A3. Then insert the formula =A3 + 1 in cell A4. Copy this formula down the column. This will automatically calculate the number of the term.

Step 3 In column B, we will record the Fibonacci numbers. Enter 1 in cells B3 and B4 since you do not have two previous terms to add. Then insert the formula =B3 + B4 in cell B5. Copy this formula down the column.

Step 4 In column C, we will find the ratio of each term to its preceding term. Enter 1 in cell C3 since there is no preceding term. Then enter =B4/B3 in cell C4. Copy this formula down the column.

Fibonacci Table

	A	B	C
1	term	Fibonacci number	ratio
2	n	F(n)	F(n+1)/F(n)
3	1	1	1
4	2	1	1
5	3	2	2
6	4	3	1.5
7	5	5	1.666666667
8	6	8	1.6
9	7	13	1.625

Sheet1 Sheet2

Exercises

1. What happens to the Fibonacci number as the number of the term increases?
2. What pattern of odd and even numbers do you notice in the Fibonacci sequence?
3. As the number of terms gets greater, what pattern do you notice in the ratio column?
4. Extend the spreadsheet to calculate fifty terms of the Fibonacci sequence. Describe any differences in the patterns you described in Exercises 1–3.

The rectangle that most humans perceive to be pleasing to the eye has a width to length ratio of about 1:1.618. This is called the *golden ratio*, and the rectangle is called the *golden rectangle*. This type of rectangle is visible in nature and architecture. The Fibonacci sequence occurs in nature in patterns that are also pleasing to the human eye, such as in sunflowers, pineapples, and tree branch structure.

5. **MAKE A CONJECTURE** How might the Fibonacci sequence relate to the golden ratio?

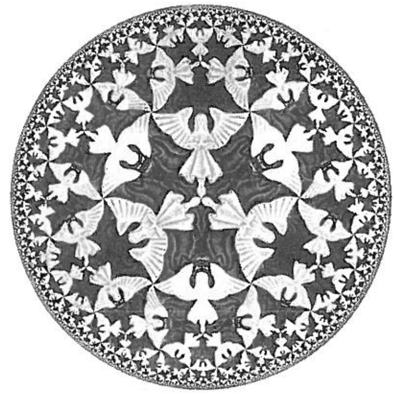
6-2 Similar Polygons

What You'll Learn

- Identify similar figures.
- Solve problems involving scale factors.

How do artists use geometric patterns?

M.C. Escher (1898–1972) was a Dutch graphic artist known for drawing impossible structures, spatial illusions, and repeating interlocking geometric patterns. The image at the right is a print of Escher's *Circle Limit IV*, which is actually a woodcutting. It includes winged images that have the same shape, but are different in size. Also note that there are not only similar dark images but also similar light images.



Circle Limit IV,
M.C. Escher (1960)

Vocabulary

- similar polygons
- scale factor

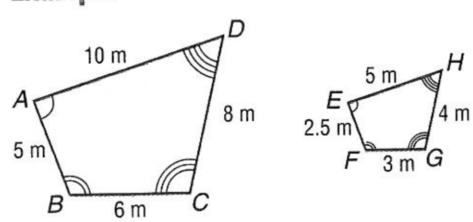
IDENTIFY SIMILAR FIGURES When polygons have the same shape but may be different in size, they are called **similar polygons**.

Study Tip

Similarity and Congruence
If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

Key Concept — Similar Polygons

- **Words** Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
- **Symbol** ~ is read *is similar to*
- **Example**



The order of the vertices in a similarity statement is important. It identifies the corresponding angles and the corresponding sides.

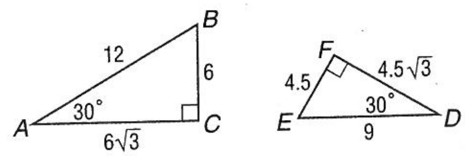
similarity statement	congruent angles	corresponding sides
$ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Like congruent polygons, similar polygons may be repositioned so that corresponding parts are easy to identify.

Example 1 Similar Polygons

Determine whether each pair of figures is similar. Justify your answer.

a.



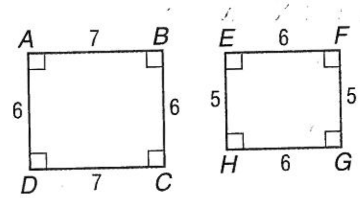
All right angles are congruent, so $\angle C \cong \angle F$.
Since $m\angle A = m\angle D$, $\angle A \cong \angle D$.
By the Third Angle Theorem, $\angle B \cong \angle E$.
Thus, all corresponding angles are congruent.

Now determine whether corresponding sides are proportional.

Sides opposite 90° angle	Sides opposite 30° angle	Sides opposite 60° angle
$\frac{AB}{DE} = \frac{12}{9}$ or $1.\overline{3}$	$\frac{BC}{EF} = \frac{6}{4.5}$ or $1.\overline{3}$	$\frac{AC}{DF} = \frac{6\sqrt{3}}{4.5\sqrt{3}}$ or $1.\overline{3}$

The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so $\triangle ABC \sim \triangle DEF$.

b.



Both rectangles have all right angles and right angles are congruent.
$\frac{AB}{EF} = \frac{7}{6}$ and $\frac{BC}{FG} = \frac{6}{5}$, but $\frac{AB}{EF} \neq \frac{BC}{FG}$ because $\frac{7}{6} \neq \frac{6}{5}$. The rectangles are not similar.

Study Tip

Identifying Corresponding Parts
Using different colors to circle letters of congruent angles may help you identify corresponding parts.

Study Tip

Common Misconception
When two figures have vertices that are in alphabetical order, this does not mean that the corresponding vertices in the similarity statement will follow alphabetical order.

SCALE FACTORS When you compare the lengths of corresponding sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures. Scale factors are often given for models of real-life objects.

Example 2 Scale Factor

MOVIES Some special effects in movies are created using miniature models. In a recent movie, a model sports-utility vehicle (SUV) 22 inches long was created to look like a real $14\frac{2}{3}$-foot SUV. What is the scale factor of the model compared to the real SUV?

Before finding the scale factor you must make sure that both measurements use the same unit of measure.

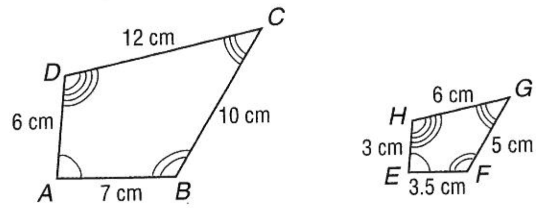
$$14\tfrac{2}{3}(12) = 176 \text{ inches}$$

$$\frac{\text{length of model}}{\text{length of real SUV}} = \frac{22 \text{ inches}}{176 \text{ inches}}$$

$$= \frac{1}{8}$$

The ratio comparing the two lengths is $\frac{1}{8}$ or 1:8. The scale factor is $\frac{1}{8}$, which means that the model is $\frac{1}{8}$ the length of the real SUV.

When finding the scale factor for two similar polygons, the scale factor will depend on the order of comparison.

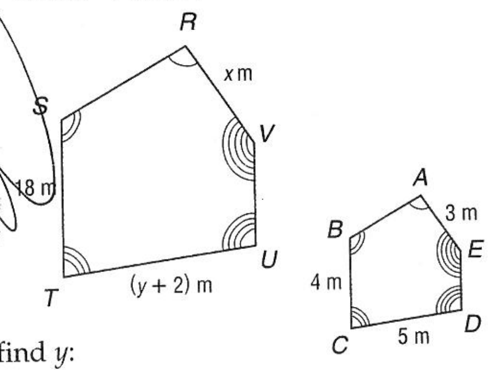


- The scale factor of quadrilateral $ABCD$ to quadrilateral $EFGH$ is 2.
- The scale factor of quadrilateral $EFGH$ to quadrilateral $ABCD$ is $\frac{1}{2}$.

Example 3 Proportional Parts and Scale Factor

The two polygons are similar.

a. Write a similarity statement. Then find x, y, and UT.



Use the congruent angles to write the corresponding vertices in order.

polygon $RSTUV \sim$ polygon $ABCDE$

Now write proportions to find x and y.

To find x:

$\frac{ST}{BC} = \frac{VR}{EA}$	Similarity proportion
$\frac{18}{4} = \frac{x}{3}$	$ST = 18$, $BC = 4$ $VR = x$, $EA = 3$
$18(3) = 4(x)$	Cross products
$54 = 4x$	Multiply.
$13.5 = x$	Divide each side by 4.

To find y:

$\frac{ST}{BC} = \frac{UT}{DC}$	Similarity proportion
$\frac{18}{4} = \frac{y+2}{5}$	$ST = 18$, $BC = 4$ $UT = y + 2$, $EA = 3$
$18(5) = 4(y + 2)$	Cross products
$90 = 4y + 8$	Multiply.
$82 = 4y$	Subtract 8 from each side.
$20.5 = y$	Divide each side by 4.

$UT = y + 2$, so $UT = 20.5 + 2$ or 22.5.

b. Find the scale factor of polygon $RSTUV$ to polygon $ABCDE$.

The scale factor is the ratio of the lengths of any two corresponding sides.

$\frac{ST}{BC} = \frac{18}{4}$ or $\frac{9}{2}$

Study Tip

Checking Solutions
To verify the scale factor, find the ratio of two other corresponding sides.

You can use scale factors to produce similar figures.

Example 4 Enlargement of a Figure

Triangle ABC is similar to $\triangle XYZ$ with a scale factor of $\frac{2}{3}$. If the lengths of the sides of $\triangle ABC$ are 6, 8, and 10 inches, what are the lengths of the sides of $\triangle XYZ$?

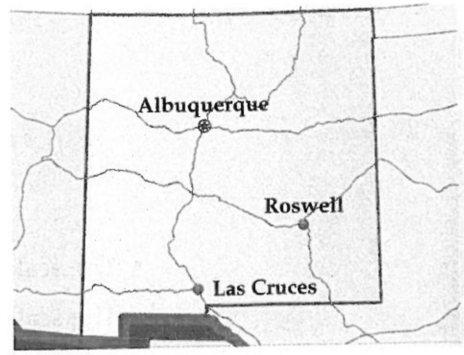
Write proportions for finding side measures.

$\begin{matrix}\triangle ABC \rightarrow \\ \triangle XYZ \rightarrow\end{matrix} \frac{6}{x} = \frac{2}{3}$	$\begin{matrix}\triangle ABC \rightarrow \\ \triangle XYZ \rightarrow\end{matrix} \frac{8}{y} = \frac{2}{3}$	$\begin{matrix}\triangle ABC \rightarrow \\ \triangle XYZ \rightarrow\end{matrix} \frac{10}{z} = \frac{2}{3}$
$18 = 2x$	$24 = 2y$	$30 = 2z$
$9 = x$	$12 = y$	$15 = z$

The lengths of the sides of $\triangle XYZ$ are 9, 12, and 15 inches.

Example 5 Scale Factors on Maps

MAPS **The scale on the map of New Mexico is 2 centimeters = 160 miles. The distance on the map across New Mexico from east to west through Albuquerque is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour?**



Explore Every 2 centimeters represents 160 miles. The distance across the map is 4.1 centimeters.

Plan Create a proportion relating the measurements to the scale to find the distance in miles. Then use the formula $d = rt$ to find the time.

Solve

$$\begin{array}{rcl} \text{centimeters} \rightarrow & \dfrac{2}{160} = \dfrac{4.1}{x} & \leftarrow \text{centimeters} \\ \text{miles} \rightarrow & & \leftarrow \text{miles} \end{array}$$

$2x = 656$ Cross products

$x = 328$ Divide each side by 2.

The distance across New Mexico is approximately 328 miles.

$d = rt$

$328 = 60t$ $d = 328$ and $r = 60$

$\frac{328}{60} = t$ Divide each side by 60.

$5\frac{7}{15} = t$ Simplify.

It would take $5\frac{7}{15}$ hours or about 5 hours and 28 minutes to drive across New Mexico at an average of 60 miles per hour.

Examine Reexamine the scale. If 2 centimeters = 160 miles, then 4 centimeters = 320 miles. The map is about 4 centimeters wide, so the distance across New Mexico is about 320 miles. The answer is about 5.5 hours and at 60 miles per hour, the trip would be 330 miles. The two distances are close estimates, so the answer is reasonable.

Study Tip

Units of Time

Remember that there are 60 minutes in an hour. When rewriting $\frac{328}{60}$ as a mixed number, you could also write $5\frac{28}{60}$, which means 5 hours 28 minutes.

Check for Understanding

Concept Check

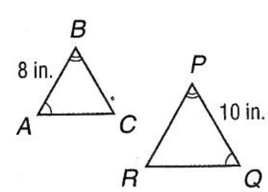
1. **FIND THE ERROR** Roberto and Garrett have calculated their scale factor for two similar triangles.

Roberto

$\frac{AB}{PQ} = \frac{8}{10} = \frac{4}{5}$

Garrett

$\frac{PQ}{AB} = \frac{10}{8} = \frac{5}{4}$



Who is correct? Explain your reasoning.

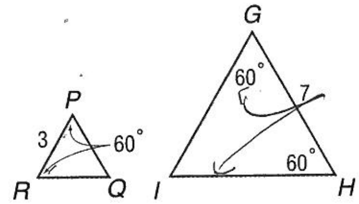
2. **Find a counterexample** for the statement *All rectangles are similar.*

3. **OPEN ENDED** Explain whether two polygons that are congruent are also similar. Then explain whether two polygons that are similar are also congruent.

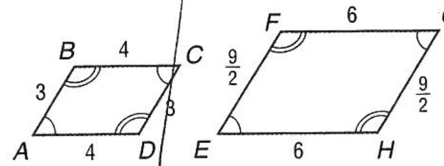
Guided Practice

Determine whether each pair of figures is similar. Justify your answer.

4.

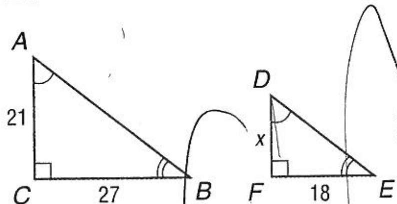


5.

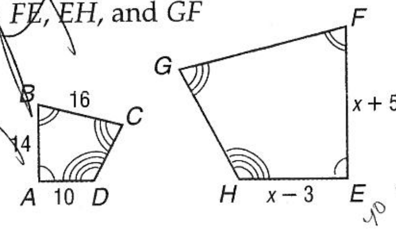


Each pair of polygons is similar. Write a similarity statement, and find x, the measure(s) of the indicated side(s), and the scale factor.

6. $\overline{DF}$



7. $\overline{FE}$, $\overline{EH}$, and $\overline{GF}$



8. A rectangle with length 60 centimeters and height 40 centimeters is reduced so that the new rectangle is similar to the original and the scale factor is $\frac{1}{4}$. Find the length and height of the new rectangle.

9. A triangle has side lengths of 3 meters, 5 meters, and 4 meters. The triangle is enlarged so that the larger triangle is similar to the original and the scale factor is 5. Find the perimeter of the larger triangle.

Application

10. **MAPS** Refer to Example 5 on page 292. Draw the state of New Mexico using a scale of 2 centimeters = 100 miles. Is your drawing similar to the one in Example 4? Explain how you know.

Practice and Apply

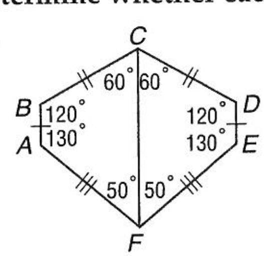
Homework Help

For Exercises	See Examples
11–14	1
15–20, 34–39	2, 3
21–23	4
24–26	5

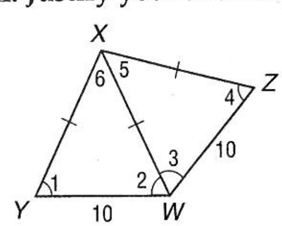
Extra Practice
See page 765.

Determine whether each pair of figures is similar. Justify your answer.

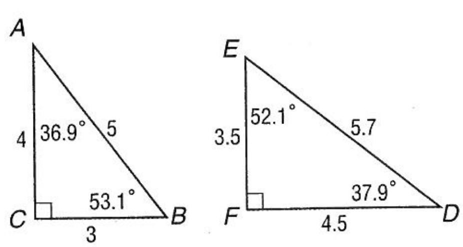
11.



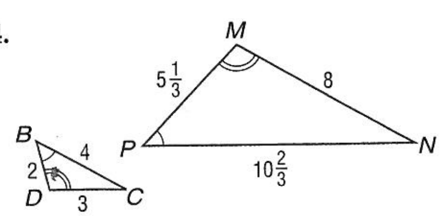
12.



13.



14.

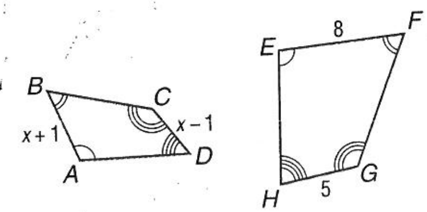


15. **ARCHITECTURE** The replica of the Eiffel Tower at an amusement park is $350\frac{2}{3}$ feet tall. The actual Eiffel Tower is 1052 feet tall. What is the scale factor comparing the amusement park tower to the actual tower?

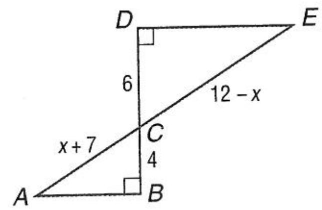
16. **PHOTOCOPYING** Mr. Richardson walked to a copier in his office, made a copy of his proposal, and sent the original to one of his customers. When Mr. Richardson looked at his copy before filing it, he saw that the copy had been made at an 80% reduction. He needs his filing copy to be the same size as the original. What enlargement scale factor must he use on the first copy to make a second copy the same size as the original?

Each pair of polygons is similar. Write a similarity statement, and find x, the measures of the indicated sides, and the scale factor.

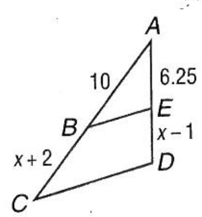
17. $\overline{AB}$ and $\overline{CD}$



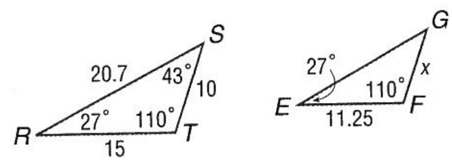
18. $\overline{AC}$ and $\overline{CE}$



19. $\overline{BC}$ and $\overline{ED}$



20. $\overline{GF}$ and $\overline{EG}$



PHOTOGRAPHY **For Exercises 21–23, use the following information.**
A picture is enlarged by a scale factor of $\frac{5}{4}$ and then enlarged again by the same factor.

21. If the original picture was 2.5 inches by 4 inches, what were its dimensions after both enlargements?
22. Write an equation describing the enlargement process.
23. By what scale factor was the original picture enlarged?

More About. . .

Sports

Crew Stadium in Columbus, Ohio, was specifically built for Major League Soccer. The dimensions of the field are about 69 meters by 105 meters.

Source: www.MLSnet.com

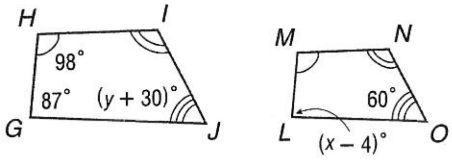
SPORTS **Make a scale drawing of each playing field using the given scale.**

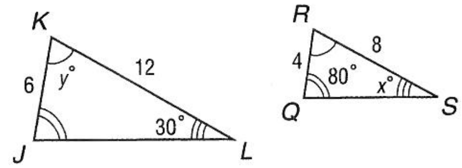
24. Use the information about the soccer field in Crew Stadium. Use the scale 1 millimeter = 1 meter.
25. A basketball court is 84 feet by 50 feet. Use the scale $\frac{1}{4}$ inch = 4 feet.
26. A tennis court is 36 feet by 78 feet. Use the scale $\frac{1}{8}$ inch = 1 foot.

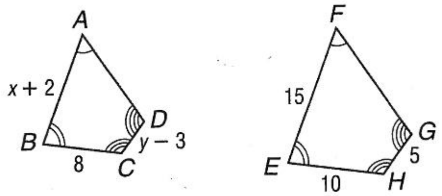
Determine whether each statement is *always*, *sometimes*, or *never* true.

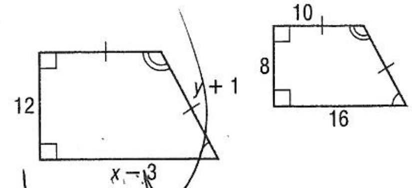
27. Two congruent triangles are similar.
28. Two squares are similar.
29. A triangle is similar to a quadrilateral.
30. Two isosceles triangles are similar.
31. Two rectangles are similar.
32. Two obtuse triangles are similar.
33. Two equilateral triangles are similar.

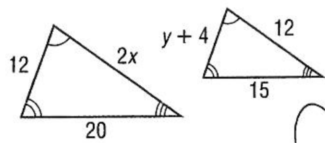
Each pair of polygons is similar. Find x and y. Round to the nearest hundredth if necessary.

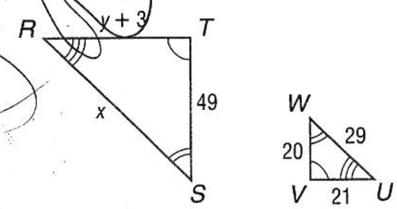
34. 

35. 

36. 

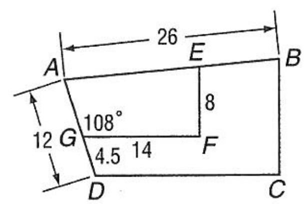
37. 

38. 

39. 

For Exercises 40–47, use the following information to find each measure.

Polygon $ABCD \sim$ polygon $AEFG$, $m\angle AGF = 108$, $GF = 14$, $AD = 12$, $DG = 4.5$, $EF = 8$, and $AB = 26$.



40. scale factor of trapezoid $ABCD$ to trapezoid $AEFG$

41. AG

42. DC

43. $m\angle ADC$

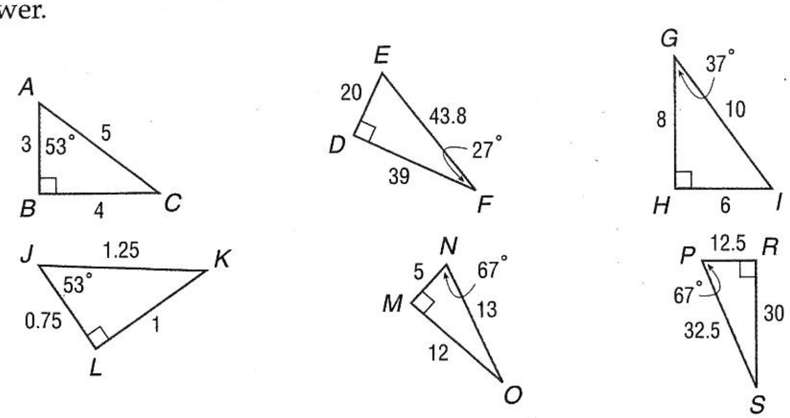
44. BC

45. perimeter of trapezoid $ABCD$

46. perimeter of trapezoid $AEFG$

47. ratio of the perimeter of polygon $ABCD$ to the perimeter of polygon $AEFG$

48. Determine which of the following right triangles are similar. Justify your answer.

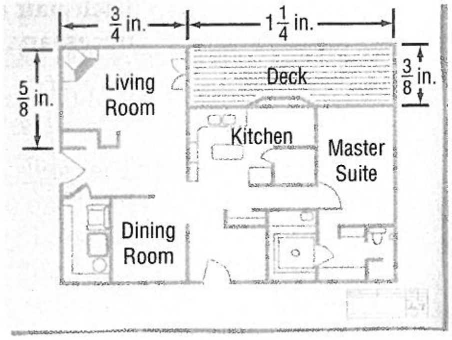


COORDINATE GEOMETRY **Graph the given points. Draw polygon $ABCD$ and $\overline{MN}$. Find the coordinates for vertices L and P such that $ABCD \sim NLPM$.**

49. $A(2, 0)$, $B(4, 4)$, $C(0, 4)$, $D(-2, 0)$; $M(4, 0)$, $N(12, 0)$

50. $A(-7, 1)$, $B(2, 5)$, $C(7, 0)$, $D(-2, -4)$; $M(-3, 1)$, $N\left(-\frac{11}{2}, \frac{7}{2}\right)$

CONSTRUCTION **For Exercises 51 and 52, use the following information.**
A floor plan is given for the first floor of a new house. One inch represents 24 feet. Use the information in the plan to find the dimensions.



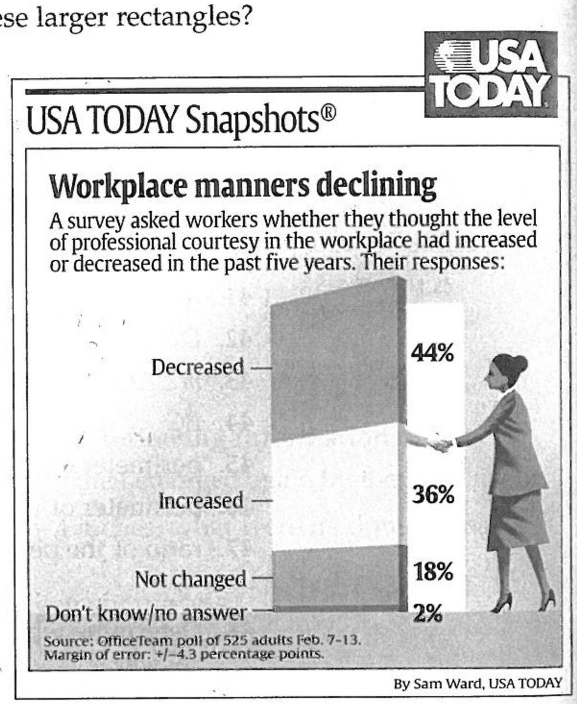
51. living room
52. deck

CRITICAL THINKING **For Exercises 53–55, use the following information.**
The area A of a rectangle is the product of its length ℓ and width w. Rectangle $ABCD$ is similar to rectangle $WXYZ$ with sides in a ratio of 4:1.

53. What is the ratio of the areas of the two rectangles?
54. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?
55. What is the ratio of the areas of these larger rectangles?

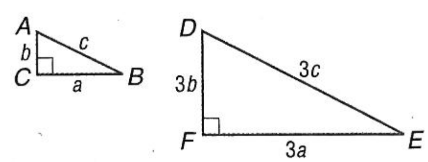
STATISTICS **For Exercises 56–58, refer to the graphic, which uses rectangles to represent percents.**

56. Are the rectangles representing 36% and 18% similar? Explain.
57. What is the ratio of the areas of the rectangles representing 36% and 18% if area = length × width? Compare the ratio of the areas to the ratio of the percents.
58. Use the graph to make a conjecture about the overall changes in the level of professional courtesy in the workplace in the past five years.



CRITICAL THINKING **For Exercises 59 and 60, $\triangle ABC \sim \triangle DEF$.**

59. Show that the perimeters of $\triangle ABC$ and $\triangle DEF$ have the same ratio as their corresponding sides.
60. If 6 units are added to the lengths of each side, are the new triangles similar?



61. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do artists use geometric patterns?

Include the following in your answer:
- why Escher called the picture *Circle Limit IV*, and
- how one of the light objects and one of the dark objects compare in size.

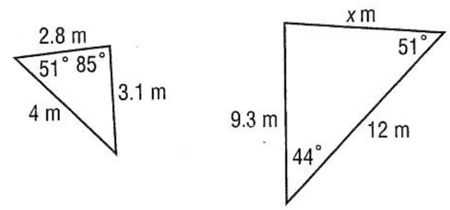
62. In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

Standardized Test Practice

Ⓐ 2 Ⓑ 8 Ⓒ 12 Ⓓ 15

63. ALGEBRA Find x.

Ⓐ 4.2 Ⓑ 4.65
Ⓒ 5.6 Ⓓ 8.4



Extending the Lesson

Scale factors can be used to produce similar figures. The resulting figure is an enlargement or reduction of the original figure depending on the scale factor.

Triangle ABC has vertices $A(0, 0)$, $B(8, 0)$, and $C(2, 7)$. Suppose the coordinates of each vertex are multiplied by 2 to create the similar triangle $A'B'C'$.

64. Find the coordinates of the vertices of $\triangle A'B'C'$.

65. Graph $\triangle ABC$ and $\triangle A'B'C'$.

66. Use the Distance Formula to find the measures of the sides of each triangle.

67. Find the ratios of the sides that appear to correspond.

68. How could you use slope to determine if angles are congruent?

69. Is $\triangle ABC \sim \triangle A'B'C'$? Explain your reasoning.

Maintain Your Skills

Mixed Review

Solve each proportion. *(Lesson 6-1)*

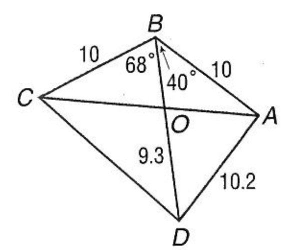
70. $\frac{b}{7.8} = \frac{2}{3}$ **71.** $\frac{c-2}{c+3} = \frac{5}{4}$ **72.** $\frac{2}{4y+5} = \frac{-4}{y}$

Use the figure to write an inequality relating each pair of angle or segment measures. *(Lesson 5-5)*

73. OC, AO

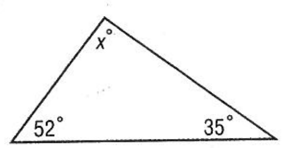
74. $m\angle AOD, m\angle AOB$

75. $m\angle ABD, m\angle ADB$

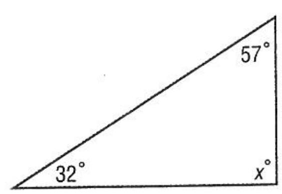


Find x. *(Lesson 4-2)*

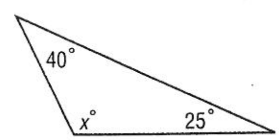
76.



77.



78.



79. Suppose two parallel lines are cut by a transversal and $\angle 1$ and $\angle 2$ are alternate interior angles. Find $m\angle 1$ and $m\angle 2$ if $m\angle 1 = 10x - 9$ and $m\angle 2 = 9x + 3$. *(Lesson 3-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL In the figure, $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \parallel \overline{BD}$, and $m\angle 4 = 118$. Find the measure of each angle. *(To review **angles and parallel lines**, see Lesson 3-2.)*

80. $\angle 1$ **81.** $\angle 2$
82. $\angle 3$ **83.** $\angle 5$
84. $\angle ABD$ **85.** $\angle 6$
86. $\angle 7$ **87.** $\angle 8$