Chapter

5 Relationships in Triangles

What You'll Learn

- **Lesson 5-1** Identify and use perpendicular bisectors, angle bisectors, medians, and altitudes of triangles.
- **Lesson 5-2** Apply properties of inequalities relating to the measures of angles and sides of triangles.
- **Lesson 5-3** Use indirect proof with algebra and geometry.
- **Lessons 5-4 and 5-5** Apply the Triangle Inequality Theorem and SAS and SSS inequalities.

Key Vocabulary

- **perpendicular bisector** (p. 238)
- **median** (p. 240)
- **altitude** (p. 241)
- **indirect proof** (p. 255)

Why It's Important

There are several relationships among the sides and angles of triangles. These relationships can be used to compare the length of a person's stride and the rate at which that person is walking or running. *In Lesson 5-5, you will learn how to use the measure of the sides of a triangle to compare stride and rate.*

Mike Powell/Getty Images

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lesson 5-1 — Midpoint of a Segment

Find the coordinates of the midpoint of a segment with the given endpoints.
(For review, see Lesson 1-3.)

1. $A(-12, -5)$, $B(4, 15)$ **2.** $C(-22, -25)$, $D(10, 10)$ **3.** $E(19, -7)$, $F(-20, -3)$

For Lesson 5-2 — Exterior Angle Theorem

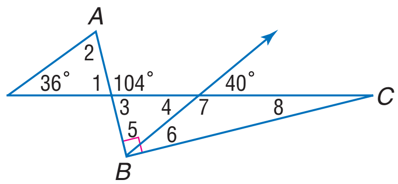
Find the measure of each numbered angle if $\overline{AB} \perp \overline{BC}$. *(For review, see Lesson 4-2.)*

4. $\angle 1$ **5.** $\angle 2$

6. $\angle 3$ **7.** $\angle 4$

8. $\angle 5$ **9.** $\angle 6$

10. $\angle 7$ **11.** $\angle 8$



For Lesson 5-3 — Deductive Reasoning

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment. If a valid conclusion is possible, state it. If a valid conclusion does not follow, write *no conclusion*. *(For review, see Lesson 2-4.)*

12. (1) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.
(2) $\triangle ABC$ and $\triangle PQR$ are congruent.

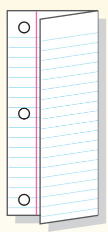
13. (1) The sum of the measures of the angles of a triangle is 180.
(2) Polygon JKL is a triangle.

FOLDABLES™ Study Organizer

Relationships in Triangles Make this Foldable to help you organize your notes. Begin with one sheet of notebook paper.

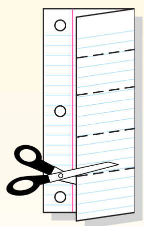
Step 1 Fold

Fold lengthwise to the holes.



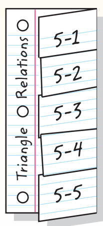
Step 2 Cut

Cut 5 tabs.



Step 3 Label

Label the edge. Then label the tabs using lesson numbers.



Reading and Writing As you read and study each lesson, write notes and examples under the appropriate tab.

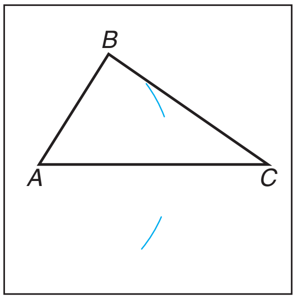
Geometry Activity

Bisectors, Medians, and Altitudes

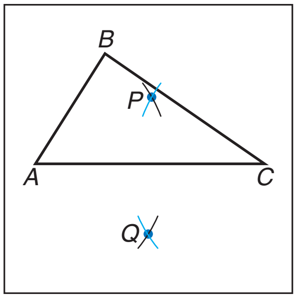
You can use the constructions for midpoint, perpendiculars, and angle bisectors to construct special segments in triangles.

Construction 1 Construct the bisector of a side of a triangle.

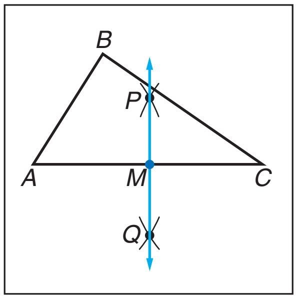
1. Draw a triangle like $\triangle ABC$. Adjust the compass to an opening greater than $\frac{1}{2}AC$. Place the compass at vertex A, and draw an arc above and below $\overline{AC}$.



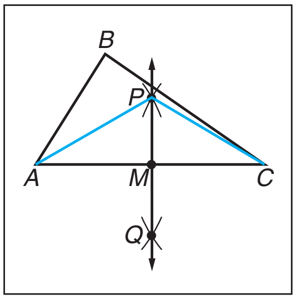
2. Using the same compass settings, place the compass at vertex C. Draw an arc above and below $\overline{AC}$. Label the points of intersection of the arcs P and Q.



3. Use a straightedge to draw $\overleftrightarrow{PQ}$. Label the point where $\overleftrightarrow{PQ}$ bisects $\overline{AC}$ as M.



$\overline{AM} \cong \overline{MC}$ by construction and $\overline{PM} \cong \overline{PM}$ by the Reflexive Property. $\overline{AP} \cong \overline{CP}$ because the arcs were drawn with the same compass setting. Thus, $\triangle APM \cong \triangle CPM$ by SSS. By CPCTC, $\angle PMA \cong \angle PMC$. A linear pair of congruent angles are right angles. So $\overleftrightarrow{PQ}$ is not only a bisector of $\overline{AC}$, but a perpendicular bisector.

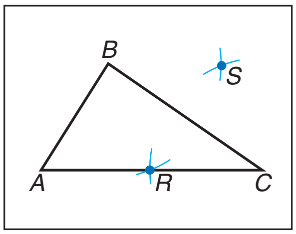


1. Construct the perpendicular bisectors for the other two sides.

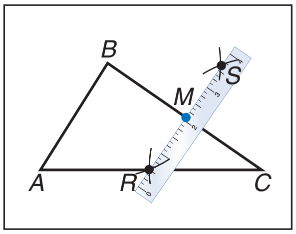
2. What do you notice about the intersection of the perpendicular bisectors?

Construction 2 Construct a median of a triangle.

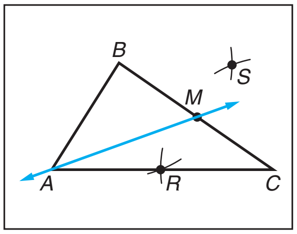
1. Draw intersecting arcs above and below $\overline{BC}$. Label the points of intersection R and S.



2. Use a straightedge to find the point where $\overline{RS}$ intersects $\overline{BC}$. Label the midpoint M.



3. Draw a line through A and M. $\overline{AM}$ is a median of $\triangle ABC$.

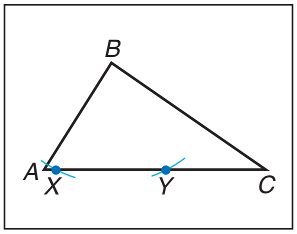


3. Construct the medians of the other two sides.

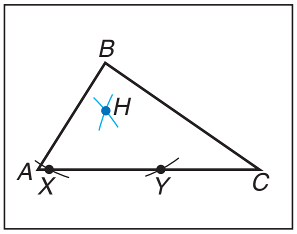
4. What do you notice about the medians of a triangle?

Construction 3 Construct an altitude of a triangle.

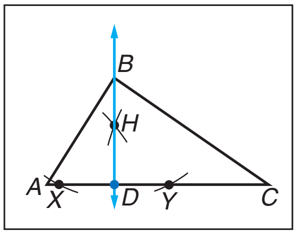
1 Place the compass at vertex *B* and draw two arcs intersecting $\overleftrightarrow{AC}$. Label the points where the arcs intersect the side *X* and *Y*.



2 Adjust the compass to an opening greater than $\frac{1}{2}XY$. Place the compass on *X* and draw an arc above $\overline{AC}$. Using the same setting, place the compass on *Y* and draw an arc above $\overline{AC}$. Label the intersection of the arcs *H*.



3 Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overleftrightarrow{BH}$ intersects $\overline{AC}$ as *D*. $\overline{BD}$ is an altitude of $\triangle ABC$ and is perpendicular to $\overline{AC}$.

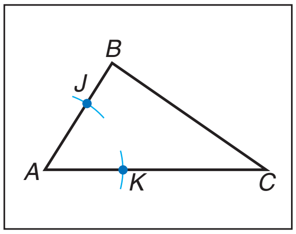


5. Construct the altitudes to the other two sides. (*Hint:* You may need to extend the lines containing the sides of your triangle.)

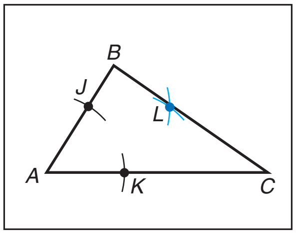
6. What observation can you make about the altitudes of your triangle?

Construction 4 Construct an angle bisector of a triangle.

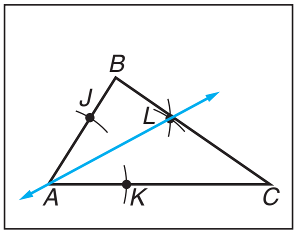
1 Place the compass on vertex *A*, and draw arcs through $\overline{AB}$ and $\overline{AC}$. Label the points where the arcs intersect the sides as *J* and *K*.



2 Place the compass on *J*, and draw an arc. Then place the compass on *K* and draw an arc intersecting the first arc. Label the intersection *L*.



3 Use a straightedge to draw $\overrightarrow{AL}$. $\overrightarrow{AL}$ is an angle bisector of $\triangle ABC$.



7. Construct the angle bisectors for the other two angles.

8. What do you notice about the angle bisectors?

Analyze

9. Repeat the four constructions for each type of triangle.

a. obtuse scalene **b.** right scalene **c.** isosceles **d.** equilateral

Make a Conjecture

10. Where do the lines intersect for acute, obtuse, and right triangles?

11. Under what circumstances do the special lines of triangles coincide with each other?

5-1 Bisectors, Medians, and Altitudes

What You'll Learn

- Identify and use perpendicular bisectors and angle bisectors in triangles.
- Identify and use medians and altitudes in triangles.

How can you balance a paper triangle on a pencil point?

Acrobats and jugglers often balance objects while performing their acts. These skilled artists need to find the center of gravity for each object or body position in order to keep balanced. The center of gravity for any triangle can be found by drawing the *medians* of a triangle and locating the point where they intersect.

Vocabulary

- perpendicular bisector
- concurrent lines
- point of concurrency
- circumcenter
- incenter
- median
- centroid
- altitude
- orthocenter

PERPENDICULAR BISECTORS AND ANGLE BISECTORS The first construction you made in the Geometry Activity on pages 236 and 237 was the perpendicular bisector of a side of a triangle. A **perpendicular bisector** of a side of a triangle is a line, segment, or ray that passes through the midpoint of the side and is perpendicular to that side. Perpendicular bisectors of segments have some special properties.

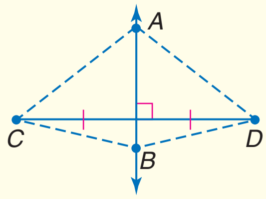
Study Tip

Common Misconception
Note that Theorem 5.2 states the point is on the perpendicular bisector. It does not say that any line containing that point is a perpendicular bisector.

Theorems *Points on Perpendicular Bisectors*

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Example: If $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$, then $AC = AD$ and $BC = BD$.



5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of $\overline{CD}$.
If $BC = BD$, then B lies on the perpendicular bisector of $\overline{CD}$.

You will prove Theorems 5.1 and 5.2 in Exercises 10 and 31, respectively.

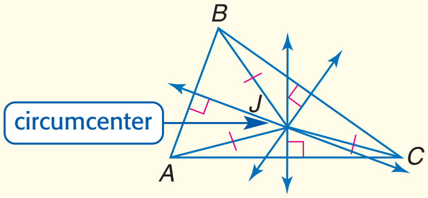
Recall that a locus is the set of all points that satisfy a given condition. A perpendicular bisector can be described as the locus of points in a plane equidistant from the endpoints of a given segment.

Since a triangle has three sides, there are three perpendicular bisectors in a triangle. The perpendicular bisectors of a triangle intersect at a common point. When three or more lines intersect at a common point, the lines are called **concurrent lines**, and their point of intersection is called the **point of concurrency**. The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter**.

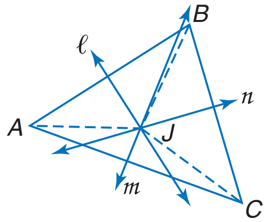
Theorem 5.3

Circumcenter Theorem The circumcenter of a triangle is equidistant from the vertices of the triangle.

Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



Proof *Theorem 5.3*



Given: ℓ, m, and n are perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.

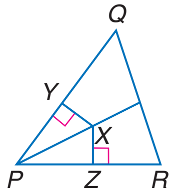
Prove: $AJ = BJ = CJ$

Paragraph Proof:

Since J lies on the perpendicular bisector of $\overline{AB}$, it is equidistant from A and B. By the definition of equidistant, $AJ = BJ$. The perpendicular bisector of $\overline{BC}$ also contains J. Thus, $BJ = CJ$. By the Transitive Property of Equality, $AJ = CJ$. Thus, $AJ = BJ = CJ$.

Another special line, segment, or ray in triangles is an angle bisector.

Example 1 *Use Angle Bisectors*



Given: $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.

Prove: $\overline{XY} \cong \overline{XZ}$

Proof:

Statements	Reasons
1. $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.	**1.** Given
2. $\angle YPX \cong \angle ZPX$	**2.** Definition of angle bisector
3. $\angle PYX$ and $\angle PZX$ are right angles.	**3.** Definition of perpendicular
4. $\angle PYX \cong \angle PZX$	**4.** Right angles are congruent.
5. $\overline{PX} \cong \overline{PX}$	**5.** Reflexive Property
6. $\triangle PYX \cong \triangle PZX$	**6.** AAS
7. $\overline{XY} \cong \overline{XZ}$	**7.** CPCTC

Study Tip

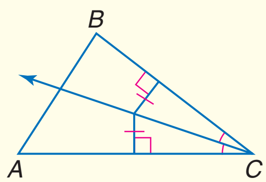
Locus
An angle bisector can be described as the locus of points in a plane equidistant from the sides of an angle. Since the sides of the angle are contained in intersecting lines, the locus of points in a plane equidistant from two intersecting lines is the angle bisector of the vertical angles formed by the lines.

In Example 1, XY and XZ are lengths representing the distance from X to each side of $\angle QPR$. This is a proof of Theorem 5.4.

Theorems *Points on Angle Bisectors*

5.4 Any point on the angle bisector is equidistant from the sides of the angle.

5.5 Any point equidistant from the sides of an angle lies on the angle bisector.



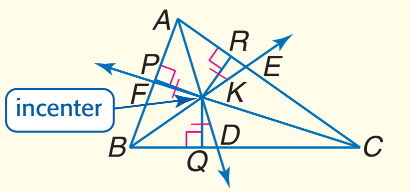
You will prove Theorem 5.5 in Exercise 32.

As with perpendicular bisectors, there are three angle bisectors in any triangle. The angle bisectors of a triangle are concurrent, and their point of concurrency is called the **incenter** of a triangle.

Theorem 5.6

Incenter Theorem The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.



You will prove Theorem 5.6 in Exercise 33.

Study Tip

Medians as Bisectors
Because the median contains the midpoint, it is also a bisector of the side of the triangle.

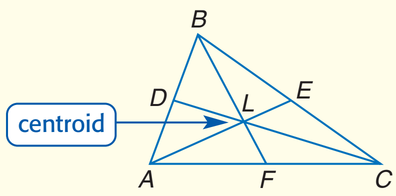
MEDIANS AND ALTITUDES A **median** is a segment whose endpoints are a vertex of a triangle and the midpoint of the side opposite the vertex. Every triangle has three medians.

The medians of a triangle also intersect at a common point. The point of concurrency for the medians of a triangle is called a **centroid**. The centroid is the point of balance for any triangle.

Theorem 5.7

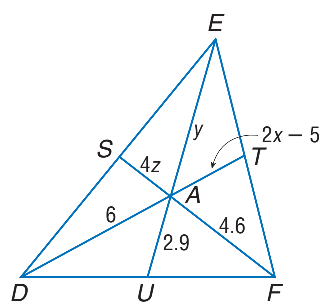
Centroid Theorem The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

Example: If L is the centroid of $\triangle ABC$, $AL = \frac{2}{3}AE$, $BL = \frac{2}{3}BF$, and $CL = \frac{2}{3}CD$.



Example 2 Segment Measures

ALGEBRA **Points S, T, and U are the midpoints of $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$, respectively. Find x, y, and z.**



- Find x.

$DT = DA + AT$	Segment Addition Postulate
$= 6 + (2x - 5)$	Substitution
$= 2x + 1$	Simplify.
$DA = \frac{2}{3}DT$	Centroid Theorem
$6 = \frac{2}{3}[2x + 1]$	$DA = 6$, $DT = 2x + 1$
$18 = 4x + 2$	Multiply each side by 3 and simplify.
$16 = 4x$	Subtract 2 from each side.
$4 = x$	Divide each side by 4.

Study Tip

Eliminating Fractions
You could also multiply the equation $DA = \frac{2}{3}DT$ by 3 to eliminate the denominator.

- Find y.

$$EA = \frac{2}{3}EU \quad \text{Centroid Theorem}$$

$$y = \frac{2}{3}(y + 2.9) \quad EA = y,\ EU = y + 2.9$$

$$3y = 2y + 5.8 \quad \text{Multiply each side by 3 and simplify.}$$

$$y = 5.8 \quad \text{Subtract } 2y \text{ from each side.}$$

- Find z.

$$FA = \frac{2}{3}FS \quad \text{Centroid Theorem}$$

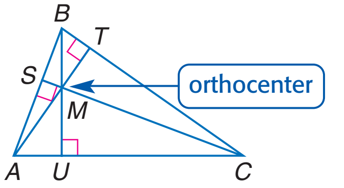
$$4.6 = \frac{2}{3}(4.6 + 4z) \quad FA = 4.6,\ FS = 4.6 + 4z$$

$$13.8 = 9.2 + 8z \quad \text{Multiply each side by 3 and simplify.}$$

$$4.6 = 8z \quad \text{Subtract 9.2 from each side.}$$

$$0.575 = z \quad \text{Divide each side by 8.}$$

An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three altitudes. The intersection point of the altitudes of a triangle is called the **orthocenter**.



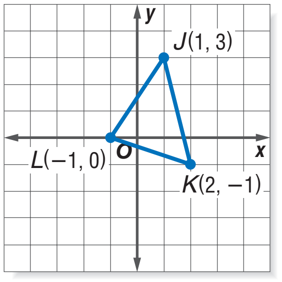
WebQuest

Finding the orthocenter can be used to help you construct your own nine-point circle. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

If the vertices of a triangle are located on a coordinate plane, you can use a system of equations to find the coordinates of the orthocenter.

Example 3 Use a System of Equations to Find a Point

COORDINATE GEOMETRY **The vertices of $\triangle JKL$ are $J(1, 3)$, $K(2, -1)$, and $L(-1, 0)$. Find the coordinates of the orthocenter of $\triangle JKL$.**



- Find an equation of the altitude from J to $\overline{KL}$.

The slope of $\overline{KL}$ is $-\frac{1}{3}$, so the slope of the altitude is 3.

$$(y - y_1) = m(x - x_1) \quad \text{Point-slope form}$$

$$(y - 3) = 3(x - 1) \quad x_1 = 1,\ y_1 = 3,\ m = 3$$

$$y - 3 = 3x - 3 \quad \text{Distributive Property}$$

$$y = 3x \quad \text{Add 3 to each side.}$$

- Next, find an equation of the altitude from K to $\overline{JL}$. The slope of $\overline{JL}$ is $\frac{3}{2}$, so the slope of the altitude to $\overline{JL}$ is $-\frac{2}{3}$.

$$(y - y_1) = m(x - x_1) \quad \text{Point-slope form}$$

$$(y + 1) = -\frac{2}{3}(x - 2) \quad x_1 = 2,\ y_1 = -1,\ m = -\frac{2}{3}$$

$$y + 1 = -\frac{2}{3}x + \frac{4}{3} \quad \text{Distributive Property}$$

$$y = -\frac{2}{3}x + \frac{1}{3} \quad \text{Subtract 1 from each side.}$$

(continued on the next page)

Study Tip

Graphing Calculator
Once you have two equations, you can graph the two lines and use the Intersect option on the Calc menu to determine where the two lines meet.

- Then, solve a system of equations to find the point of intersection of the altitudes.

Find x.

$y = -\frac{2}{3}x + \frac{1}{3}$	Equation of altitude from K
$3x = -\frac{2}{3}x + \frac{1}{3}$	Substitution, $y = 3x$
$9x = -2x + 1$	Multiply each side by 3.
$11x = 1$	Add $2x$ to each side.
$x = \frac{1}{11}$	Divide each side by 11.

Replace x with $\frac{1}{11}$ in one of the equations to find the y-coordinate.

$y = 3\left(\frac{1}{11}\right)$	$x = \frac{1}{11}$
$y = \frac{3}{11}$	Multiply.

The coordinates of the orthocenter of $\triangle JKL$ are $\left(\frac{1}{11}, \frac{3}{11}\right)$.

You can also use systems of equations to find the coordinates of the circumcenter and the centroid of a triangle graphed on a coordinate plane.

Concept Summary — *Special Segments in Triangles*

Name	Type	Point of Concurrency
perpendicular bisector	line, segment, or ray	circumcenter
angle bisector	line, segment, or ray	incenter
median	segment	centroid
altitude	segment	orthocenter

Check for Understanding

Concept Check

1. **Compare and contrast** a perpendicular bisector and a median of a triangle.

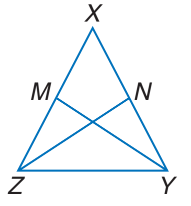
2. **OPEN ENDED** Draw a triangle in which the circumcenter lies outside the triangle.

3. **Find a counterexample** to the statement *An altitude and an angle bisector of a triangle are never the same segment.*

Guided Practice

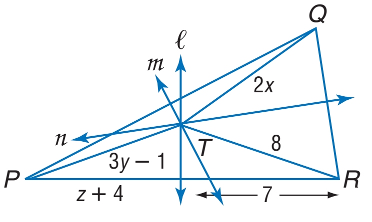
4. **COORDINATE GEOMETRY** The vertices of $\triangle ABC$ are $A(-3, 3)$, $B(3, 2)$, and $C(1, -4)$. Find the coordinates of the circumcenter.

5. **PROOF** Write a two-column proof.
Given: $\overline{XY} \cong \overline{XZ}$
$\overline{YM}$ and $\overline{ZN}$ are medians.
Prove: $\overline{YM} \cong \overline{ZN}$



Application

6. **ALGEBRA** Lines ℓ, m, and n are perpendicular bisectors of $\triangle PQR$ and meet at T. If $TQ = 2x$, $PT = 3y - 1$, and $TR = 8$, find x, y, and z.



Practice and Apply

Homework Help

For Exercises	See Examples
10–12, 31–33	1
13–16, 21–26	2
7–9, 27–30	3

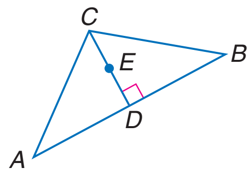
Extra Practice
See page 763.

COORDINATE GEOMETRY **The vertices of $\triangle DEF$ are $D(4, 0)$, $E(-2, 4)$, and $F(0, 6)$. Find the coordinates of the points of concurrency of $\triangle DEF$.**

7. centroid
8. orthocenter
9. circumcenter

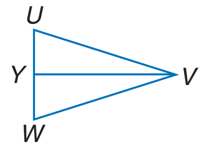
10. **PROOF** Write a paragraph proof of Theorem 5.1.

Given: $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.
E is a point on $\overline{CD}$.
Prove: $EB = EA$

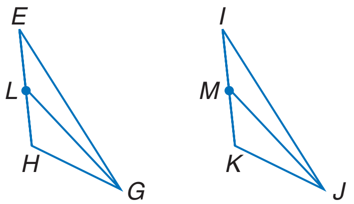


PROOF **Write a two-column proof.**

11. **Given:** $\triangle UVW$ is isosceles with vertex angle UVW.
$\overline{YV}$ is the bisector of $\angle UVW$.
Prove: $\overline{YV}$ is a median.

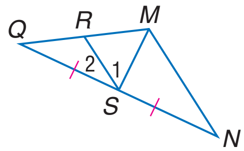


12. **Given:** $\overline{GL}$ is a median of $\triangle EGH$.
$\overline{JM}$ is a median of $\triangle IJK$.
$\triangle EGH \cong \triangle IJK$
Prove: $\overline{GL} \cong \overline{JM}$



13. **ALGEBRA** Find x and $m\angle 2$ if $\overline{MS}$ is an altitude of $\triangle MNQ$, $m\angle 1 = 3x + 11$, and $m\angle 2 = 7x + 9$.

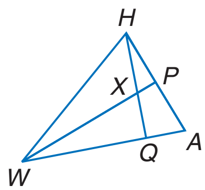
14. **ALGEBRA** If $\overline{MS}$ is a median of $\triangle MNQ$, $QS = 3a - 14$, $SN = 2a + 1$, and $m\angle MSQ = 7a + 1$, find the value of a. Is $\overline{MS}$ also an altitude of $\triangle MNQ$? Explain.



Exercises 13 and 14

15. **ALGEBRA** If $\overline{WP}$ is a median and an angle bisector, $AP = 3y + 11$, $PH = 7y - 5$, $m\angle HWP = x + 12$, $m\angle PAW = 3x - 2$, and $m\angle HWA = 4x - 16$, find x and y. Is $\overline{WP}$ also an altitude? Explain.

16. **ALGEBRA** If $\overline{WP}$ is a perpendicular bisector, $m\angle WHA = 8q + 17$, $m\angle HWP = 10 + q$, $AP = 6r + 4$, and $PH = 22 + 3r$, find r, q, and $m\angle HWP$.



Exercises 15 and 16

State whether each sentence is *always*, *sometimes*, or *never* true.

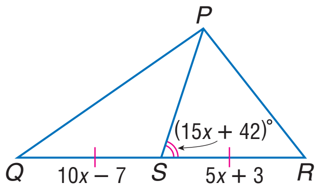
17. The three medians of a triangle intersect at a point in the interior of the triangle.

18. The three altitudes of a triangle intersect at a vertex of the triangle.

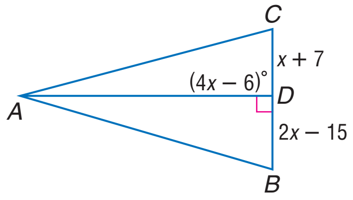
19. The three angle bisectors of a triangle intersect at a point in the exterior of the triangle.

20. The three perpendicular bisectors of a triangle intersect at a point in the exterior of the triangle.

21. **ALGEBRA** Find x if $\overline{PS}$ is a median of $\triangle PQR$.

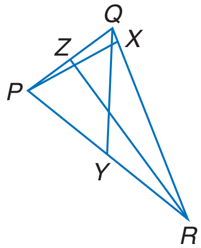


22. **ALGEBRA** Find x if $\overline{AD}$ is an altitude of $\triangle ABC$.



ALGEBRA **For Exercises 23–26, use the following information.**

In $\triangle PQR$, $ZQ = 3a - 11$, $ZP = a + 5$, $PY = 2c - 1$, $YR = 4c - 11$, $m\angle PRZ = 4b - 17$, $m\angle ZRQ = 3b - 4$, $m\angle QYR = 7b + 6$, and $m\angle PXR = 2a + 10$.



23. $\overline{PX}$ is an altitude of $\triangle PQR$. Find a.

24. If $\overline{RZ}$ is an angle bisector, find $m\angle PRZ$.

25. Find PR if $\overline{QY}$ is a median.

26. If $\overleftrightarrow{QY}$ is a perpendicular bisector of $\overline{PR}$, find b.

COORDINATE GEOMETRY **For Exercises 27–30, use the following information.**

$R(3, 3)$, $S(-1, 6)$, and $T(1, 8)$ are the vertices of $\triangle RST$, and $\overline{RX}$ is a median.

27. What are the coordinates of X?

28. Find RX.

29. Determine the slope of $\overleftrightarrow{RX}$.

30. Is $\overline{RX}$ an altitude of $\triangle RST$? Explain.

More About. . .

Orienteering

The International Orienteering Federation World Cup consists of a series of nine races held throughout the world, in which the runners compete for points based on their completion times.

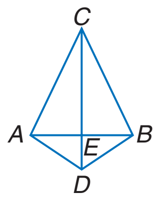
Source: www.orienteering.org

PROOF **Write a two-column proof for each theorem.**

31. Theorem 5.2

Given: $\overline{CA} \cong \overline{CB}$
$\overline{AD} \cong \overline{BD}$

Prove: C and D are on the perpendicular bisector of $\overline{AB}$.



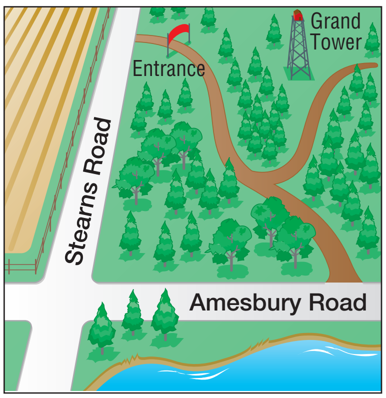
32. Theorem 5.5

33. Theorem 5.6

34. **ORIENTEERING** Orienteering is a competitive sport, originating in Sweden, that tests the skills of map reading and cross-country running. Competitors race through an unknown area to find various checkpoints using only a compass and topographical map. On an amateur course, clues were given to locate the first flag.

- The flag is as far from the Grand Tower as it is from the park entrance.
- If you run from Stearns Road to the flag or from Amesbury Road to the flag, you would run the same distance.

Describe how to find the first flag.



Getty Images

STATISTICS **For Exercises 35–38, use the following information.**

The *mean* of a set of data is an average value of the data. Suppose $\triangle ABC$ has vertices $A(16, 8)$, $B(2, 4)$, and $C(-6, 12)$.

35. Find the mean of the x-coordinates of the vertices.
36. Find the mean of the y-coordinates of the vertices.
37. Graph $\triangle ABC$ and its medians.
38. Make a conjecture about the centroid and the means of the coordinates of the vertices.

39. **CRITICAL THINKING** Draw any $\triangle XYZ$ with median $\overline{XN}$ and altitude $\overline{XO}$. Recall that the area of a triangle is one-half the product of the measures of the base and the altitude. What conclusion can you make about the relationship between the areas of $\triangle XYN$ and $\triangle XZN$?

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you balance a paper triangle on a pencil point?

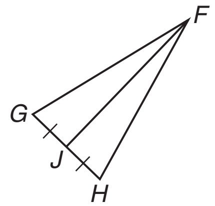
Include the following in your answer:
- which special point is the center of gravity, and
- a construction showing how to find this point.

Standardized Test Practice

41. In $\triangle FGH$, which type of segment is $\overline{FJ}$?

Ⓐ angle bisector　Ⓑ perpendicular bisector

Ⓒ median　Ⓓ altitude



42. **ALGEBRA** If $xy \neq 0$ and $3x = 0.3y$, then $\frac{y}{x} = $ ___?___.

Ⓐ 0.1　Ⓑ 1.0　Ⓒ 3.0　Ⓓ 10.0

Maintain Your Skills

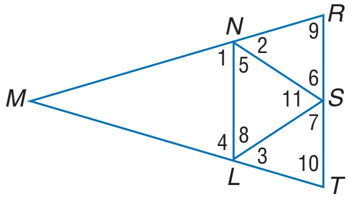
Mixed Review

Position and label each triangle on the coordinate plane. *(Lesson 4-7)*

43. equilateral $\triangle ABC$ with base $\overline{AB}$ n units long
44. isosceles $\triangle DEF$ with congruent sides $2a$ units long and base a units long
45. right $\triangle GHI$ with hypotenuse $\overline{GI}$, HI is three times GH, and GH is x units long

For Exercises 46–49, refer to the figure at the right. *(Lesson 4-6)*

46. If $\angle 9 \cong \angle 10$, name two congruent segments.
47. If $\overline{NL} \cong \overline{SL}$, name two congruent angles.
48. If $\overline{LT} \cong \overline{LS}$, name two congruent angles.
49. If $\angle 1 \cong \angle 4$, name two congruent segments.



50. **INTERIOR DESIGN** Stacey is installing a curtain rod on the wall above the window. To ensure that the rod is parallel to the ceiling, she measures and marks 6 inches below the ceiling in several places. If she installs the rod at these markings centered over the window, how does she know the curtain rod will be parallel to the ceiling? *(Lesson 3-6)*

Getting Ready for the Next Lesson

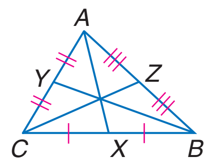
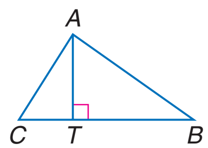
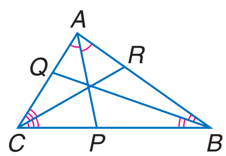
BASIC SKILL **Replace each ● with < or > to make each sentence true.**

51. $\frac{3}{8}$ ● $\frac{5}{16}$　52. 2.7 ● $\frac{5}{3}$　53. -4.25 ● $-\frac{19}{4}$　54. $-\frac{18}{25}$ ● $-\frac{19}{27}$

Reading Mathematics

Math Words and Everyday Words

Several of the words and terms used in mathematics are also used in everyday language. The everyday meaning can help you to better understand the mathematical meaning and help you remember each meaning. This table shows some words used in this chapter with the everyday meanings and the mathematical meanings.

Word	Everyday Meaning	Geometric Meaning	
median	a paved or planted strip dividing a highway into lanes according to direction of travel	a segment of a triangle that connects the vertex to the midpoint of the opposite side	
altitude	the vertical elevation of an object above a surface	a segment from a vertex of a triangle that is perpendicular to the line containing the opposite side	
bisector	something that divides into two usually equal parts	a segment that divides an angle or a side into two parts of equal measure	

Source: *Merriam-Webster Collegiate Dictionary*

Notice that the geometric meaning is more specific, but related to the everyday meaning. For example, the everyday definition of *altitude* is elevation, or height. In geometry, an altitude is a segment of a triangle perpendicular to the base through the vertex. The length of an altitude is the height of the triangle.

Reading to Learn

1. How does the mathematical meaning of *median* relate to the everyday meaning?

2. **RESEARCH** Use a dictionary or other sources to find alternate definitions of *vertex*.

3. **RESEARCH** *Median* has other meanings in mathematics. Use the Internet or other sources to find alternate definitions of this term.

4. **RESEARCH** Use a dictionary or other sources to investigate definitions of *segment*.

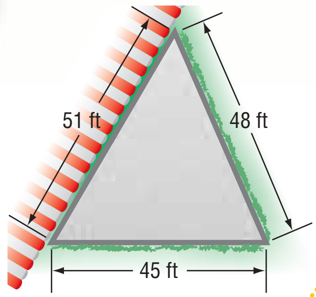
5-2 Inequalities and Triangles

What You'll Learn

- Recognize and apply properties of inequalities to the measures of angles of a triangle.
- Recognize and apply properties of inequalities to the relationships between angles and sides of a triangle.

How can you tell which corner is bigger?

Sam is delivering two potted trees to be used on a patio. The instructions say for the trees to be placed in the two largest corners of the patio. All Sam has is a diagram of the triangular patio that shows the measurements 45 feet, 48 feet, and 51 feet. Sam can find the largest corner because the measures of the angles of a triangle are related to the measures of the sides opposite them.



ANGLE INEQUALITIES In algebra, you learned about the inequality relationship between two real numbers. This relationship is often used in proofs.

Key Concept — *Definition of Inequality*

For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.

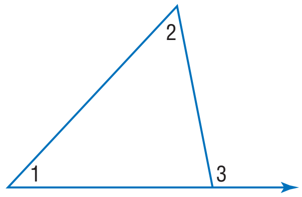
Example: If $6 = 4 + 2$, $6 > 4$ and $6 > 2$.

The properties of inequalities you studied in algebra can be applied to the measures of angles and segments.

Properties of Inequalities for Real Numbers	
	For all numbers a, b, and c
Comparison Property	$a < b$, $a = b$, or $a > b$
Transitive Property	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition and Subtraction Properties	**1.** If $a > b$, then $a + c > b + c$ and $a - c > b - c$. **2.** If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
Multiplication and Division Properties	**1.** If $c > 0$ and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. **2.** If $c > 0$ and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **3.** If $c < 0$ and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **4.** If $c < 0$ and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

Example 1 Compare Angle Measures

Determine which angle has the greatest measure.



Explore Compare the measure of $\angle 3$ to the measures of $\angle 1$ and $\angle 2$.

Plan Use properties and theorems of real numbers to compare the angle measures.

Solve Compare $m\angle 1$ to $m\angle 3$.
By the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$. Since angle measures are positive numbers and from the definition of inequality, $m\angle 3 > m\angle 1$.

Compare $m\angle 2$ to $m\angle 3$.
Again, by the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$. The definition of inequality states that if $m\angle 3 = m\angle 1 + m\angle 2$, then $m\angle 3 > m\angle 2$.

Examine $m\angle 3$ is greater than $m\angle 1$ and $m\angle 2$. Therefore, $\angle 3$ has the greatest measure.

The results from Example 1 suggest that the measure of an exterior angle is always greater than either of the measures of the remote interior angles.

Study Tip

Symbols for Angles and Inequalities

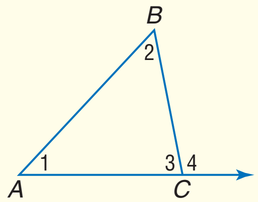
The symbol for angle ($\angle$) looks similar to the symbol for less than ($<$), especially when handwritten. Be careful to write the symbols correctly in situations where both are used.

Theorem 5.8

Exterior Angle Inequality Theorem If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

Example: $m\angle 4 > m\angle 1$

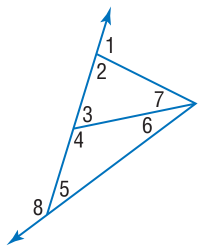
$m\angle 4 > m\angle 2$



The proof of Theorem 5.8 is in Lesson 5-3.

Example 2 Exterior Angles

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



a. all angles whose measures are less than $m\angle 8$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 4$, $m\angle 8 > m\angle 6$, $m\angle 8 > m\angle 2$, and $m\angle 8 > m\angle 6 + m\angle 7$. Thus, the measures of $\angle 4$, $\angle 6$, $\angle 2$, and $\angle 7$ are all less than $m\angle 8$.

b. all angles whose measures are greater than $m\angle 2$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 2$ and $m\angle 4 > m\angle 2$. Thus, the measures of $\angle 4$ and $\angle 8$ are greater than $m\angle 2$.

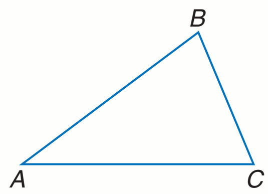
ANGLE-SIDE RELATIONSHIPS Recall that if two sides of a triangle are congruent, then the angles opposite those sides are congruent. In the following Geometry Activity, you will investigate the relationship between sides and angles when they are not congruent.

Geometry Activity

Inequalities for Sides and Angles of Triangles

Model

- Draw an acute scalene triangle, and label the vertices *A*, *B*, and *C*.



- Measure each side of the triangle. Record the measures in a table.

Side	Measure
$\overline{BC}$	
$\overline{AC}$	
$\overline{AB}$	

- Measure each angle of the triangle. Record each measure in a table.

Angle	Measure
$\angle A$	
$\angle B$	
$\angle C$	

Analyze

1. Describe the measure of the angle opposite the longest side in terms of the other angles.
2. Describe the measure of the angle opposite the shortest side in terms of the other angles.
3. Repeat the activity using other triangles.

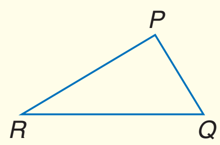
Make a Conjecture

4. What can you conclude about the relationship between the measures of sides and angles of a triangle?

The Geometry Activity suggests the following theorem.

Theorem 5.9

If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.



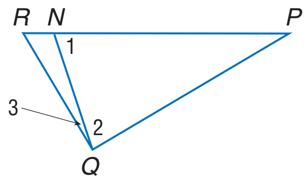
Study Tip

Theorem 5.9
The longest side in a triangle is opposite the largest angle in that triangle.

Proof Theorem 5.9

Given: $\triangle PQR$
$PQ < PR$
$\overline{PN} \cong \overline{PQ}$

Prove: $m\angle R < m\angle PQR$



(continued on the next page)

Proof:

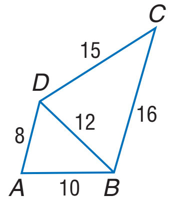
Statements	Reasons
1. $\triangle PQR$, $PQ < PR$, $\overline{PN} \cong \overline{PQ}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Isosceles Triangle Theorem
3. $m\angle 1 = m\angle 2$	3. Definition of congruent angles
4. $m\angle R < m\angle 1$	4. Exterior Angle Inequality Theorem
5. $m\angle 2 + m\angle 3 = m\angle PQR$	5. Angle Addition Postulate
6. $m\angle 2 < m\angle PQR$	6. Definition of inequality
7. $m\angle 1 < m\angle PQR$	7. Substitution Property of Equality
8. $m\angle R < m\angle PQR$	8. Transitive Property of Inequality

Example 3 Side-Angle Relationships

Determine the relationship between the measures of the given angles.

a. $\angle ADB$, $\angle DBA$

The side opposite $\angle ADB$ is longer than the side opposite $\angle DBA$, so $m\angle ADB > m\angle DBA$.



b. $\angle CDA$, $\angle CBA$

$$m\angle DBA < m\angle ADB$$
$$m\angle CBD < m\angle CDB$$
$$m\angle DBA + m\angle CBD < m\angle ADB + m\angle CDB$$
$$m\angle CBA < m\angle CDA$$

More About...

Treehouses

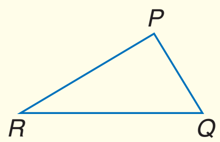
The strength of the tree is the most important concern when building a treehouse. It is important to look for a tree that has branches thick and strong.

Source: www.treehouses.com

The converse of Theorem 5.9 is also true.

Theorem 5.10

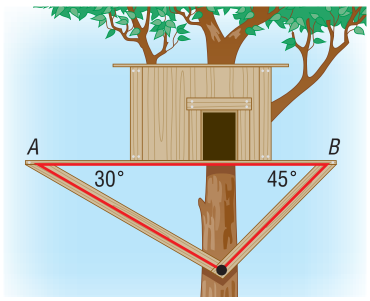
If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.



You will prove Theorem 5.10 in Lesson 5-3, Exercise 26.

Example 4 Angle-Side Relationships

TREEHOUSES **Mr. Jackson is constructing the framework for part of a treehouse for his daughter. He plans to install braces at the ends of a certain floor support as shown. Which supports should he attach to A and B?**

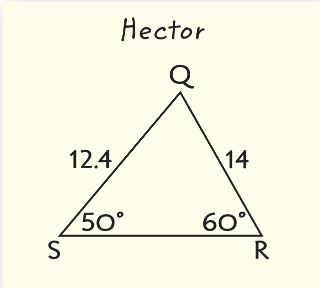


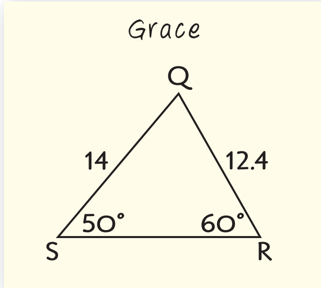
Theorem 5.9 states that if one angle of a triangle has a greater measure, then the side opposite that angle is longer than the side opposite the other angle. Therefore, Mr. Jackson should attach the longer brace at the end marked A and the shorter brace at the end marked B.

Check for Understanding

Concept Check

1. **State** whether the following statement is *always*, *sometimes*, or *never* true. *In $\triangle JKL$ with right angle J, if $m\angle J$ is twice $m\angle K$, then the side opposite $\angle J$ is twice the length of the side opposite $\angle K$.*
2. **OPEN ENDED** Draw $\triangle ABC$. List the angle measures and side lengths of your triangle from greatest to least.
3. **FIND THE ERROR** Hector and Grace each labeled $\triangle QRS$.

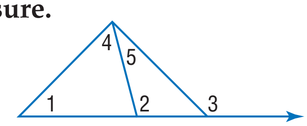




Who is correct? Explain.

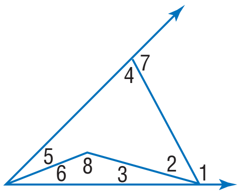
Guided Practice

Determine which angle has the greatest measure.



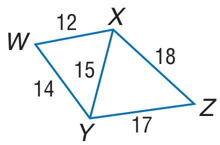
4. $\angle 1, \angle 2, \angle 4$
5. $\angle 2, \angle 3, \angle 5$
6. $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.



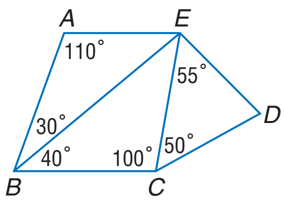
7. all angles whose measures are less than $m\angle 1$
8. all angles whose measures are greater than $m\angle 6$
9. all angles whose measures are less than $m\angle 7$

Determine the relationship between the measures of the given angles.



10. $\angle WXY, \angle XYW$
11. $\angle XZY, \angle XYZ$
12. $\angle WYX, \angle XWY$

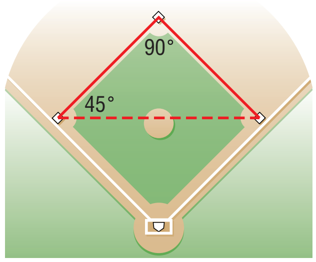
Determine the relationship between the lengths of the given sides.



13. $\overline{AE}, \overline{EB}$
14. $\overline{CE}, \overline{CD}$
15. $\overline{BC}, \overline{EC}$

Application

16. **BASEBALL** During a baseball game, the batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain.



Practice and Apply

Homework Help

For Exercises	See Examples
17–22	1
23–28	2
30–35	3
38–43	4

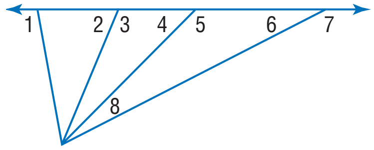
Extra Practice
See page 763.

Determine which angle has the greatest measure.

17. $\angle 1, \angle 2, \angle 4$ **18.** $\angle 2, \angle 4, \angle 6$

19. $\angle 3, \angle 5, \angle 7$ **20.** $\angle 1, \angle 2, \angle 6$

21. $\angle 5, \angle 7, \angle 8$ **22.** $\angle 2, \angle 6, \angle 8$

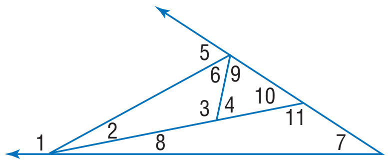


Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.

23. all angles whose measures are less than $m\angle 5$

24. all angles whose measures are greater than $m\angle 6$

25. all angles whose measures are greater than $m\angle 10$

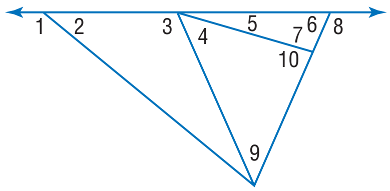


Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.

26. all angles whose measures are less than $m\angle 1$

27. all angles whose measures are greater than $m\angle 9$

28. all angles whose measures are less than $m\angle 8$

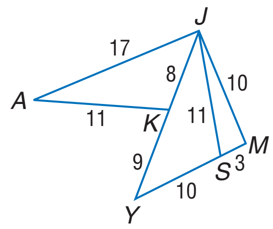


Determine the relationship between the measures of the given angles.

29. $\angle KAJ, \angle AJK$ **30.** $\angle MJY, \angle JYM$

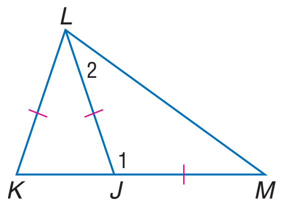
31. $\angle SMJ, \angle MJS$ **32.** $\angle AKJ, \angle JAK$

33. $\angle MYJ, \angle JMY$ **34.** $\angle JSY, \angle JYS$

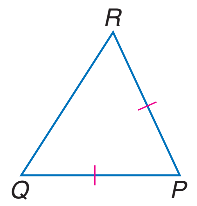


PROOF **Write a two-column proof.**

35. Given: $\overline{JM} \cong \overline{JL}$
$\overline{JL} \cong \overline{KL}$
Prove: $m\angle 1 > m\angle 2$



36. Given: $\overline{PR} \cong \overline{PQ}$
$QR > QP$
Prove: $m\angle P > m\angle Q$

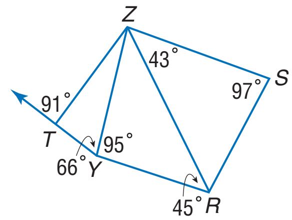


Determine the relationship between the lengths of the given sides.

37. $\overline{ZY}, \overline{YR}$ **38.** $\overline{SR}, \overline{ZS}$

39. $\overline{RZ}, \overline{SR}$ **40.** $\overline{ZY}, \overline{RZ}$

41. $\overline{TY}, \overline{ZY}$ **42.** $\overline{TY}, \overline{ZT}$



43. COORDINATE GEOMETRY Triangle KLM has vertices $K(3, 2)$, $L(-1, 5)$, and $M(-3, -7)$. List the angles in order from the least to the greatest measure.

44. If $AB > AC > BC$ in $\triangle ABC$ and $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are the medians of the triangle, list AM, BN, and CO in order from least to greatest.

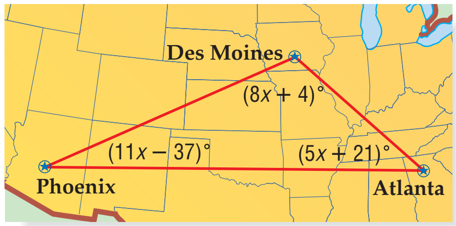
More About...

Travel

One sixth of adult Americans have never flown in a commercial aircraft.

Source: U.S. Bureau of Transportation Statistics

45. **TRAVEL** A plane travels from Des Moines to Phoenix, on to Atlanta, and then completes the trip directly back to Des Moines as shown in the diagram. Write the lengths of the legs of the trip in order from greatest to least.



ALGEBRA **Find the value of n. List the sides of $\triangle PQR$ in order from shortest to longest for the given angle measures.**

46. $m\angle P = 9n + 29$, $m\angle Q = 93 - 5n$, $m\angle R = 10n + 2$

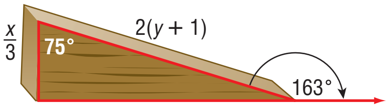
47. $m\angle P = 12n - 9$, $m\angle Q = 62 - 3n$, $m\angle R = 16n + 2$

48. $m\angle P = 9n - 4$, $m\angle Q = 4n - 16$, $m\angle R = 68 - 2n$

49. $m\angle P = 3n + 20$, $m\angle Q = 2n + 37$, $\angle R = 4n + 15$

50. $m\angle P = 4n + 61$, $m\angle Q = 67 - 3n$, $\angle R = n + 74$

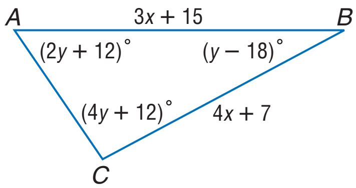
51. **DOORS** The wedge at the right is used as a door stopper. The values of x and y are in inches. Write an inequality relating x and y. Then solve the inequality for y in terms of x.



52. **PROOF** Write a paragraph proof for the following statement.

If a triangle is not isosceles, then the measure of the median to any side of the triangle is greater than the measure of the altitude to that side.

53. **CRITICAL THINKING** Write and solve an inequality for x.



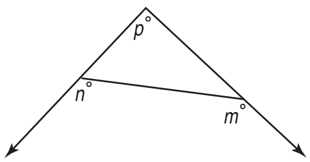
54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you tell which corner is bigger?

Include the following in your answer:
- the name of the theorem or postulate that lets you determine the comparison of the angle measures, and
- which angles in the diagram are the largest.

Standardized Test Practice

55. In the figure at the right, what is the value of p in terms of m and n?



(A) $m + n - 180$

(B) $m + n + 180$

(C) $m - n + 360$

(D) $360 - (m - n)$

56. **ALGEBRA** If $\frac{1}{2}x - 3 = 2\left(\frac{x-1}{5}\right)$, then $x =$ __?__.

(A) 11 (B) 13 (C) 22 (D) 26

Maintain Your Skills

Mixed Review

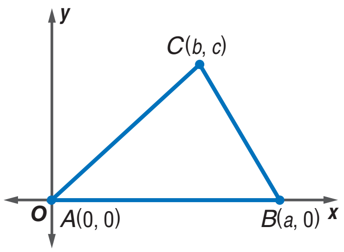
ALGEBRA **For Exercises 57–59, use the following information.** *(Lesson 5-1)*
Two vertices of $\triangle ABC$ are $A(3, 8)$ and $B(9, 12)$. $\overline{AD}$ is a median with D at $(12, 3)$.

57. What are the coordinates of C?

58. Is $\overline{AD}$ an altitude of $\triangle ABC$? Explain.

59. The graph of point E is at $(6, 6)$. $\overline{EF}$ intersects $\overline{BD}$ at F. If F is at $\left(10\frac{1}{2}, 7\frac{1}{2}\right)$, is $\overline{EF}$ a perpendicular bisector of $\overline{BD}$? Explain.

For Exercises 60 and 61, refer to the figure. *(Lesson 4-7)*



60. Find the coordinates of D if the x-coordinate of D is the mean of the x-coordinates of the vertices of $\triangle ABC$ and the y-coordinate is the mean of the y-coordinates of the vertices of $\triangle ABC$.

61. Prove that D is the intersection of the medians of $\triangle ABC$.

Name the corresponding congruent angles and sides for each pair of congruent triangles. *(Lesson 4-3)*

62. $\triangle TUV \cong \triangle XYZ$ **63.** $\triangle CDG \cong \triangle RSW$ **64.** $\triangle BCF \cong \triangle DGH$

65. Find the value of x so that the line containing points at $(x, 2)$ and $(-4, 5)$ is perpendicular to the line containing points at $(4, 8)$ and $(2, -1)$. *(Lesson 3-3)*

Getting Ready for the Next Lesson

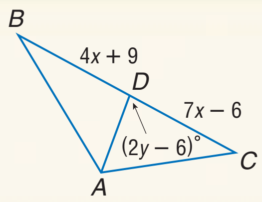
BASIC SKILL **Determine whether each equation is *true* or *false* if $a = 2$, $b = 5$, and $c = 6$.** *(To review **evaluating expressions**, see page 736.)*

66. $2ab = 20$ **67.** $c(b - a) = 15$ **68.** $a + c > a + b$

Practice Quiz 1

Lessons 5-1 and 5-2

ALGEBRA **Use $\triangle ABC$.** *(Lesson 5-1)*



1. Find x if $\overline{AD}$ is a median of $\triangle ABC$.

2. Find y if $\overline{AD}$ is an altitude of $\triangle ABC$.

State whether each statement is *always*, *sometimes*, or *never* true. *(Lesson 5-1)*

3. The medians of a triangle intersect at one of the vertices of the triangle.

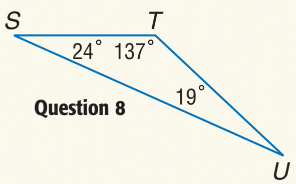
4. The angle bisectors of a triangle intersect at a point in the interior of the triangle.

5. The altitudes of a triangle intersect at a point in the exterior of the triangle.

6. The perpendicular bisectors of a triangle intersect at a point on the triangle.

7. Describe a triangle in which the angle bisectors all intersect in a point outside the triangle. If no triangle exists, write *no triangle*. *(Lesson 5-1)*

8. List the sides of $\triangle STU$ in order from longest to shortest. *(Lesson 5-2)*



Question 8

ALGEBRA **In $\triangle QRS$, $m\angle Q = 3x + 20$, $m\angle R = 2x + 37$, and $m\angle S = 4x + 15$.** *(Lesson 5-2)*

9. Determine the measure of each angle.

10. List the sides in order from shortest to longest.

5-3 Indirect Proof

Why

- Use indirect proof with algebra.
- Use indirect proof with geometry.

How is indirect proof used in literature?

In *The Adventure of the Blanched Soldier*, Sherlock Holmes describes his detective technique, stating, "That process starts upon the supposition that when you have eliminated all which is impossible, then whatever remains, . . . must be the truth." The method Sherlock Holmes uses is an example of *indirect reasoning*.

Vocabulary

- indirect reasoning
- indirect proof
- proof by contradiction

INDIRECT PROOF WITH ALGEBRA The proofs you have written so far use direct reasoning, in which you start with a true hypothesis and prove that the conclusion is true. When using **indirect reasoning**, you assume that the conclusion is false and then show that this assumption leads to a contradiction of the hypothesis, or some other accepted fact, such as a definition, postulate, theorem, or corollary. Since all other steps in the proof are logically correct, the assumption has been proven false, so the original conclusion must be true. A proof of this type is called an **indirect proof** or a **proof by contradiction**.

Study Tip

Truth Value of a Statement
Recall that a statement must be either true or false. To review **truth values**, see Lesson 2-2.

The following steps summarize the process of an indirect proof.

Key Concept *Steps for Writing an Indirect Proof*

1. Assume that the conclusion is false.
2. Show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.
3. Point out that because the false conclusion leads to an incorrect statement, the original conclusion must be true.

Example 1 Stating Conclusions

State the assumption you would make to start an indirect proof of each statement.

a. $AB \neq MN$
$AB = MN$

b. $\triangle PQR$ **is an isosceles triangle.**
$\triangle PQR$ is not an isosceles triangle.

c. $x < 4$
If $x < 4$ is false, then $x = 4$ or $x > 4$. In other words, $x \geq 4$.

d. If 9 is a factor of n**, then 3 is a factor of** n**.**
The conclusion of the conditional statement is *3 is a factor of n*. The negation of the conclusion is *3 is not a factor of n*.

Joshua Ets-Hokin/PhotoDisc

Indirect proofs can be used to prove algebraic concepts.

Example 2 Algebraic Proof

Given: $2x - 3 > 7$

Prove: $x > 5$

Indirect Proof:

Step 1 Assume that $x \leq 5$. That is, assume that $x < 5$ or $x = 5$.

Step 2 Make a table with several possibilities for x given that $x < 5$ or $x = 5$.
This is a contradiction because when $x < 5$ or $x = 5$, $2x - 3 \leq 7$.

x	$2x - 3$
1	−1
2	1
3	4
4	5
5	7

Step 3 In both cases, the assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 5$ must be false, which means that $x > 5$ must be true.

Indirect reasoning and proof can be used in everyday situations.

Example 3 Use Indirect Proof

SHOPPING Lawanda bought two skirts for just over \$60, before tax. A few weeks later, her friend Tiffany asked her how much each skirt cost. Lawanda could not remember the individual prices. Use indirect reasoning to show that at least one of the skirts cost more than \$30.

Given: The two skirts cost more than \$60.

Prove: At least one of the skirts cost more than \$30.
That is, if $x + y > 60$, then either $x > 30$ or $y > 30$.

Indirect Proof:

Step 1 Assume that neither skirt costs more than \$30. That is, $x \leq 30$ and $y \leq 30$.

Step 2 If $x \leq 30$ and $y \leq 30$, then $x + y \leq 60$. This is a contradiction because we know that the two skirts cost more than \$60.

Step 3 The assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 30$ and $y \leq 30$ must be false. Thus, at least one of the skirts had to have cost more than \$30.

More About. . .

Shopping

The West Edmonton Mall in Edmonton, Alberta, Canada, is the world's largest entertainment and shopping center, with an area of 5.3 million square feet. The mall houses an amusement park, water park, ice rink, and aquarium, along with over 800 stores and services.

Source: www.westedmall.com

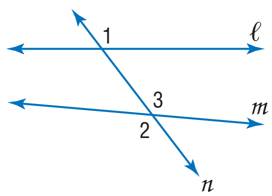
INDIRECT PROOF WITH GEOMETRY

Indirect reasoning can be used to prove statements in geometry.

Example 4 Geometry Proof

Given: $\ell \nparallel m$

Prove: $\angle 1 \not\cong \angle 3$



Indirect Proof:

Step 1 Assume that $\angle 1 \cong \angle 3$.

James Marshall/CORBIS

Step 2 $\angle 1$ and $\angle 3$ are corresponding angles. If two lines are cut by a transversal so that corresponding angles are congruent, the lines are parallel. This means that $\ell \parallel m$. However, this contradicts the given statement.

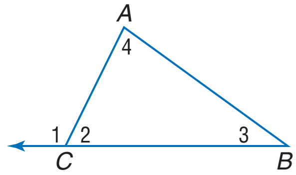
Step 3 Since the assumption leads to a contradiction, the assumption must be false. Therefore, $\angle 1 \not\cong \angle 3$.

Indirect proofs can also be used to prove theorems.

Proof *Exterior Angle Inequality Theorem*

Given: $\angle 1$ is an exterior angle of $\triangle ABC$.

Prove: $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$



Indirect Proof:

Step 1 Make the assumption that $m\angle 1 \not> m\angle 3$ or $m\angle 1 \not> m\angle 4$. In other words, $m\angle 1 \leq m\angle 3$ or $m\angle 1 \leq m\angle 4$.

Step 2 You only need to show that the assumption $m\angle 1 \leq m\angle 3$ leads to a contradiction as the argument for $m\angle 1 \leq m\angle 4$ follows the same reasoning.

$m\angle 1 \leq m\angle 3$ means that either $m\angle 1 = m\angle 3$ or $m\angle 1 < m\angle 3$.

Case 1: $m\angle 1 = m\angle 3$

$m\angle 1 = m\angle 3 + m\angle 4$ Exterior Angle Theorem

$m\angle 3 = m\angle 3 + m\angle 4$ Substitution

$0 = m\angle 4$ Subtract $m\angle 3$ from each side.

This contradicts the fact that the measure of an angle is greater than 0, so $m\angle 1 < m\angle 3$.

Case 2: $m\angle 1 < m\angle 3$

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$. Since angle measures are positive, the definition of inequality implies $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$. This contradicts the assumption.

Step 3 In both cases, the assumption leads to the contradiction of a theorem or definition. Therefore, the assumption that $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$ must be true.

Check for Understanding

Concept Check

1. **Explain** how contradicting a known fact means that an assumption is false.
2. **Compare and contrast** indirect proof and direct proof. See margin.
3. **OPEN ENDED** State a conjecture. Then write an indirect proof to prove your conjecture.

Guided Practice **Write the assumption you would make to start an indirect proof of each statement.**

4. If $5x < 25$, then $x < 5$.
5. Two lines that are cut by a transversal so that alternate interior angles are congruent are parallel.
6. If the alternate interior angles formed by two lines and a transversal are congruent, the lines are parallel.

PROOF **Write an indirect proof.**

7. **Given:** $a > 0$
   Prove: $\frac{1}{a} > 0$
8. **Given:** n is odd.
   Prove: n^2 is odd.
9. **Given:** $\triangle ABC$
   Prove: There can be no more than one obtuse angle in $\triangle ABC$.
10. **Given:** $m \not\parallel n$
    Prove: Lines m and n intersect at exactly one point.
11. **PROOF** Use an indirect proof to show that the hypotenuse of a right triangle is the longest side.

Application

12. **BICYCLING** The Tour de France bicycle race takes place over several weeks in various stages throughout France. During two stages of the 2002 Tour de France, riders raced for just over 270 miles. Prove that at least one of the stages was longer than 135 miles.

Practice and Apply

Homework Help

For Exercises	See Examples
13–18	1
19, 20, 23	2, 3
21, 22, 24, 25	4

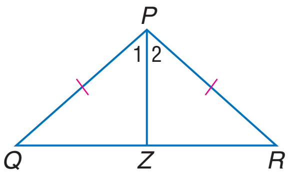
Extra Practice
See page 763.

Write the assumption you would make to start an indirect proof of each statement.

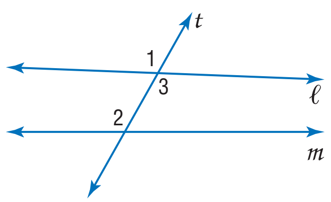
13. $\overline{PQ} \cong \overline{ST}$
14. If $3x > 12$, then $x > 4$.
15. If a rational number is any number that can be expressed as $\frac{a}{b}$, where a and b are integers, and $b \neq 0$, 6 is a rational number.
16. A median of an isosceles triangle is also an altitude.
17. Points P, Q, and R are collinear.
18. The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

PROOF **Write an indirect proof.**

19. **Given:** $\frac{1}{a} < 0$
    Prove: a is negative.
20. **Given:** n^2 is even.
    Prove: n^2 is divisible by 4.
21. **Given:** $\overline{PQ} \cong \overline{PR}$
    $\angle 1 \not\cong \angle 2$
    Prove: $\overline{PZ}$ is not a median of $\triangle PQR$.



22. **Given:** $m\angle 2 \neq m\angle 1$
    Prove: $\ell \not\parallel m$

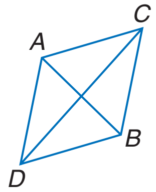


PROOF **Write an indirect proof.**

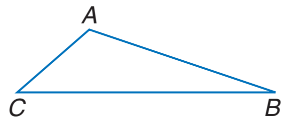
23. If $a > 0$, $b > 0$, and $a > b$, then $\frac{a}{b} > 1$.

24. If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.

25. **Given:** $\triangle ABC$ and $\triangle ABD$ are equilateral.
$\triangle ACD$ is not equilateral.
Prove: $\triangle BCD$ is not equilateral.

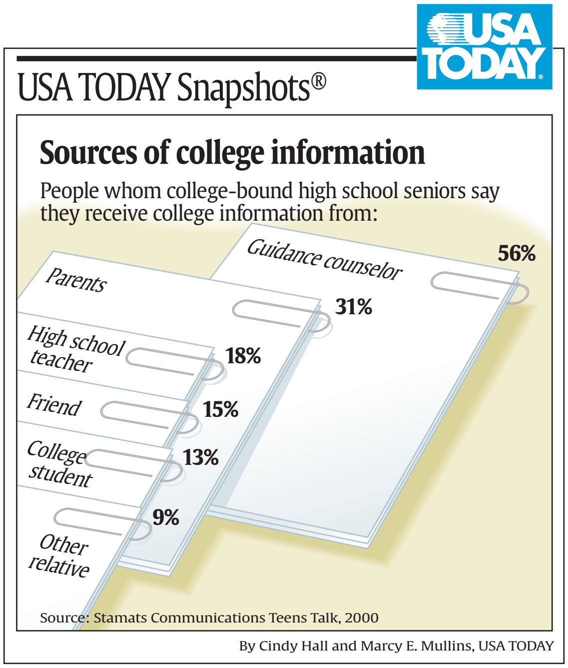


26. Theorem 5.10
Given: $m\angle A > m\angle ABC$
Prove: $BC > AC$



27. **TRAVEL** Ramon drove 175 miles from Seattle, Washington, to Portland, Oregon. It took him three hours to complete the trip. Prove that his average driving speed was less than 60 miles per hour.

EDUCATION **For Exercises 28–30, refer to the graphic at the right.**



28. Prove the following statement.
The majority of college-bound seniors stated that they received college information from a guidance counselor.

29. If 1500 seniors were polled for this survey, verify that 225 said they received college information from a friend.

30. Did more seniors receive college information from their parents or from teachers and friends? Explain.

31. **LAW** During the opening arguments of a trial, a defense attorney stated, "My client is innocent. The police report states that the crime was committed on November 6 at approximately 10:15 A.M. in San Diego. I can prove that my client was on vacation in Chicago with his family at this time. A verdict of not guilty is the only possible verdict." Explain whether this is an example of indirect reasoning.

32. **RESEARCH** Use the Internet or other resource to write an indirect proof for the following statement.
In the Atlantic Ocean, the percent of tropical storms that developed into hurricanes over the past five years varies from year to year.

33. **CRITICAL THINKING** Recall that a rational number is any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers with no common factors and $b \neq 0$, or as a terminating or repeating decimal. Use indirect reasoning to prove that $\sqrt{2}$ is not a rational number.

34. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

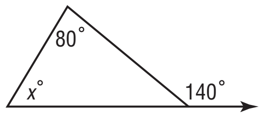
How is indirect proof used in literature?

Include the following in your answer:

- an explanation of how Sherlock Holmes used indirect proof, and
- an example of indirect proof used every day.

Standardized Test Practice

35. Which statement about the value of x is not true?



(A) $x = 60$ (B) $x < 140$

(C) $x + 80 = 140$ (D) $x < 60$

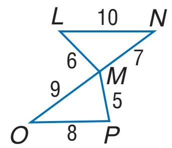
36. **PROBABILITY** A bag contains 6 blue marbles, 8 red marbles, and 2 white marbles. If three marbles are removed at random and no marble is returned to the bag after removal, what is the probability that all three marbles will be red?

(A) $\frac{1}{10}$ (B) $\frac{1}{8}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$

Maintain Your Skills

Mixed Review

For Exercises 37 and 38, refer to the figure at the right.
(Lesson 5-2)



37. Which angle in $\triangle MOP$ has the greatest measure?

38. Name the angle with the least measure in $\triangle LMN$.

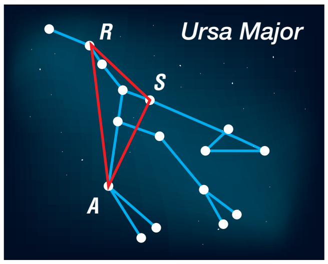
PROOF **Write a two-column proof.** *(Lesson 5-1)*

39. If an angle bisector of a triangle is also an altitude of the triangle, then the triangle is isosceles.

40. The median to the base of an isosceles triangle bisects the vertex angle.

41. Corresponding angle bisectors of congruent triangles are congruent.

42. **ASTRONOMY** The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$.
(Lesson 4-2)



Write an equation in point-slope form of the line having the given slope that contains the given point.
(Lesson 3-4)

43. $m = 2$, $(4, 3)$ 44. $m = -3$, $(2, -2)$ 45. $m = 11$, $(-4, -9)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Determine whether each inequality is *true* or *false*.**
*(To review the **meaning of inequalities**, see pages 739 and 740.)*

46. $19 - 10 < 11$ 47. $31 - 17 < 12$ 48. $38 + 76 > 109$

CONTENTS

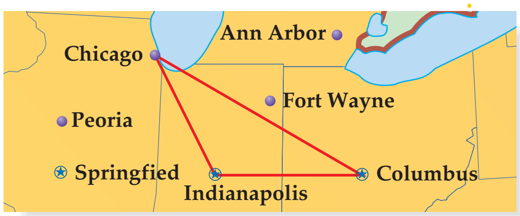
5-4 The Triangle Inequality

What You'll Learn

- Apply the Triangle Inequality Theorem.
- Determine the shortest distance between a point and a line.

How can you use the Triangle Inequality Theorem when traveling?

Chuck Noland travels between Chicago, Indianapolis, and Columbus as part of his job. Mr. Noland lives in Chicago and needs to get to Columbus as quickly as possible. Should he take a flight that goes from Chicago to Columbus, or a flight that goes from Chicago to Indianapolis, then to Columbus?

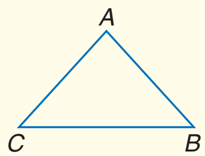


THE TRIANGLE INEQUALITY In the example above, if you chose to fly directly from Chicago to Columbus, you probably reasoned that a straight route is shorter. This is an example of the Triangle Inequality Theorem.

Theorem 5.11

Triangle Inequality Theorem The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Examples:
$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



You will prove Theorem 5.11 in Exercise 40.

The Triangle Inequality Theorem can be used to determine whether three segments can form a triangle.

Study Tip

Inequality
If the sum of the smallest number and the middle number is greater than the largest number, then each combination of inequalities are true.

Example 1 Identify Sides of a Triangle

Determine whether the given measures can be the lengths of the sides of a triangle.

a. 2, 4, 5

Check each inequality.

$2 + 4 \stackrel{?}{>} 5$ $2 + 5 \stackrel{?}{>} 4$ $4 + 5 \stackrel{?}{>} 2$

$6 > 5\ \checkmark$ $7 > 4\ \checkmark$ $9 > 2\ \checkmark$

All of the inequalities are true, so 2, 4, and 5 can be the lengths of the sides of a triangle.

b. 6, 8, 14

$6 + 8 \stackrel{?}{>} 14$

$14 \not> 14$

Because the sum of two measures equals the measure of the third side, the sides cannot form a triangle.

When you know the lengths of two sides of a triangle, you can determine the range of possible lengths for the third side.

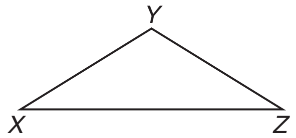
Standardized Test Practice

Example 2 Determine Possible Side Length

Multiple-Choice Test Item

In $\triangle XYZ$, $XY = 8$, and $XZ = 14$.
Which measure cannot be YZ?

Ⓐ 6 Ⓑ 10
Ⓒ 14 Ⓓ 18



Read the Test Item

You need to determine which value is not valid.

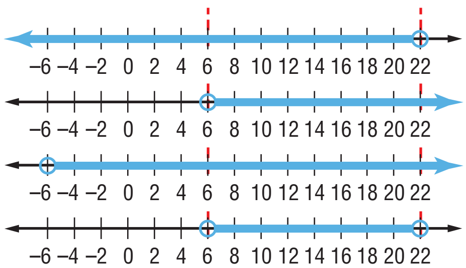
Solve the Test Item

Solve each inequality to determine the range of values for YZ.

Let $YZ = n$.

$XY + XZ > YZ$	$XY + YZ > XZ$	$YZ + XZ > XY$
$8 + 14 > n$	$8 + n > 14$	$n + 14 > 8$
$22 > n$ or $n < 22$	$n > 6$	$n > -6$

Graph the inequalities on the same number line.



Graph $n < 22$.

Graph $n > 6$.

Graph $n > -6$.

Find the intersection.

The range of values that fit all three inequalities is $6 < n < 22$.

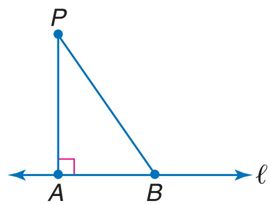
Examine the answer choices. The only value that does not satisfy the compound inequality is 6 since $6 = 6$. Thus, the answer is choice A.

Test-Taking Tip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

DISTANCE BETWEEN A POINT AND A LINE

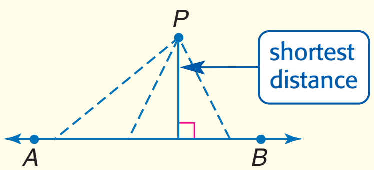
Recall that the distance between point P and line ℓ is measured along a perpendicular segment from the point to the line. It was accepted without proof that $\overline{PA}$ was the shortest segment from P to ℓ. The theorems involving the relationships between the angles and sides of a triangle can now be used to prove that a perpendicular segment is the shortest distance between a point and a line.



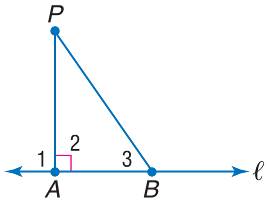
Theorem 5.12

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Example: $\overline{PQ}$ is the shortest segment from P to $\overleftrightarrow{AB}$.



Example 3 Prove Theorem 5.12



Given: $\overline{PA} \perp \ell$
$\overline{PB}$ is any nonperpendicular segment from P to ℓ.

Prove: $PB > PA$

Proof:

Statements	Reasons
1. $\overline{PA} \perp \ell$	1. Given
2. $\angle 1$ and $\angle 2$ are right angles.	2. $\perp$ lines form right angles.
3. $\angle 1 \cong \angle 2$	3. All right angles are congruent.
4. $m\angle 1 = m\angle 2$	4. Definition of congruent angles
5. $m\angle 1 > m\angle 3$	5. Exterior Angle Inequality Theorem
6. $m\angle 2 > m\angle 3$	6. Substitution Property
7. $PB > PA$	7. If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Study Tip

Shortest Distance to a Line
If a line is horizontal, the shortest distance from a point to that line will be along a vertical line. Likewise, the shortest distance from a point to a vertical line lies along a horizontal line.

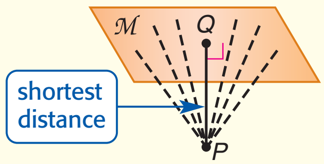
Corollary 5.1 follows directly from Theorem 5.12.

Corollary 5.1

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Example:

$\overline{QP}$ is the shortest segment from P to Plane $\mathcal{M}$.



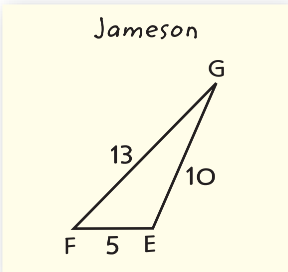
You will prove Corollary 5.1 in Exercise 12.

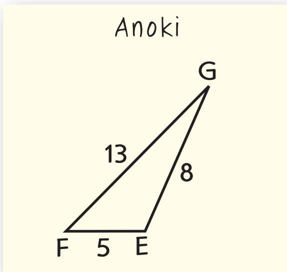
Check for Understanding

Concept Check

1. **Explain** why the distance between two nonhorizontal parallel lines on a coordinate plane cannot be found using the distance between their y-intercepts.

2. **FIND THE ERROR** Jameson and Anoki drew $\triangle EFG$ with $FG = 13$ and $EF = 5$. They each chose a possible measure for GE.





Who is correct? Explain.

3. **OPEN ENDED** Find three numbers that can be the lengths of the sides of a triangle and three numbers that cannot be the lengths of the sides of a triangle. Justify your reasoning with a drawing.

Guided Practice

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

4. 5, 4, 3
5. 5, 15, 10
6. 30.1, 0.8, 31
7. 5.6, 10.1, 5.2

Find the range for the measure of the third side of a triangle given the measures of two sides.

8. 7 and 12
9. 14 and 23
10. 22 and 34
11. 15 and 18

12. **PROOF** Write a proof for Corollary 5.1.
Given: $\overline{PQ} \perp$ plane $\mathcal{M}$
Prove: $\overline{PQ}$ is the shortest segment from P to plane $\mathcal{M}$.

Standardized Test Practice

13. An isosceles triangle has a base 10 units long. If the congruent sides have whole number measures, what is the least possible length of the sides?
(A) 5 (B) 6 (C) 17 (D) 21

Practice and Apply

Homework Help

For Exercises	See Examples
14–25	1
26–37	2
38–40	3

Extra Practice
See page 764.

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

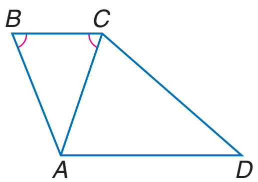
14. 1, 2, 3
15. 2, 6, 11
16. 8, 8, 15
17. 13, 16, 29
18. 18, 32, 21
19. 9, 21, 20
20. 5, 17, 9
21. 17, 30, 30
22. 8.4, 7.2, 3.5
23. 4, 0.9, 4.1
24. 14.3, 12, 2.2
25. 0.18, 0.21, 0.52

Find the range for the measure of the third side of a triangle given the measures of two sides.

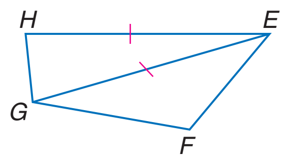
26. 5 and 11
27. 7 and 9
28. 10 and 15
29. 12 and 18
30. 21 and 47
31. 32 and 61
32. 30 and 30
33. 64 and 88
34. 57 and 55
35. 75 and 75
36. 78 and 5
37. 99 and 2

PROOF Write a two-column proof.

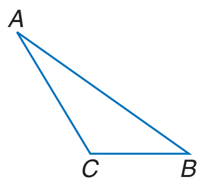
38. **Given:** $\angle B \cong \angle ACB$
Prove: $AD + AB > CD$



39. **Given:** $\overline{HE} \cong \overline{EG}$
Prove: $HE + FG > EF$



40. **Given:** $\angle ABC$
Prove: $AC + BC > AB$ (Triangle Inequality Theorem)
(*Hint:* Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



ALGEBRA **Determine whether the given coordinates are the vertices of a triangle. Explain.**

41. $A(5, 8), B(2, -4), C(-3, -1)$

42. $L(-24, -19), M(-22, 20), N(-5, -7)$

43. $X(0, -8), Y(16, -12), Z(28, -15)$

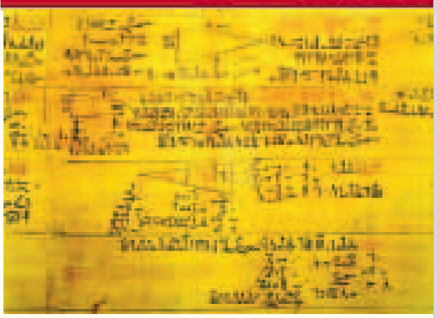
44. $R(1, -4), S(-3, -20), T(5, 12)$

CRAFTS **For Exercises 45 and 46, use the following information.**
Carlota has several strips of trim she wishes to use as a triangular border for a section of a decorative quilt she is going to make. The strips measure 3 centimeters, 4 centimeters, 5 centimeters, 6 centimeters, and 12 centimeters.

45. How many different triangles could Carlota make with the strips?

46. How many different triangles could Carlota make that have a perimeter that is divisible by 3?

More About. . .



History

Ancient Egyptians used pieces of flattened, dried papyrus reed as paper. Surviving examples include the Rhind Papyrus and the Moscow Papyrus, from which we have attained most of our knowledge about Egyptian mathematics.

Source: www.aldokkan.com

47. **HISTORY** The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below?

PROBABILITY **For Exercises 48 and 49, use the following information.**
One side of a triangle is 2 feet long. Let m represent the measure of the second side and n represent the measure of the third side. Suppose m and n are whole numbers and that $14 < m < 17$ and $13 < n < 17$.

48. List the measures of the sides of the triangles that are possible.

49. What is the probability that a randomly chosen triangle that satisfies the given conditions will be isosceles?

50. **CRITICAL THINKING** State and prove a theorem that compares the measures of each side of a triangle with the differences of the measures of the other two sides.

51. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

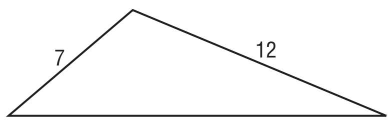
How can you use the Triangle Inequality when traveling?

Include the following in your answer:

- an example of a situation in which you might want to use the greater measures, and
- an explanation as to why it is not always possible to apply the Triangle Inequality when traveling.

Standardized Test Practice

52. If two sides of a triangle measure 12 and 7, which of the following cannot be the perimeter of the triangle?



Ⓐ 29 Ⓑ 34

Ⓒ 37 Ⓓ 38

53. **ALGEBRA** How many points of intersection exist if the equations $(x - 5)^2 + (y - 5)^2 = 4$ and $y = -x$ are graphed on the same coordinate plane?

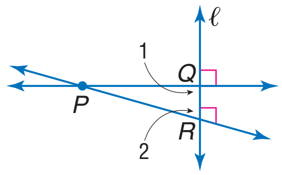
Ⓐ none Ⓑ one Ⓒ two Ⓓ three

Maintain Your Skills

Mixed Review

54. **PROOF** Write an indirect proof. *(Lesson 5-3)*

Given: P is a point not on line ℓ.
Prove: $\overline{PQ}$ is the only line through P perpendicular to ℓ.



ALGEBRA **List the sides of $\triangle PQR$ in order from longest to shortest if the angles of $\triangle PQR$ have the given measures.** *(Lesson 5-2)*

55. $m\angle P = 7x + 8$, $m\angle Q = 8x - 10$, $m\angle R = 7x + 6$

56. $m\angle P = 3x + 44$, $m\angle Q = 68 - 3x$, $m\angle R = x + 61$

Determine whether $\triangle JKL \cong \triangle PQR$ given the coordinates of the vertices. Explain. *(Lesson 4-4)*

57. $J(0, 5)$, $K(0, 0)$, $L(-2, 0)$, $P(4, 8)$, $Q(4, 3)$, $R(6, 3)$

58. $J(6, 4)$, $K(1, -6)$, $L(-9, 5)$, $P(0, 7)$, $Q(5, -3)$, $R(15, 8)$

59. $J(-6, -3)$, $K(1, 5)$, $L(2, -2)$, $P(2, -11)$, $Q(5, -4)$, $R(10, -10)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each inequality.**
*(To review **solving inequalities**, see pages 739 and 740.)*

60. $3x + 54 < 90$ **61.** $8x - 14 < 3x + 19$ **62.** $4x + 7 < 180$

Practice Quiz 2

Lessons 5-3 and 5-4

Write the assumption you would make to start an indirect proof of each statement. *(Lesson 5-3)*

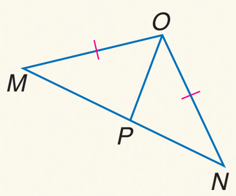
1. The number 117 is divisible by 13.

2. $m\angle C < m\angle D$

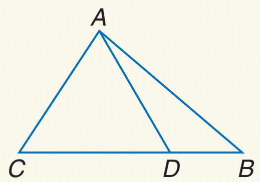
Write an indirect proof. *(Lesson 5-3)*

3. If $7x > 56$, then $x > 8$.

4. Given: $\overline{MO} \cong \overline{ON}$, $\overline{MP} \not\cong \overline{NP}$
Prove: $\angle MOP \not\cong \angle NOP$



5. Given: $m\angle ADC \neq m\angle ADB$
Prove: $\overline{AD}$ is not an altitude of $\triangle ABC$.



Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain. *(Lesson 5-4)*

6. 7, 24, 25 **7.** 25, 35, 60 **8.** 20, 3, 18 **9.** 5, 10, 6

10. If the measures of two sides of a triangle are 57 and 32, what is the range of possible measures of the third side? *(Lesson 5-4)*

CONTENTS

5-5 Inequalities Involving Two Triangles

What You'll Learn

- Apply the SAS Inequality.
- Apply the SSS Inequality.

How does a backhoe work?

Many objects, like a backhoe, have two fixed arms connected by a joint or hinge. This allows the angle between the arms to increase and decrease. As the angle changes, the distance between the endpoints of the arms changes as well.

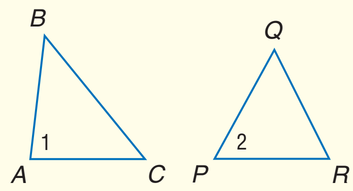
SAS INEQUALITY The relationship of the arms and the angle between them illustrates the following theorem.

Study Tip

SAS Inequality
The SAS Inequality Theorem is also called the *Hinge Theorem*.

Theorem 5.13

SAS Inequality/Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle.

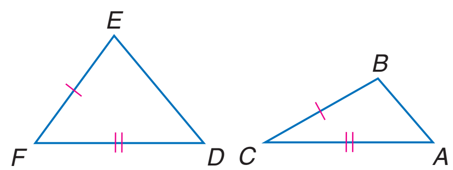


Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $m\angle 1 > m\angle 2$, then $BC > QR$.

Proof SAS Inequality Theorem

Given: $\triangle ABC$ and $\triangle DEF$
$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
$m\angle F > m\angle C$

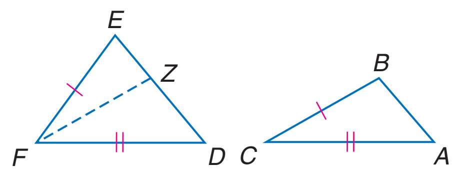
Prove: $DE > AB$



Proof:

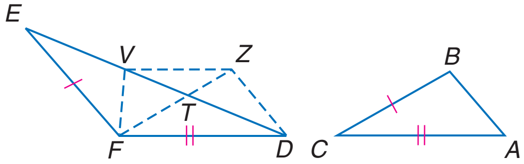
We are given that $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. We also know that $m\angle F > m\angle C$. Draw auxiliary ray FZ such that $m\angle DFZ = m\angle C$ and that $\overline{ZF} \cong \overline{BC}$. This leads to two cases.

Case 1: If Z lies on $\overline{DE}$, then $\triangle FZD \cong \triangle CBA$ by SAS. Thus, $ZD = BA$ by CPCTC and the definition of congruent segments. By the Segment Addition Postulate, $DE = EZ + ZD$. Also, $DE > ZD$ by the definition of inequality. Therefore, $DE > AB$ by the Substitution Property.



Jeremy Walker/Getty Images

Case 2: If Z does not lie on $\overline{DE}$, then let the intersection of $\overline{FZ}$ and $\overline{ED}$ be point T. Now draw another auxiliary segment $\overline{FV}$ such that V is on $\overline{DE}$ and $\angle EFV \cong \angle VFZ$.



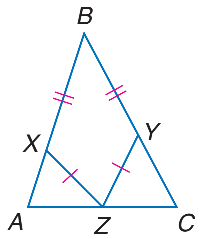
Since $\overline{FZ} \cong \overline{BC}$ and $\overline{BC} \cong \overline{EF}$, we have $\overline{FZ} \cong \overline{EF}$ by the Transitive Property. Also $\overline{VF}$ is congruent to itself by the Reflexive Property. Thus, $\triangle EFV \cong \triangle ZFV$ by SAS. By CPCTC, $\overline{EV} \cong \overline{ZV}$ or $EV = ZV$. Also, $\triangle FZD \cong \triangle CBA$ by SAS. So, $\overline{ZD} \cong \overline{BA}$ by CPCTC or $ZD = BA$. In $\triangle VZD$, $VD + ZV > ZD$ by the Triangle Inequality Theorem. By substitution, $VD + EV > ZD$. Since $ED = VD + EV$ by the Segment Addition Postulate, $ED > ZD$. Using substitution, $ED > BA$ or $DE > AB$.

Example 1 Use SAS Inequality in a Proof

Write a two-column proof.

Given: $\overline{YZ} \cong \overline{XZ}$
Z is the midpoint of $\overline{AC}$.
$m\angle CZY > m\angle AZX$
$\overline{BY} \cong \overline{BX}$

Prove: $BC > AB$



Proof:

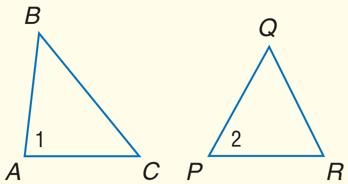
Statements	Reasons
1. $\overline{YZ} \cong \overline{XZ}$; Z is the midpoint of $\overline{AC}$.; $m\angle CZY > m\angle AZX$; $\overline{BY} \cong \overline{BX}$	1. Given
2. $CZ = AZ$	2. Definition of midpoint
3. $CY > AX$	3. SAS Inequality
4. $BY = BX$	4. Definition of congruent segments
5. $CY + BY > AX + BX$	5. Addition Property
6. $BC = CY + BY$; $AB = AX + BX$	6. Segment Addition Postulate
7. $BC > AB$	7. Substitution Property

SSS INEQUALITY

The converse of the SAS Inequality Theorem is the SSS Inequality Theorem.

Theorem 5.14

SSS Inequality If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.



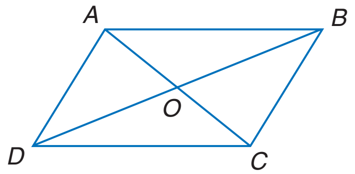
Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $BC > QR$, then $m\angle 1 > m\angle 2$.

You will prove Theorem 5.14 in Exercise 24.

Example 2 Prove Triangle Relationships

Given: $\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{CD}$
$CD > AD$

Prove: $m\angle AOB > m\angle BOC$



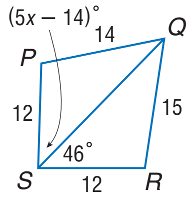
Flow Proof:

- $\overline{AB} \cong \overline{CD}$ — **Given**
- $\overline{AB} \parallel \overline{CD}$ — **Given**
- $\angle BAC \cong \angle ACD$, $\angle ABD \cong \angle BDC$ — Alt. Int. ∠s Th.
- $\triangle AOB \cong \triangle COD$ — ASA
- $\overline{AO} \cong \overline{CO}$ — CPCTC
- $\overline{DO} \cong \overline{DO}$ — Reflexive Property (≅)
- $CD > AD$ — **Given**
- $m\angle COD > m\angle AOD$ — SSS Inequality
- $\angle COD \cong \angle AOB$, $\angle AOD \cong \angle BOC$ — Vert. ∠s are ≅
- $m\angle COD = m\angle AOB$, $m\angle AOD = m\angle BOC$ — Def. of ≅ ∠s
- $m\angle AOB > m\angle BOC$ — Substitution (=)

You can use algebra to relate the measures of the angles and sides of two triangles.

Example 3 Relationships Between Two Triangles

Write an inequality using the information in the figure.



a. Compare $m\angle QSR$ and $m\angle QSP$.

In $\triangle PQS$ and $\triangle RQS$, $\overline{PS} \cong \overline{RS}$, $\overline{QS} \cong \overline{QS}$, and $QR > QP$. The SAS Inequality allows us to conclude that $m\angle QSR > m\angle QSP$.

b. Find the range of values containing x.

By the SSS Inequality, $m\angle QSR > m\angle QSP$, or $m\angle QSP < m\angle QSR$.

$m\angle QSP < m\angle QSR$ — SSS Inequality

$5x - 14 < 46$ — Substitution

$5x < 60$ — Add 14 to each side.

$x < 12$ — Divide each side by 5.

Also, recall that the measure of any angle is always greater than 0.

$5x - 14 > 0$

$5x > 14$ — Add 14 to each side.

$x > \frac{14}{5}$ or 2.8 — Divide each side by 5.

The two inequalities can be written as the compound inequality $2.8 < x < 12$.

More About. . .

Health

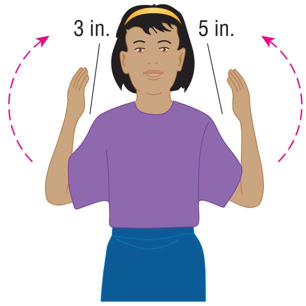
Physical therapists help their patients regain range of motion after an illness or injury.

Source: www.apta.org

Inequalities involving triangles can be used to describe real-world situations.

Example 4 Use Triangle Inequalities

HEALTH **Range of motion describes the amount that a limb can be moved from a straight position. To determine the range of motion of a person's forearm, determine the distance from his or her wrist to the shoulder when the elbow is bent as far as possible. Suppose Jessica can bend her left arm so her wrist is 5 inches from her shoulder and her right arm so her right wrist is 3 inches from her shoulder. Which of Jessica's arms has the greater range of motion? Explain.**



The distance between the wrist and shoulder is smaller on her right arm. Assuming that both her arms have the same measurements, the SSS inequality tells us that the angle formed at the elbow is smaller on the right arm. This means that the right arm has a greater range of motion.

Check for Understanding

Concept Check

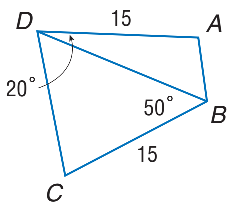
1. **OPEN ENDED** **Describe** a real-world object that illustrates either SAS or SSS inequality.

2. **Compare and contrast** the SSS Inequality Theorem to the SSS Postulate for triangle congruence.

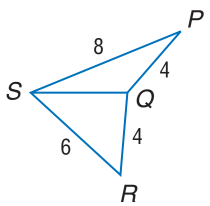
Guided Practice

Write an inequality relating the given pair of angles or segment measures.

3. AB, CD

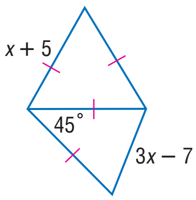


4. $m\angle PQS$, $m\angle RQS$

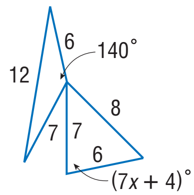


Write an inequality to describe the possible values of x.

5.

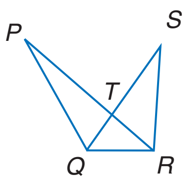


6.

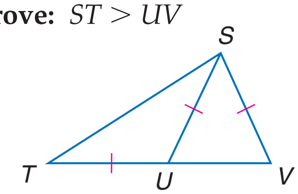


PROOF **Write a two-column proof.**

7. **Given:** $\overline{PQ} \cong \overline{SQ}$
Prove: $PR > SR$

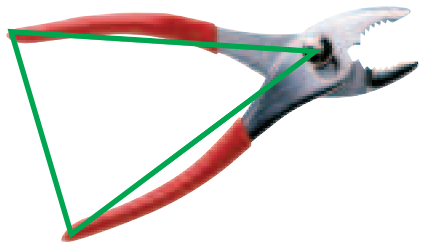


8. **Given:** $\overline{TU} \cong \overline{US}$
$\overline{US} \cong \overline{SV}$
Prove: $ST > UV$



Application

9. **TOOLS** A lever is used to multiply the force applied to an object. One example of a lever is a pair of pliers. Use the SAS or SSS Inequality to explain how to use a pair of pliers.



Practice and Apply

Homework Help

For Exercises	See Examples
20–24	1–2
10–19	3
25, 26	4

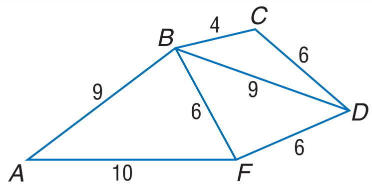
Extra Practice
See page 764.

Write an inequality relating the given pair of angles or segment measures.

10. AB, FD

11. $m\angle BDC$, $m\angle FDB$

12. $m\angle FBA$, $m\angle DBF$

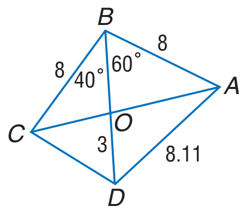


Write an inequality relating the given pair of angles or segment measures.

13. AD, DC

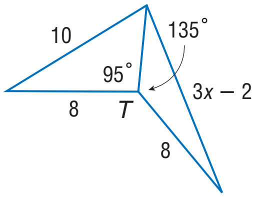
14. OC, OA

15. $m\angle AOD$, $m\angle AOB$

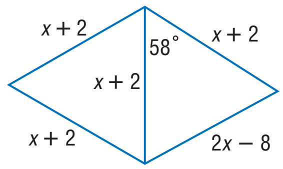


Write an inequality to describe the possible values of x.

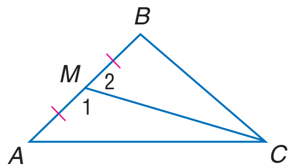
16.



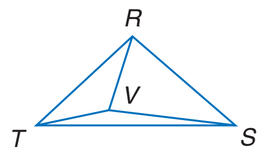
17.



18. In the figure, $\overline{AM} \cong \overline{MB}$, $AC > BC$, $m\angle 1 = 5x + 20$ and $m\angle 2 = 8x - 100$. Write an inequality to describe the possible values of x.

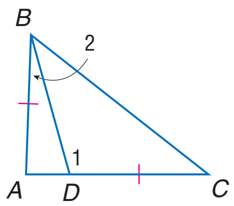


19. In the figure, $m\angle RVS = 15 + 5x$, $m\angle SVT = 10x - 20$, $RS < ST$, and $\angle RTV \cong \angle TRV$. Write an inequality to describe the possible values of x.

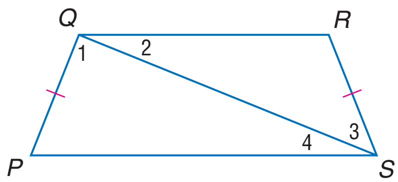


PROOF **Write a two-column proof.**

20. **Given:** $\triangle ABC$
$\overline{AB} \cong \overline{CD}$
Prove: $BC > AD$

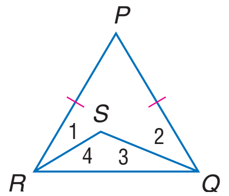


21. **Given:** $\overline{PQ} \cong \overline{RS}$
$QR < PS$
Prove: $m\angle 3 < m\angle 1$

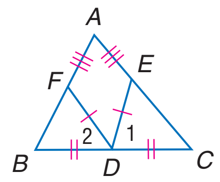


PROOF **Write a two-column proof.**

22. Given: $\overline{PR} \cong \overline{PQ}$
$SQ > SR$
Prove: $m\angle 1 < m\angle 2$

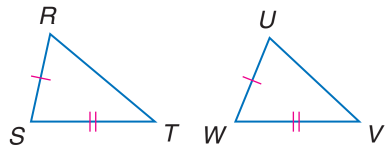


23. Given: $\overline{ED} \cong \overline{DF}$
$m\angle 1 > m\angle 2$
D is the midpoint of $\overline{CB}$.
$\overline{AE} \cong \overline{AF}$
Prove: $AC > AB$



24. **PROOF** Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

Given: $\overline{RS} \cong \overline{UW}$
$\overline{ST} \cong \overline{WV}$
$RT > UV$
Prove: $m\angle S > m\angle W$



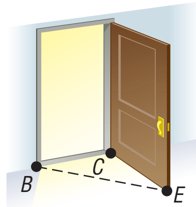
Career Choices

Landscape Architect

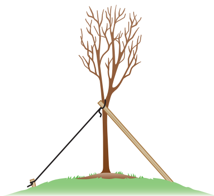
Landscape architects design the settings of buildings and parklands by arranging both the location of the buildings and the placement of plant life so the site is functional, beautiful, and environmentally friendly.

Online Research
For information about a career as a landscape architect, visit: www.geometryonline.com/careers

25. DOORS Open a door slightly. With the door open, measure the angle made by the door and the door frame. Measure the distance from the end of the door to the door frame. Open the door wider, and measure again. How do the measures compare?



26. LANDSCAPING When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for supporting a newly planted tree.



27. CRITICAL THINKING The SAS Inequality states that the base of an isosceles triangle gets longer as the measure of the vertex angle increases. Describe the effect of changing the measure of the vertex angle on the measure of the altitude.

BIOLOGY **For Exercises 28–30, use the following information.**

The velocity of a person walking or running can be estimated using the formula $v = \frac{0.78s^{1.67}}{h^{1.17}}$, where v is the velocity of the person in meters per second, s is the length of the stride in meters, and h is the height of the hip in meters.

28. Find the velocities of two people that each have a hip height of 0.85 meters and whose strides are 1.0 meter and 1.2 meters.

29. Copy and complete the table at the right for a person whose hip height is 1.1 meters.

30. Discuss how the stride length is related to either the SAS Inequality of the SSS Inequality.

Stride (m)	Velocity (m/s)
0.25	
0.50	
0.75	
1.00	
1.25	
1.50	

Rachel Epstein/PhotoEdit

31. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

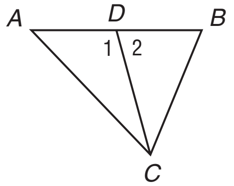
How does a backhoe work?

Include the following in your answer:

- a description of the angle between the arms as the backhoe operator digs, and
- an explanation of how the distance between the ends of the arms is related to the angle between them.

Standardized Test Practice

32. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is *not* true?



Ⓐ $AD = BD$ Ⓑ $m\angle ADC = m\angle BDC$

Ⓒ $AC > BC$ Ⓓ $m\angle 1 > m\angle B$

33. **ALGEBRA** A student bought four college textbooks that cost \$99.50, \$88.95, \$95.90, and \$102.45. She paid one half of the total amount herself and borrowed the rest from her mother. She repaid her mother in 4 equal monthly payments. How much was each of the monthly payments?

Ⓐ \$24.18 Ⓑ \$48.35 Ⓒ \$96.70 Ⓓ \$193.40

Maintain Your Skills

Mixed Review

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain. *(Lesson 5-4)*

34. 25, 1, 21 **35.** 16, 6, 19 **36.** 8, 7, 15

Write the assumption you would make to start an indirect proof of each statement. *(Lesson 5-3)*

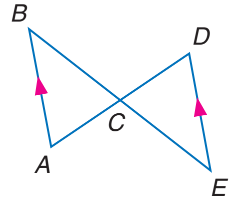
37. $\overline{AD}$ is a median of $\triangle ABC$.

38. If two altitudes of a triangle are congruent, then the triangle is isosceles.

Write a proof. *(Lesson 4-5)*

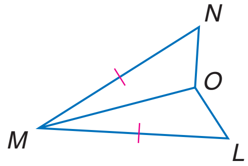
39. Given: $\overline{AD}$ bisects $\overline{BE}$.
$\overline{AB} \parallel \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$



40. Given: $\overline{OM}$ bisects $\angle LMN$.
$\overline{LM} \cong \overline{MN}$

Prove: $\triangle MOL \cong \triangle MON$



Find the measures of the sides of $\triangle EFG$ with the given vertices and classify each triangle by its sides. *(Lesson 4-1)*

41. $E(4, 6)$, $F(4, 11)$, $G(9, 6)$ **42.** $E(-7, 10)$, $F(15, 0)$, $G(-2, -1)$

43. $E(16, 14)$, $F(7, 6)$, $G(-5, -14)$ **44.** $E(9, 9)$, $F(12, 14)$, $G(14, 6)$

45. **ADVERTISING** An ad for Wildflowers Gift Boutique says *When it has to be special, it has to be Wildflowers.* Catalina needs a special gift. Does it follow that she should go to Wildflowers? Explain. *(Lesson 2-4)*

Chapter 5 Study Guide and Review

Vocabulary and Concept Check

altitude (p. 241)	incenter (p. 240)	orthocenter (p. 241)
centroid (p. 240)	indirect proof (p. 255)	perpendicular bisector (p. 238)
circumcenter (p. 238)	indirect reasoning (p. 255)	point of concurrency (p. 238)
concurrent lines (p. 238)	median (p. 240)	proof by contradiction (p. 255)

For a complete list of postulates and theorems, see pages R1–R8.

Exercises **Choose the correct term to complete each sentence.**

1. All of the angle bisectors of a triangle meet at the *(incenter, circumcenter)*.
2. In $\triangle RST$, if point P is the midpoint of $\overline{RS}$, then $\overline{PT}$ is a(n) *(angle bisector, median)*.
3. The theorem that the sum of the lengths of two sides of a triangle is greater than the length of the third side is the *(Triangle Inequality Theorem, SSS Inequality)*.
4. The three medians of a triangle intersect at the *(centroid, orthocenter)*.
5. In $\triangle JKL$, if point H is equidistant from $\overrightarrow{KJ}$ and $\overrightarrow{KL}$, then $\overleftrightarrow{HK}$ is an *(angle bisector, altitude)*.
6. The circumcenter of a triangle is the point where all three *(perpendicular bisectors, medians)* of the sides of the triangle intersect.
7. In $\triangle ABC$, if $\overleftrightarrow{AK} \perp \overleftrightarrow{BC}$, $\overleftrightarrow{BK} \perp \overleftrightarrow{AC}$, and $\overleftrightarrow{CK} \perp \overleftrightarrow{AB}$, then K is the *(orthocenter, incenter)* of $\triangle ABC$.

Lesson-by-Lesson Review

5-1 Bisectors, Medians, and Altitudes

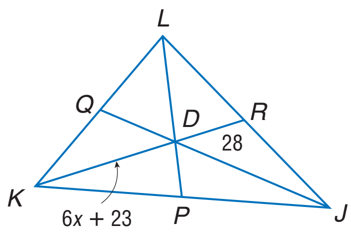
See pages 238–245.

Concept Summary

- The perpendicular bisectors, angle bisectors, medians, and altitudes of a triangle are all special segments in triangles.

Example **Points P, Q, and R are the midpoints of $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$, respectively. Find x.**

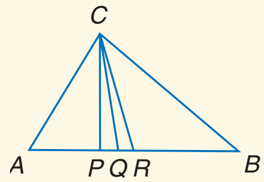
$KD = \frac{2}{3}(KR)$	Centroid Theorem
$6x + 23 = \frac{2}{3}(6x + 51)$	Substitution
$6x + 23 = 4x + 34$	Simplify.
$2x = 11$	Subtract $4x + 23$ from each side.
$x = \frac{11}{2}$	Divide each side by 2.



Exercises **In the figure, $\overline{CP}$ is an altitude, $\overline{CQ}$ is the angle bisector of $\angle ACB$, and R is the midpoint of $\overline{AB}$.**

See Example 2 on pages 240 and 241.

8. Find $m\angle ACQ$ if $m\angle ACB = 123 - x$ and $m\angle QCB = 42 + x$.
9. Find AB if $AR = 3x + 6$ and $RB = 5x - 14$.
10. Find x if $m\angle APC = 72 + x$.



www.geometryonline.com/vocabulary_review

5-2 Inequalities and Triangles

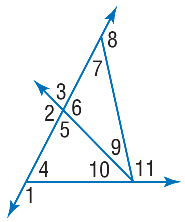
See pages 247–254.

Concept Summary

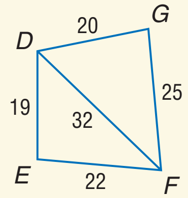
- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.

Example **Use the Exterior Angle Theorem to list all angles with measures less than $m\angle 1$.**

By the Exterior Angle Theorem, $m\angle 5 < m\angle 1$, $m\angle 10 < m\angle 1$, $m\angle 7 < m\angle 1$, and $m\angle 9 + m\angle 10 < m\angle 1$. Thus, the measures of $\angle 5$, $\angle 10$, $\angle 7$, and $\angle 9$ are all less than $m\angle 1$.



Exercises **Determine the relationship between the measures of the given angles.** *See Example 3 on page 250.*

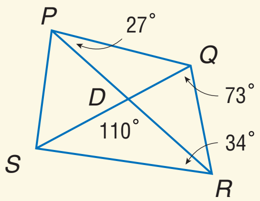


11. $\angle DEF$ and $\angle DFE$

12. $\angle GDF$ and $\angle DGF$

13. $\angle DEF$ and $\angle FDE$

Determine the relationship between the lengths of the given sides. *See Example 4 on page 250.*



14. $\overline{SR}, \overline{SD}$

15. $\overline{DQ}, \overline{DR}$

16. $\overline{PQ}, \overline{QR}$

17. $\overline{SR}, \overline{SQ}$

5-3 Indirect Proof

See pages 255–260.

Concept Summary

- In an indirect proof, the conclusion is assumed to be false, and a contradiction is reached.

Example **State the assumption you would make to start an indirect proof of the statement $AB < AC + BC$.**

If AB is not less than $AC + BC$, then either $AB > AC + BC$ or $AB = AC + BC$. In other words, $AB \geq AC + BC$.

Exercises **State the assumption you would make to start an indirect proof of each statement.** *See Example 1 on page 255.*

18. $\sqrt{2}$ is an irrational number.

19. If two sides and the included angle are congruent in two triangles, then the triangles are congruent.

20. **FOOTBALL** Miguel plays quarterback for his high school team. This year, he completed 101 passes in the five games in which he played. Prove that, in at least one game, Miguel completed more than 20 passes.

• Extra Practice, see pages 763 and 764.
• Mixed Problem Solving, see page 786.

5-4 The Triangle Inequality

See pages 261–266.

Concept Summary

- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example **Determine whether 7, 6, and 14 can be the measures of the sides of a triangle.**

Check each inequality.

$7 + 6 \overset{?}{>} 14$ $\qquad$ $7 + 14 \overset{?}{>} 6$ $\qquad$ $6 + 14 \overset{?}{>} 7$

$13 \not> 14$ $\qquad$ $21 > 6 \checkmark$ $\qquad$ $20 > 7 \checkmark$

Because the inequalities are not true in all cases, the sides cannot form a triangle.

Exercises **Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.** *See Example 1 on page 261.*

21. 7, 20, 5 **22.** 16, 20, 5 **23.** 18, 20, 6

5-5 Inequalities Involving Two Triangles

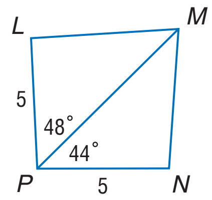
See pages 267–273.

Concept Summary

- SAS Inequality: In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- SSS Inequality: In two triangles, if two sides are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

Example **Write an inequality relating LM and MN.**

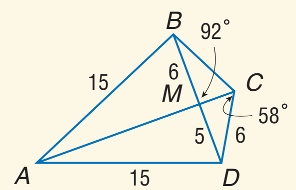
In $\triangle LMP$ and $\triangle NMP$, $\overline{LP} \cong \overline{NP}$, $\overline{PM} \cong \overline{PM}$, and $m\angle LPM > m\angle NPM$. The SAS Inequality allows us to conclude that $LM > MN$.



Exercises **Write an inequality relating the given pair of angles or segment measures.** *See Example 3 on page 269.*

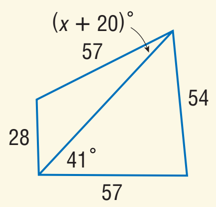
24. $m\angle BAC$ and $m\angle DAC$

25. BC and MD

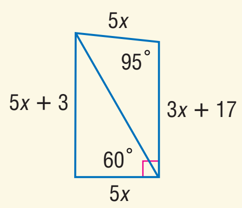


Write an inequality to describe the possible values of x.
See Example 3 on page 269.

26.



27.



Chapter 5

Practice Test

Vocabulary and Concepts

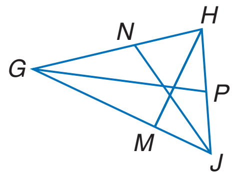
Choose the letter that best matches each description.

1. point of concurrency of the angle bisectors of a triangle
2. point of concurrency of the altitudes of a triangle
3. point of concurrency of the perpendicular bisectors of a triangle

a. circumcenter
b. incenter
c. orthocenter
d. centroid

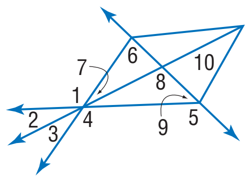
Skills and Applications

In $\triangle GHJ$, $HP = 5x - 16$, $PJ = 3x + 8$, $m\angle GJN = 6y - 3$, $m\angle NJH = 4y + 23$, and $m\angle HMG = 4z + 14$.



4. $\overline{GP}$ is a median of $\triangle GHJ$. Find HJ.
5. Find $m\angle GJH$ if $\overline{JN}$ is an angle bisector.
6. If $\overline{HM}$ is an altitude of $\angle GHJ$, find the value of z.

Refer to the figure at the right. Determine which angle has the greatest measure.



7. $\angle 8$, $\angle 5$, $\angle 7$
8. $\angle 6$, $\angle 7$, $\angle 8$
9. $\angle 1$, $\angle 6$, $\angle 9$

Write the assumption you would make to start an indirect proof of each statement.

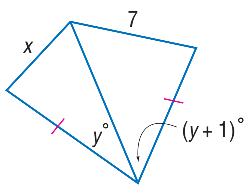
10. If n is a natural number, then $2^n + 1$ is odd.
11. Alternate interior angles are congruent.

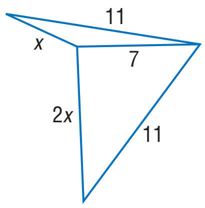
12. **BUSINESS** Over the course of three days, Marcus spent one and one-half hours on a teleconference for his marketing firm. Use indirect reasoning to show that, on at least one day, Marcus spent at least one half-hour on a teleconference.

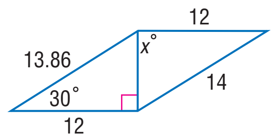
Find the range for the measure of the third side of a triangle given the measures of two sides.

13. 1 and 14
14. 14 and 11
15. 13 and 19

Write an inequality for the possible values of x.

16.


17.


18.


19. **GEOGRAPHY** The distance between Atlanta and Cleveland is about 554 miles. The distance between Cleveland and New York City is about 399 miles. Use the Triangle Inequality Theorem to find the possible values of the distance between New York and Atlanta.

20. **STANDARDIZED TEST PRACTICE** A given triangle has two sides with measures 8 and 11. Which of the following is *not* a possible measure for the third side?
Ⓐ 3 Ⓑ 7 Ⓒ 12 Ⓓ 18

Chapter 5 Standardized Test Practice

Part 1 Multiple Choice

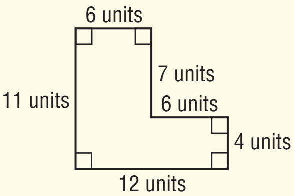
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Tamara works at a rug and tile store after school. The ultra-plush carpet has 80 yarn fibers per square inch. How many fibers are in a square yard? (Prerequisite Skill)

(A) 2,880 (B) 8,640
(C) 34,560 (D) 103,680

2. What is the perimeter of the figure? (Lesson 1-4)

(A) 20 units
(B) 46 units
(C) 90 units
(D) 132 units



3. Which is a correct statement about the conditional and its converse below? (Lesson 2-2)

Statement: If the measure of an angle is 50°, then the angle is an acute angle.

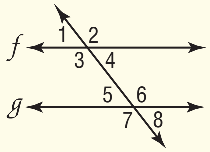
Converse: If an angle is an acute angle, then the measure of the angle is 50°.

(A) The statement and its converse are both true.
(B) The statement is true, but its converse is false.
(C) The statement and its converse are both false.
(D) The statement is false, but its converse is true.

4. Six people attend a meeting. When the meeting is over, each person exchanges business cards with each of the other people. Use noncollinear points to determine how many exchanges are made. (Lesson 2-3)

(A) 6 (B) 15 (C) 36 (D) 60

For Questions 5 and 6, refer to the figure below.



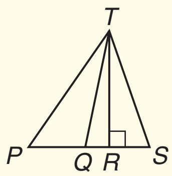
5. What is the term used to describe $\angle 4$ and $\angle 5$? (Lesson 3-1)

(A) alternate exterior angles
(B) alternate interior angles
(C) consecutive interior angles
(D) corresponding angles

6. Given that lines f and g are not parallel, what assumption can be made to prove that $\angle 3$ is not congruent to $\angle 7$? (Lesson 5-2)

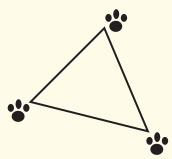
(A) $f \parallel g$ (B) $\angle 3 \cong \angle 7$
(C) $\angle 3 \cong \angle 2$ (D) $m\angle 3 \cong m\angle 7$

7. $\overline{QT}$ is a median of $\triangle PST$, and $\overline{RT}$ is an altitude of $\triangle PST$. Which of the following line segments is shortest? (Lesson 5-4)



(A) $\overline{PT}$ (B) $\overline{QT}$ (C) $\overline{RT}$ (D) $\overline{ST}$

8. A paleontologist found the tracks of an animal that is now extinct. Which of the following lengths could be the measures of the three sides of the triangle formed by the tracks? (Lesson 5-4)



(A) 2, 9, 10 (B) 5, 8, 13
(C) 7, 11, 20 (D) 9, 13, 26

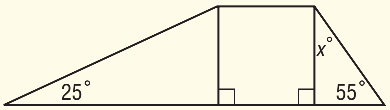
Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 795–810.

Part 2 Short Response/Grid In

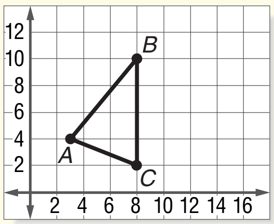
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The top of an access ramp to a building is 2 feet higher than the lower end of the ramp. If the lower end is 24 feet from the building, what is the slope of the ramp? (Lesson 3-3)

10. The ramps at a local skate park vary in steepness. Find x. (Lesson 4-2)



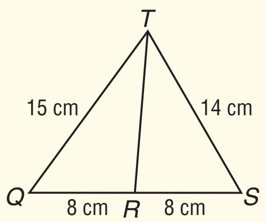
For Questions 11 and 12, refer to the graph below.



11. During a soccer game, a player stands near the goal at point A. The goalposts are located at points B and C. The goalkeeper stands at point P on the goal line $\overline{BC}$ so that $\overline{AP}$ forms a median. What is the location of the goalkeeper? (Lesson 5-1)

12. A defender positions herself on the goal line $\overline{BC}$ at point T to assist the goalkeeper. If $\overline{AT}$ forms an altitude of $\triangle ABC$, what is the location of defender T? (Lesson 5-1)

13. What postulate or theorem could you use to prove that the measure of $\angle QRT$ is greater than the measure of $\angle SRT$? (Lesson 5-5)



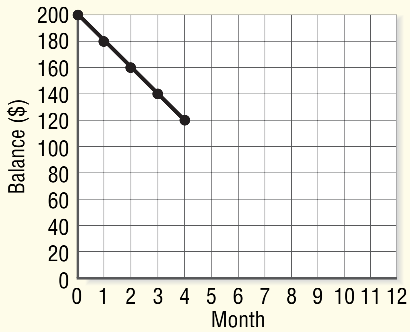
Test-Taking Tip

Questions 7, 10, and 11
Review any terms that you have learned before you take a test. Remember that a median is a segment that connects a vertex of a triangle to the midpoint of the opposite side. An altitude is a perpendicular segment from a vertex to the opposite side.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

14. Kendell is purchasing a new car stereo for \$200. He agreed to pay the same amount each month until the \$200 is paid. Kendell made the graph below to help him figure out when the amount will be paid. (Lesson 3-3)



a. Use the slope of the line to write an argument that the line intersects the x-axis at (10, 0).

b. What does the point (10, 0) represent?

15. The vertices of $\triangle ABC$ are $A(-3, 1)$, $B(0, -2)$, and $C(3, 4)$.

a. Graph $\triangle ABC$. (Prerequisite Skill)

b. Use the Distance Formula to find the length of each side to the nearest tenth. (Lesson 1-3)

c. What type of triangle is $\triangle ABC$? How do you know? (Lesson 4-1)

d. Prove $\angle A \cong \angle B$. (Lesson 4-6)

e. Prove $m\angle A > m\angle C$. (Lesson 5-3)