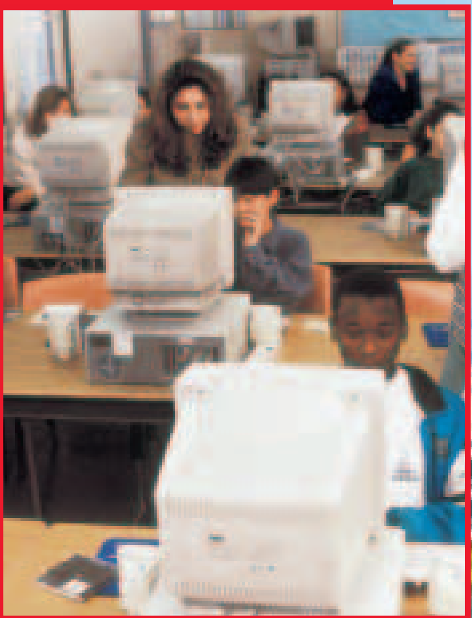
UNIT 2

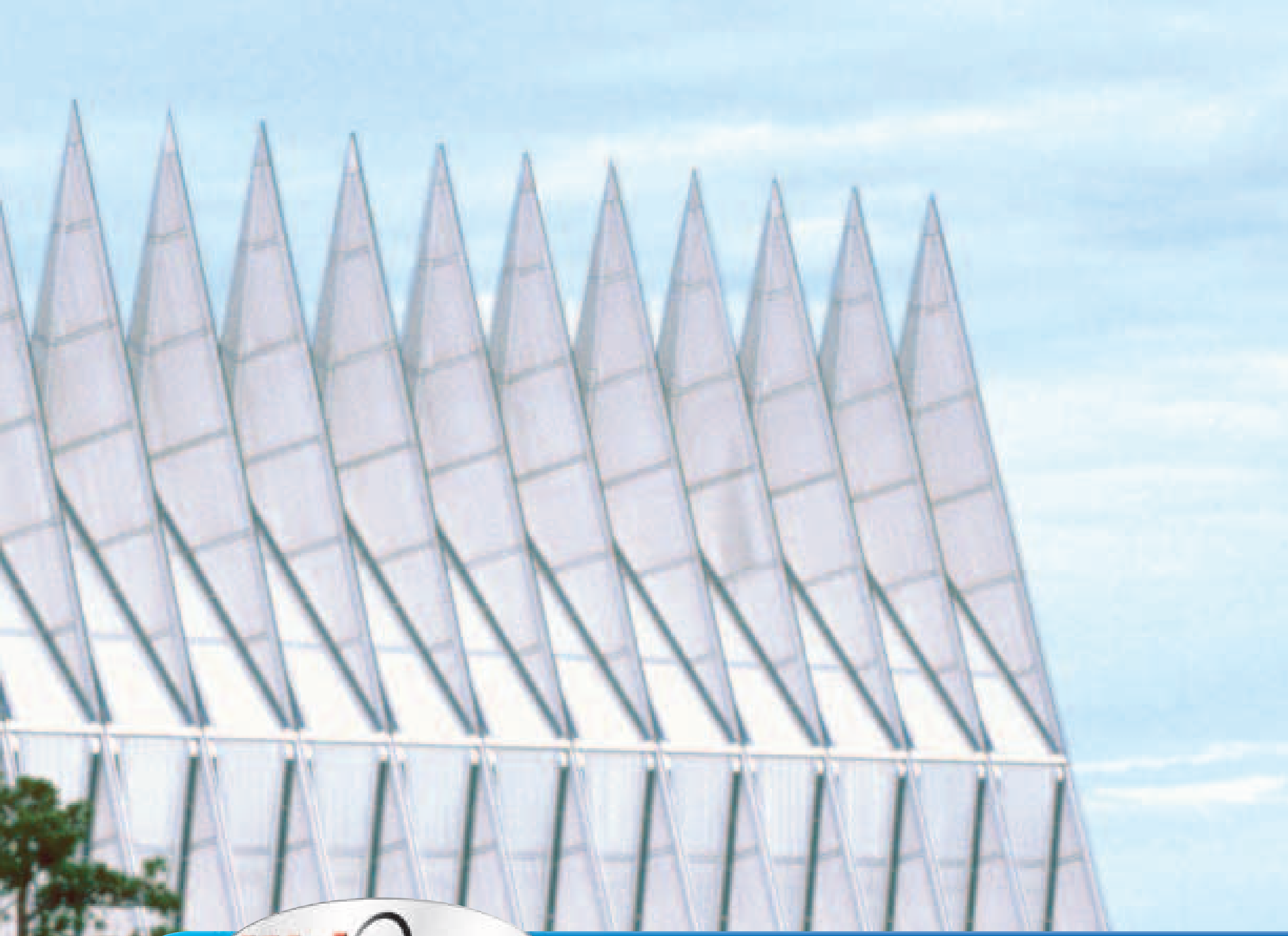
Triangles

You can use triangles and their properties to model and analyze many real-world situations. In this unit, you will learn about relationships in and among triangles, including congruence and similarity.



(l)A. Ramey/Woodfin Camp & Associates, (r)Dennis MacDonald/PhotoEdit

CONTENTS



WebQuest Internet Project

Who Is Behind This Geometry Concept Anyway?

Have you ever wondered who first developed some of the ideas you are learning in your geometry class? Today, many students use the Internet for learning and research. In this project, you will be using the Internet to research a topic in geometry. You will then prepare a portfolio or poster to display your findings.

Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Continue working on your WebQuest as you study Unit 2.

Lesson	4-6	5-1	6-6	7-1
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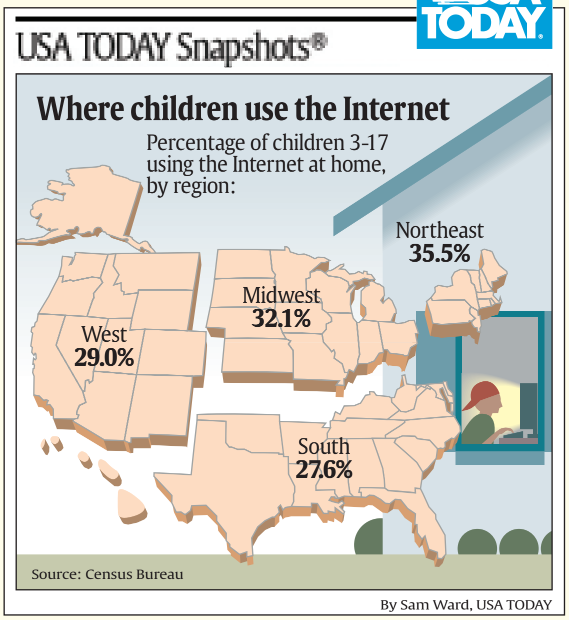


Chapter

4 Congruent Triangles

What You'll Learn

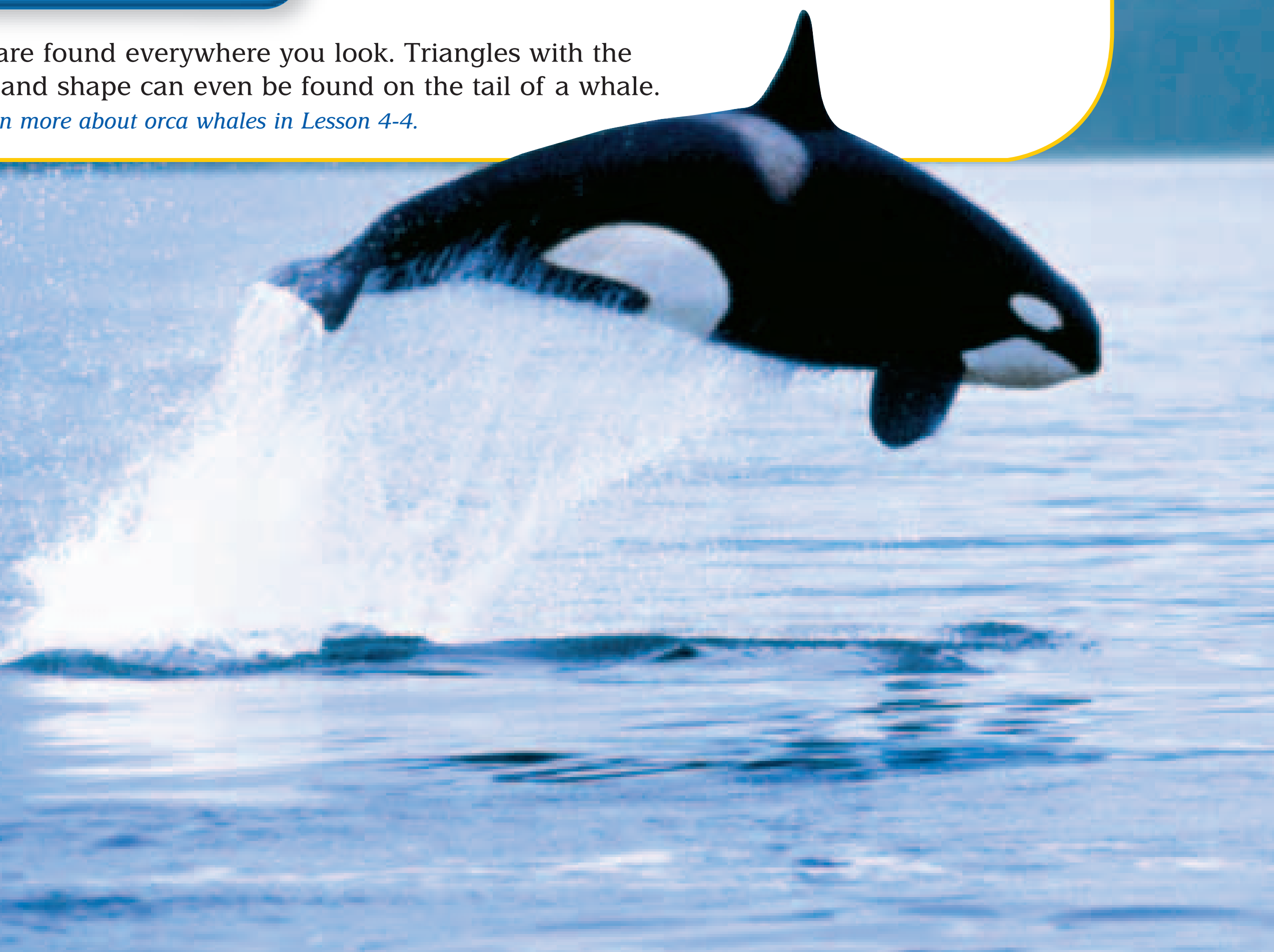
- **Lesson 4-1** Classify triangles.
- **Lesson 4-2** Apply the Angle Sum Theorem and the Exterior Angle Theorem.
- **Lesson 4-3** Identify corresponding parts of congruent triangles.
- **Lessons 4-4 and 4-5** Test for triangle congruence using SSS, SAS, ASA, and AAS.
- **Lesson 4-6** Use properties of isosceles and equilateral triangles.
- **Lesson 4-7** Write coordinate proofs.

Key Vocabulary

- exterior angle (p. 186)
- flow proof (p. 187)
- corollary (p. 188)
- congruent triangles (p. 192)
- coordinate proof (p. 222)

Why It's Important

Triangles are found everywhere you look. Triangles with the same size and shape can even be found on the tail of a whale. *You will learn more about orca whales in Lesson 4-4.*



Daniel J. Cox/Getty Images

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 4.

For Lesson 4-1 — Solve Equations

Solve each equation. *(For review, see pages 737 and 738.)*

1. $2x + 18 = 5$

2. $3m - 16 = 12$

3. $4y + 12 = 16$

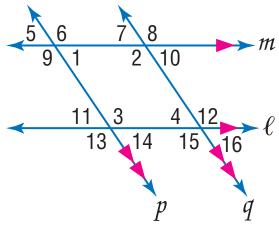
4. $10 = 8 - 3z$

5. $6 = 2a + \frac{1}{2}$

6. $\frac{2}{3}b + 9 = -15$

For Lessons 4-2, 4-4, and 4-5 — Congruent Angles

Name the indicated angles or pairs of angles if $p \parallel q$ and $m \parallel \ell$. *(For review, see Lesson 3-1.)*



7. angles congruent to $\angle 8$

8. angles congruent to $\angle 13$

9. angles supplementary to $\angle 1$

10. angles supplementary to $\angle 12$

For Lessons 4-3 and 4-7 — Distance Formula

Find the distance between each pair of points. Round to the nearest tenth. *(For review, see Lesson 1-3.)*

11. $(6, 8), (-4, 3)$

12. $(-15, 12), (6, 18)$

13. $(11, -8), (-3, -4)$

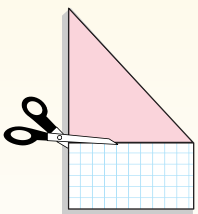
14. $(-10, 4), (8, -7)$

FOLDABLES™ Study Organizer

Triangles Make this Foldable to help you organize your notes. Begin with two sheets of grid paper and one sheet of construction paper.

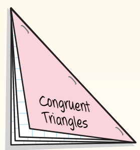
Step 1 Fold and Cut

Stack the grid paper on the construction paper. Fold diagonally as shown and cut off the excess.



Step 2 Staple and Label

Staple the edge to form a booklet. Then label each page with a lesson number and title.



Reading and Writing As you read and study the chapter, use your journal for sketches and examples of terms associated with triangles and sample proofs.

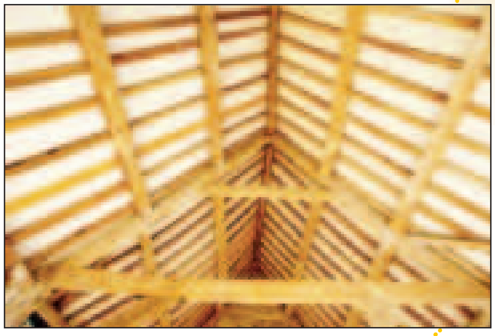
4-1 Classifying Triangles

What You'll Learn

- Identify and classify triangles by angles.
- Identify and classify triangles by sides.

Why are triangles important in construction?

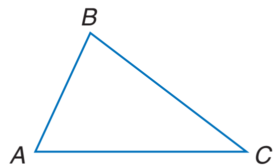
Many structures use triangular shapes as braces for construction. The roof sections of houses are made of triangular trusses that support the roof and the house.



Vocabulary

- acute triangle
- obtuse triangle
- right triangle
- equiangular triangle
- scalene triangle
- isosceles triangle
- equilateral triangle

CLASSIFY TRIANGLES BY ANGLES Recall that a triangle is a three-sided polygon. Triangle *ABC*, written $\triangle ABC$, has parts that are named using the letters *A*, *B*, and *C*.



- The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- The vertices are *A*, *B*, and *C*.
- The angles are $\angle ABC$ or $\angle B$, $\angle BCA$ or $\angle C$, and $\angle BAC$ or $\angle A$.

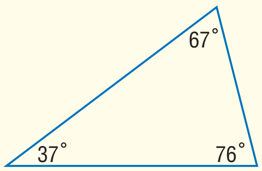
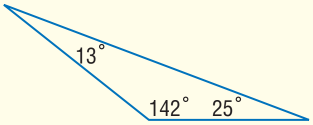
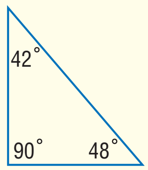
There are two ways to classify triangles. One way is by their angles. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

Study Tip

Common Misconceptions

These classifications are distinct groups. For example, a triangle cannot be right and acute.

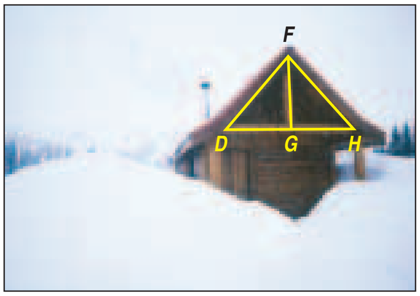
Key Concept — Classifying Triangles by Angles

In an **acute triangle**, all of the angles are acute.	In an **obtuse triangle**, one angle is obtuse.	In a **right triangle**, one angle is right.
		
all angle measures < 90	one angle measure > 90	one angle measure $= 90$

An acute triangle with all angles congruent is an **equiangular triangle**.

Example 1 Classify Triangles by Angles

ARCHITECTURE The roof of this house is made up of three different triangles. Use a protractor to classify $\triangle DFH$, $\triangle DFG$, and $\triangle HFG$ as *acute*, *equiangular*, *obtuse*, or *right*.



$\triangle DFH$ has all angles with measures less than 90, so it is an acute triangle.

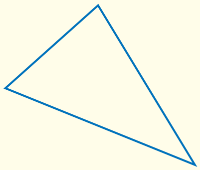
$\triangle DFG$ and $\triangle HFG$ both have one angle with measure equal to 90. Both of these are right triangles.

(t)Martin Jones/CORBIS, (b)David Scott/Index Stock

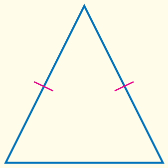
CLASSIFY TRIANGLES BY SIDES Triangles can also be classified according to the number of congruent sides they have. To indicate that sides of a triangle are congruent, an equal number of hash marks are drawn on the corresponding sides.

Key Concept *Classifying Triangles by Sides*

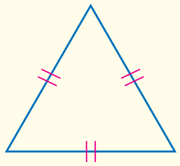
No two sides of a **scalene triangle** are congruent.



At least two sides of an **isosceles triangle** are congruent.



All of the sides of an **equilateral triangle** are congruent.



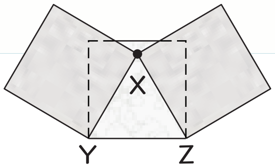
An equilateral triangle is a special kind of isosceles triangle.

Geometry Activity

Equilateral Triangles

Model

- Align three pieces of patty paper as indicated. Draw a dot at *X*.
- Fold the patty paper through *X* and *Y* and through *X* and *Z*.

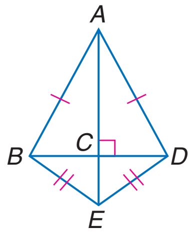


Analyze

1. Is $\triangle XYZ$ equilateral? Explain.
2. Use three pieces of patty paper to make a triangle that is isosceles, but not equilateral.
3. Use three pieces of patty paper to make a scalene triangle.

Example 2 *Classify Triangles by Sides*

Identify the indicated type of triangle in the figure.



a. isosceles triangles

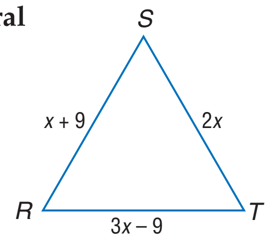
Isosceles triangles have at least two sides congruent. So, $\triangle ABD$ and $\triangle EBD$ are isosceles.

b. scalene triangles

Scalene triangles have no congruent sides. $\triangle AEB$, $\triangle AED$, $\triangle ACB$, $\triangle ACD$, $\triangle BCE$, and $\triangle DCE$ are scalene.

Example 3 *Find Missing Values*

ALGEBRA **Find x and the measure of each side of equilateral triangle RST if $RS = x + 9$, $ST = 2x$, and $RT = 3x - 9$.**



Since $\triangle RST$ is equilateral, $RS = ST$.

$x + 9 = 2x$ Substitution

$9 = x$ Subtract x from each side.

Next, substitute to find the length of each side.

$RS = x + 9$ $= 9 + 9$ or 18

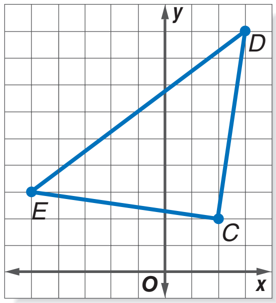
$ST = 2x$ $= 2(9)$ or 18

$RT = 3x - 9$ $= 3(9) - 9$ or 18

For $\triangle RST$, $x = 9$, and the measure of each side is 18.

Example 4 Use the Distance Formula

COORDINATE GEOMETRY **Find the measures of the sides of $\triangle DEC$. Classify the triangle by sides.**



Use the Distance Formula to find the lengths of each side.

$$EC = \sqrt{(-5-2)^2 + (3-2)^2} = \sqrt{49+1} = \sqrt{50}$$

$$ED = \sqrt{(-5-3)^2 + (3-9)^2} = \sqrt{64+36} = \sqrt{100}$$

$$DC = \sqrt{(3-2)^2 + (9-2)^2} = \sqrt{1+49} = \sqrt{50}$$

Since $\overline{EC}$ and $\overline{DC}$ have the same length, $\triangle DEC$ is isosceles.

Study Tip

Look Back
To review the **Distance Formula**, see Lesson 1-3.

Check for Understanding

Concept Check

1. **Explain** how a triangle can be classified in two ways.
2. **OPEN ENDED** Draw a triangle that is isosceles and right.

Determine whether each of the following statements is *always*, *sometimes*, or *never* true. Explain.

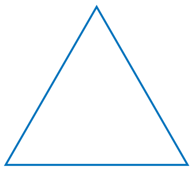
3. Equiangular triangles are also acute.
4. Right triangles are acute.

Guided Practice

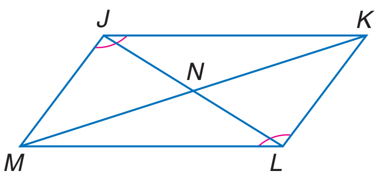
Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

5.

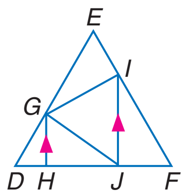
6.



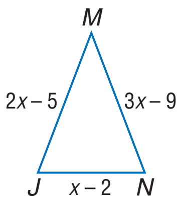
7. Identify the obtuse triangles if $\angle MJK \cong \angle KLM$, $m\angle MJK = 126$, and $m\angle JNM = 52$.



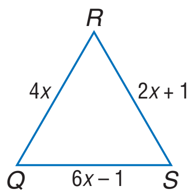
8. Identify the right triangles if $\overline{IJ} \parallel \overline{GH}$, $\overline{GH} \perp \overline{DF}$, and $\overline{GI} \perp \overline{EF}$.



9. **ALGEBRA** Find x, JM, MN, and JN if $\triangle JMN$ is an isosceles triangle with $\overline{JM} \cong \overline{MN}$.



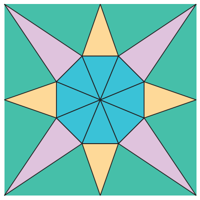
10. **ALGEBRA** Find x, QR, RS, and QS if $\triangle QRS$ is an equilateral triangle.



11. Find the measures of the sides of $\triangle TWZ$ with vertices at $T(2, 6)$, $W(4, -5)$, and $Z(-3, 0)$. Classify the triangle.

Application

12. **QUILTING** The star-shaped composite quilting square is made up of four different triangles. Use a ruler to classify the four triangles by sides.



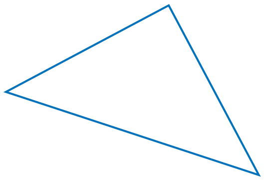
Practice and Apply

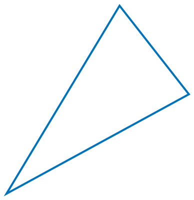
Homework Help

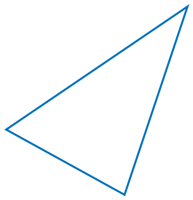
For Exercises	See Examples
13–18	1
19, 21–25	1, 2
26–29	3
30, 31	2
32–37, 40, 41	4

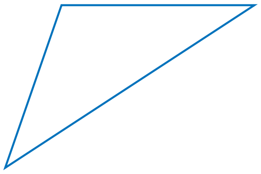
Extra Practice
See page 760.

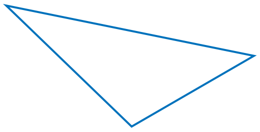
Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

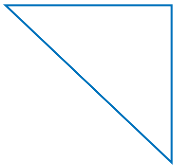
13. 

14. 

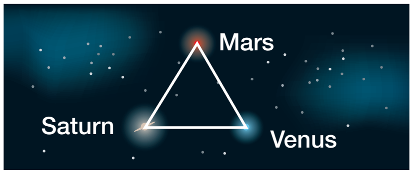
15. 

16. 

17. 

18. 

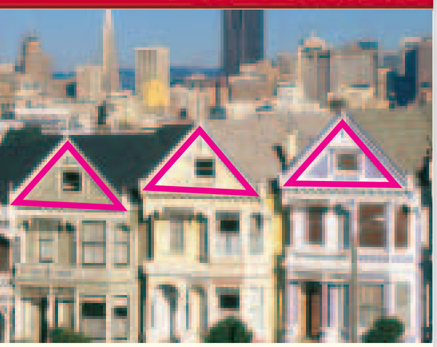
19. **ASTRONOMY** On May 5, 2002, Venus, Saturn, and Mars were aligned in a triangular formation. Use a protractor or ruler to classify the triangle formed by sides and angles.



20. **RESEARCH** Use the Internet or other resource to find out how astronomers can predict planetary alignment.

21. **ARCHITECTURE** The restored and decorated Victorian houses in San Francisco are called the "Painted Ladies." Use a protractor to classify the triangles indicated in the photo by sides and angles.

More About. . .

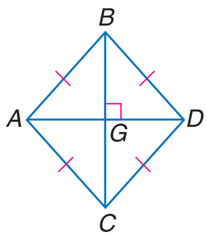


Architecture

The Painted Ladies are located in Alamo Square. The area is one of 11 designated historic districts in San Francisco.

Source: www.sfvisitor.org

Identify the indicated type of triangles in the figure if $\overline{AB} \cong \overline{BD} \cong \overline{DC} \cong \overline{CA}$ and $\overline{BC} \perp \overline{AD}$.



22. right
23. obtuse
24. scalene
25. isosceles

ALGEBRA **Find x and the measure of each side of the triangle.**

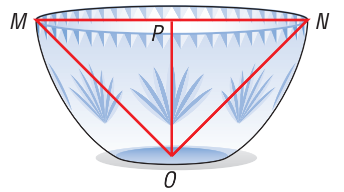
26. $\triangle GHJ$ is isosceles, with $\overline{HG} \cong \overline{JG}$, $GH = x + 7$, $GJ = 3x - 5$, and $HJ = x - 1$.

27. $\triangle MPN$ is equilateral with $MN = 3x - 6$, $MP = x + 4$, and $NP = 2x - 1$.

28. $\triangle QRS$ is equilateral. QR is two less than two times a number, RS is six more than the number, and QS is ten less than three times the number.

29. $\triangle JKL$ is isosceles with $\overline{KJ} \cong \overline{LJ}$. JL is five less than two times a number. JK is three more than the number. KL is one less than the number. Find the measure of each side.

30. **CRYSTAL** The top of the crystal bowl shown is circular. The diameter at the top of the bowl is MN. P is the midpoint of $\overline{MN}$, and $\overline{OP} \perp \overline{MN}$. If $MN = 24$ and $OP = 12$, determine whether $\triangle MPO$ and $\triangle NPO$ are equilateral.



31. **MAPS** The total distance from Nashville, Tennessee, to Cairo, Illinois, to Lexington, Kentucky, and back to Nashville, Tennessee, is 593 miles. The distance from Cairo to Lexington is 81 more miles than the distance from Lexington to Nashville. The distance from Cairo to Nashville is 40 miles less than the distance from Nashville to Lexington. Classify the triangle formed by its sides.

Find the measures of the sides of $\triangle ABC$ and classify each triangle by its sides.

32. $A(5, 4), B(3, -1), C(7, -1)$

33. $A(-4, 1), B(5, 6), C(-3, -7)$

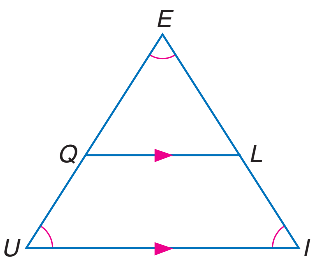
34. $A(-7, 9), B(-7, -1), C(4, -1)$

35. $A(-3, -1), B(2, 1), C(2, -3)$

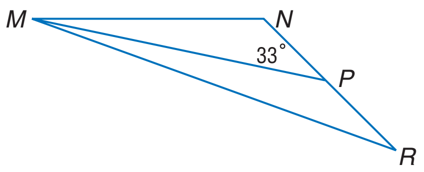
36. $A(0, 5), B(5\sqrt{3}, 2), C(0, -1)$

37. $A(-9, 0), B\left(-5, 6\sqrt{3}\right), C(-1, 0)$

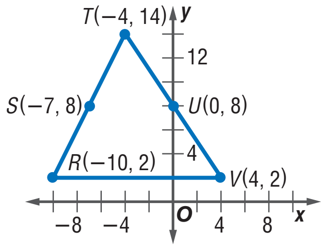
38. **PROOF** Write a two-column proof to prove that $\triangle EQL$ is equiangular.



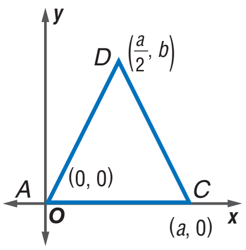
39. **PROOF** Write a paragraph proof to prove that $\triangle RPM$ is an obtuse triangle if $m\angle NPM = 33$.



40. **COORDINATE GEOMETRY** Show that S is the midpoint of $\overline{RT}$ and U is the midpoint of $\overline{TV}$.



41. **COORDINATE GEOMETRY** Show that $\triangle ADC$ is isosceles.



42. **CRITICAL THINKING** $\overline{KL}$ is a segment representing one side of isosceles right triangle KLM, with $K(2, 6)$, and $L(4, 2)$. $\angle KLM$ is a right angle, and $\overline{KL} \cong \overline{LM}$. Describe how to find the coordinates of vertex M and name these coordinates.

43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are triangles important in construction?

Include the following in your answer:

- describe how to classify triangles, and
- if one type of triangle is used more often in architecture than other types.

Standardized Test Practice

44. Classify $\triangle ABC$ with vertices $A(-1, 1)$, $B(1, 3)$, and $C(3, -1)$.

(A) scalene acute (B) equilateral (C) isosceles acute (D) isosceles right

45. **ALGEBRA** Find the value of y if the mean of x, y, 15, and 35 is 25 and the mean of x, 15, and 35 is 27.

(A) 18 (B) 19 (C) 31 (D) 36

Maintain Your Skills

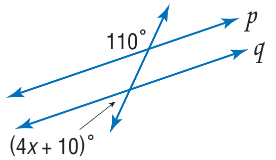
Mixed Review

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line. *(Lesson 3-6)*

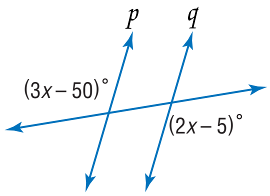
46. $y = x + 2$, $(2, -2)$ 47. $x + y = 2$, $(3, 3)$ 48. $y = 7$, $(6, -2)$

Find x so that $p \parallel q$. *(Lesson 3-5)*

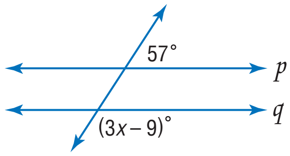
49.



50.



51.



For this proof, the reasons in the right column are not in the proper order. Reorder the reasons to properly match the statements in the left column. *(Lesson 2-6)*

52. **Given:** $3x - 4 = x - 10$

Prove: $x = -3$

Proof:

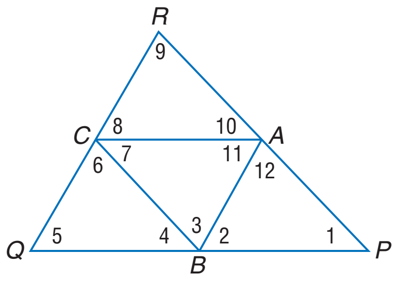
Statements	Reasons
a. $3x - 4 = x - 10$	**1.** Subtraction Property
b. $2x - 4 = -10$	**2.** Division Property
c. $2x = -6$	**3.** Given
d. $x = -3$	**4.** Addition Property

Getting Ready for the Next Lesson

PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{RQ}$, $\overline{BC} \parallel \overline{PR}$, and $\overline{AC} \parallel \overline{PQ}$. Name the indicated angles or pairs of angles.**

*(To review **angles formed by parallel lines and a transversal**, see Lessons 3-1 and 3-2.)*

53. three pairs of alternate interior angles
54. six pairs of corresponding angles
55. all angles congruent to $\angle 3$
56. all angles congruent to $\angle 7$
57. all angles congruent to $\angle 11$



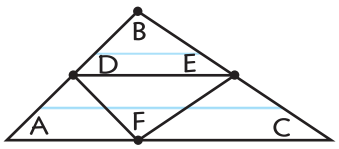
Geometry Activity

A Preview of Lesson 4-2

Angles of Triangles

There are special relationships among the angles of a triangle.

Activity 1 **Find the relationship among the measures of the interior angles of a triangle.**



Step 1 Draw an obtuse triangle and cut it out. Label the vertices A, B, and C.

Step 2 Find the midpoint of $\overline{AB}$ by matching A to B. Label this point D.

Step 3 Find the midpoint of $\overline{BC}$ by matching B to C. Label this point E.

Step 4 Draw $\overline{DE}$.

Step 5 Fold $\triangle ABC$ along $\overline{DE}$. Label the point where B touches $\overline{AC}$ as F.

Step 6 Draw $\overline{DF}$ and $\overline{FE}$. Measure each angle.

Analyze the Model

Describe the relationship between each pair.

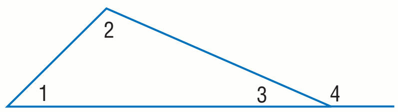
1. $\angle A$ and $\angle DFA$ **2.** $\angle B$ and $\angle DFE$ **3.** $\angle C$ and $\angle EFC$

4. What is the sum of the measures of $\angle DFA$, $\angle DFE$, and $\angle EFC$?

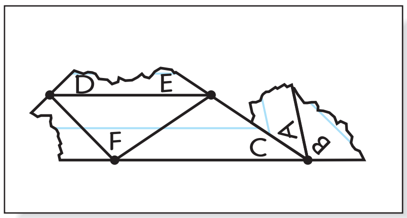
5. What is the sum of the measures of $\angle A$, $\angle B$, and $\angle C$?

6. **Make a conjecture** about the sum of the measures of the angles of any triangle.

In the figure at the right, $\angle 4$ is called an *exterior angle* of the triangle. $\angle 1$ and $\angle 2$ are the *remote interior angles* of $\angle 4$.



Activity 2 **Find the relationship among the interior and exterior angles of a triangle.**



Step 1 Trace $\triangle ABC$ from Activity 1 onto a piece of paper. Label the vertices.

Step 2 Extend $\overline{AC}$ to draw an exterior angle at C.

Step 3 Tear $\angle A$ and $\angle B$ off the triangle from Activity 1.

Step 4 Place $\angle A$ and $\angle B$ over the exterior angle.

Analyze the Model

7. **Make a conjecture** about the relationship of $\angle A$, $\angle B$, and the exterior angle at C.

8. Repeat the steps for the exterior angles of $\angle A$ and $\angle B$.

9. Is your conjecture true for all exterior angles of a triangle?

10. Repeat Activity 2 with an acute triangle.

11. Repeat Activity 2 with a right triangle.

12. **Make a conjecture** about the measure of an exterior angle and the sum of the measures of its remote interior angles.

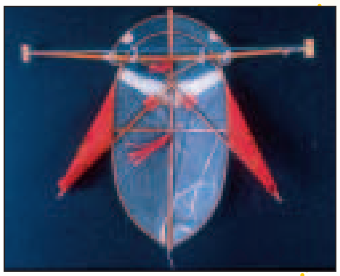
4-2 Angles of Triangles

What You'll Learn

- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

How are the angles of triangles used to make kites?

The Drachen Foundation coordinates the annual Miniature Kite Contest. This kite won second place in the Most Beautiful Kite category in 2001. The overall dimensions are 10.5 centimeters by 9.5 centimeters. The wings of the beetle are triangular.



Vocabulary

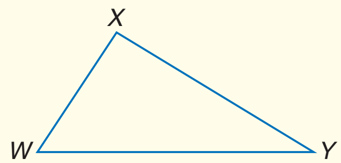
- exterior angle
- remote interior angles
- flow proof
- corollary

ANGLE SUM THEOREM If the measures of two of the angles of a triangle are known, how can the measure of the third angle be determined? The Angle Sum Theorem explains that the sum of the measures of the angles of any triangle is always 180.

Theorem 4.1

Angle Sum Theorem The sum of the measures of the angles of a triangle is 180.

Example: $m\angle W + m\angle X + m\angle Y = 180$



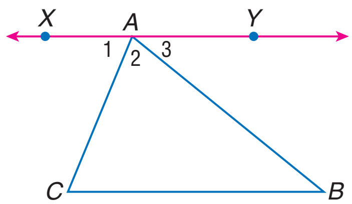
Study Tip

Look Back
Recall that sometimes extra lines have to be drawn to complete a proof. These are called *auxiliary lines*.

Proof: Angle Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle C + m\angle 2 + m\angle B = 180$



Proof:

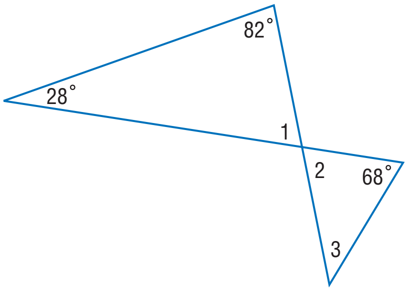
Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{XY}$ through A parallel to $\overline{CB}$.	2. Parallel Postulate
3. $\angle 1$ and $\angle CAY$ form a linear pair.	3. Def. of a linear pair
4. $\angle 1$ and $\angle CAY$ are supplementary.	4. If 2 ∠s form a linear pair, they are supplementary.
5. $m\angle 1 + m\angle CAY = 180$	5. Def. of suppl. ∠s
6. $m\angle CAY = m\angle 2 + m\angle 3$	6. Angle Addition Postulate
7. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	7. Substitution
8. $\angle 1 \cong \angle C$, $\angle 3 \cong \angle B$	8. Alt. Int. ∠s Theorem
9. $m\angle 1 = m\angle C$, $m\angle 3 = m\angle B$	9. Def. of ≅ ∠s
10. $m\angle C + m\angle 2 + m\angle B = 180$	10. Substitution

CONTENTS

If we know the measures of two angles of a triangle, we can find the measure of the third.

Example 1 *Interior Angles*

Find the missing angle measures.



Find $m\angle 1$ first because the measures of two angles of the triangle are known.

$m\angle 1 + 28 + 82 = 180$ Angle Sum Theorem

$m\angle 1 + 110 = 180$ Simplify.

$m\angle 1 = 70$ Subtract 110 from each side.

$\angle 1$ and $\angle 2$ are congruent vertical angles. So $m\angle 2 = 70$.

$m\angle 3 + 68 + 70 = 180$ Angle Sum Theorem

$m\angle 3 + 138 = 180$ Simplify.

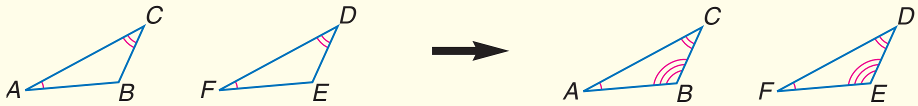
$m\angle 3 = 42$ Subtract 138 from each side.

Therefore, $m\angle 1 = 70$, $m\angle 2 = 70$, and $m\angle 3 = 42$.

The Angle Sum Theorem leads to a useful theorem about the angles in two triangles.

Theorem 4.2

Third Angle Theorem If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

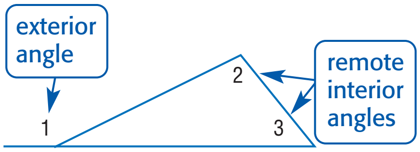


Example: If $\angle A \cong \angle F$ and $\angle C \cong \angle D$, then $\angle B \cong \angle E$.

You will prove this theorem in Exercise 44.

EXTERIOR ANGLE THEOREM

Each angle of a triangle has an exterior angle. An **exterior angle** is formed by one side of a triangle and the extension of another side. The interior angles of the triangle not adjacent to a given exterior angle are called **remote interior angles** of the exterior angle.



Study Tip

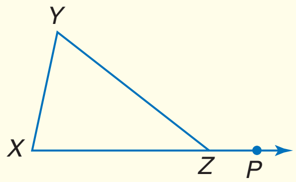
Reading Math

Remote means far away and *interior* means inside. The remote interior angles are the interior angles farthest from the exterior angle.

Theorem 4.3

Exterior Angle Theorem The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

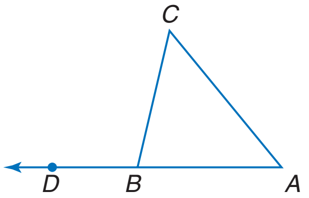
Example: $m\angle YZP = m\angle X + m\angle Y$



We will use a flow proof to prove this theorem. A **flow proof** organizes a series of statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate how the statements relate to each other.

Proof — Exterior Angle Theorem

Write a flow proof of the Exterior Angle Theorem.



Given: $\triangle ABC$

Prove: $m\angle CBD = m\angle A + m\angle C$

Flow Proof:

$\triangle ABC$
Given

$\angle CBD$ and $\angle ABC$ form a linear pair.
Definition of linear pair

$\angle CBD$ and $\angle ABC$ are supplementary.
If 2 ⦞ form a linear pair, they are supplementary.

$m\angle A + m\angle ABC + \angle C = 180$
Angle Sum Theorem

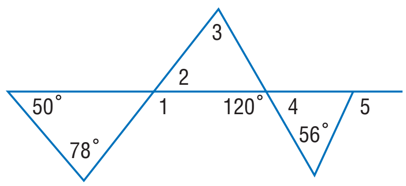
$m\angle CBD + m\angle ABC = 180$
Definition of supplementary

$m\angle A + m\angle ABC + m\angle C = m\angle CBD + m\angle ABC$
Substitution Property

$m\angle A + m\angle C = m\angle CBD$
Subtraction Property

Example 2 — Exterior Angles

Find the measure of each numbered angle in the figure.



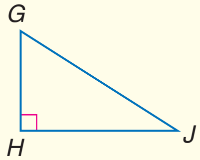
$m\angle 1 = 50 + 78$	Exterior Angle Theorem
$= 128$	Simplify.
$m\angle 1 + m\angle 2 = 180$	If 2 ⦞ form a linear pair, they are suppl.
$128 + m\angle 2 = 180$	Substitution
$m\angle 2 = 52$	Subtract 128 from each side.
$m\angle 2 + m\angle 3 = 120$	Exterior Angle Theorem
$52 + m\angle 3 = 120$	Substitution
$m\angle 3 = 68$	Subtract 52 from each side.
$120 + m\angle 4 = 180$	If 2 ⦞ form a linear pair, they are suppl.
$m\angle 4 = 60$	Subtract 120 from each side.
$m\angle 5 = m\angle 4 + 56$	Exterior Angle Theorem
$= 60 + 56$	Substitution
$= 116$	Simplify.

Therefore, $m\angle 1 = 128$, $m\angle 2 = 52$, $m\angle 3 = 68$, $m\angle 4 = 60$, and $m\angle 5 = 116$.

A statement that can be easily proved using a theorem is often called a **corollary** of that theorem. A corollary, just like a theorem, can be used as a reason in a proof.

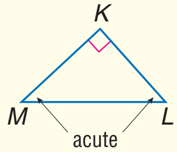
Corollaries

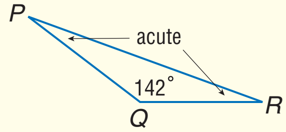
4.1 The acute angles of a right triangle are complementary.



Example: $m\angle G + m\angle J = 90$

4.2 There can be at most one right or obtuse angle in a triangle.

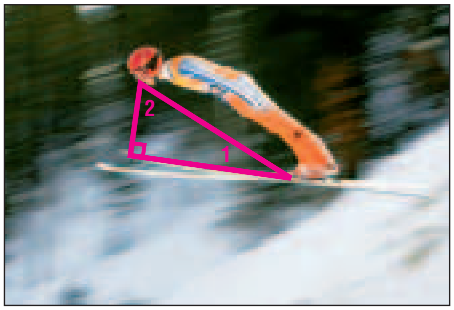




You will prove Corollaries 4.1 and 4.2 in Exercises 42 and 43.

Example 3 Right Angles

SKI JUMPING **Ski jumper Simon Ammann of Switzerland forms a right triangle with his skis and his line of sight. Find $m\angle 2$ if $m\angle 1$ is 27.**



Use Corollary 4.1 to write an equation.

$m\angle 1 + m\angle 2 = 90$

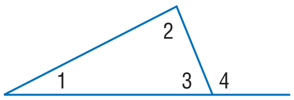
$27 + m\angle 2 = 90$ Substitution

$m\angle 2 = 63$ Subtract 27 from each side.

Check for Understanding

Concept Check

1. **OPEN ENDED** Draw a triangle. Label one exterior angle and its remote interior angles.
2. **FIND THE ERROR** Najee and Kara are discussing the Exterior Angle Theorem.



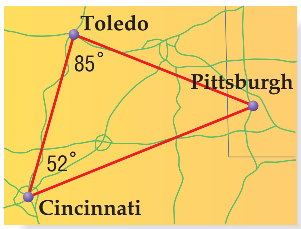
Najee	Kara
$m\angle 1 + m\angle 2 = m\angle 4$	$m\angle 1 + m\angle 2 + m\angle 4 = 180$

Who is correct? Explain your reasoning.

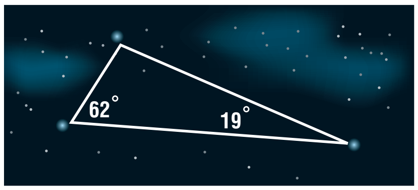
Guided Practice

Find the missing angle measure.

3.

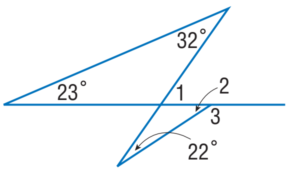


4.



Find each measure.

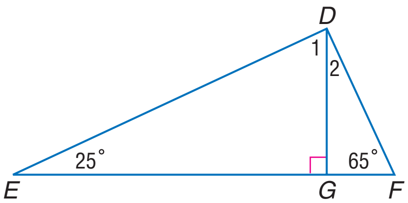
5. $m\angle 1$
6. $m\angle 2$
7. $m\angle 3$



Adam Pretty/Getty Images

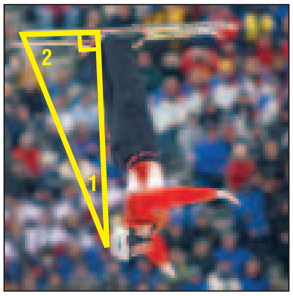
Find each measure.

8. $m\angle 1$
9. $m\angle 2$



Application

10. **SKI JUMPING** American ski jumper Eric Bergoust forms a right angle with his skis. If $m\angle 2 = 70$, find $m\angle 1$.



Practice and Apply

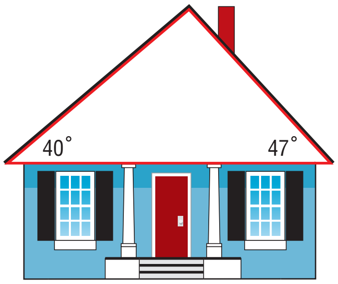
Homework Help

For Exercises	See Examples
11–17	1
18–31	2
32–35	3
36–38	2

Extra Practice
See page 761.

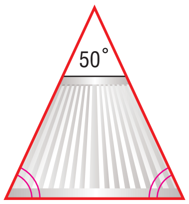
Find the missing angle measures.

11.

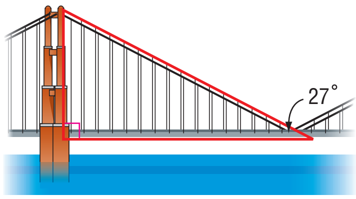


12.

13.

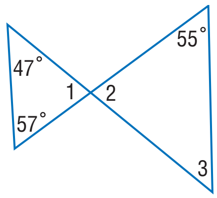


14.



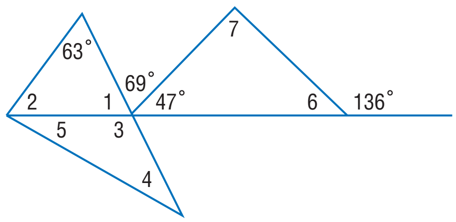
Find each measure.

15. $m\angle 1$
16. $m\angle 2$
17. $m\angle 3$



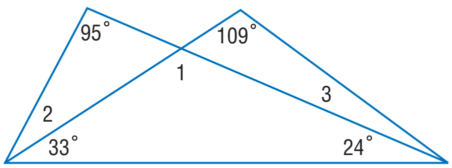
Find each measure if $m\angle 4 = m\angle 5$.

18. $m\angle 1$
19. $m\angle 2$
20. $m\angle 3$
21. $m\angle 4$
22. $m\angle 5$
23. $m\angle 6$
24. $m\angle 7$



Find each measure.

25. $m\angle 1$
26. $m\angle 2$
27. $m\angle 3$



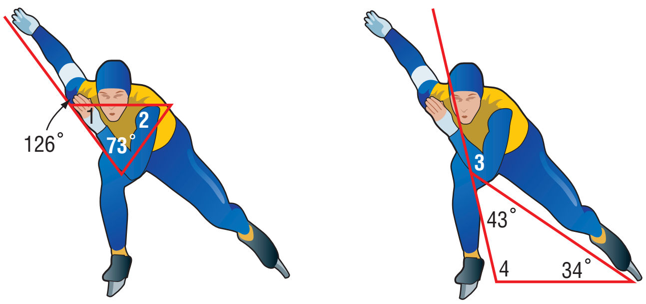
SPEED SKATING **For Exercises 28–31, use the following information.**
Speed skater Catriona Lemay Doan of Canada forms at least two sets of triangles and exterior angles as she skates. Use the measures of given angles to find each measure.

28. $m\angle 1$

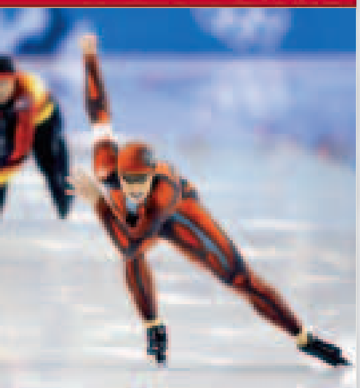
29. $m\angle 2$

30. $m\angle 3$

31. $m\angle 4$



More About. . .



Speed Skating

Catriona Lemay Doan is the first Canadian to win a Gold medal in the same event in two consecutive Olympic games.

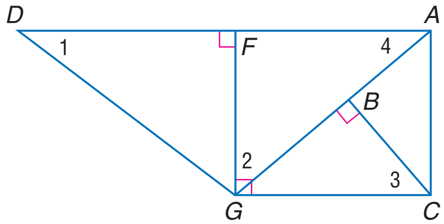
Source: www.catrionalemaydoan.com

Online Research **Data Update** Use the Internet or other resource to find the world record in speed skating. Visit www.geometryonline.com/data_update to learn more.

Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$.

32. $m\angle 1$ **33.** $m\angle 2$

34. $m\angle 3$ **35.** $m\angle 4$

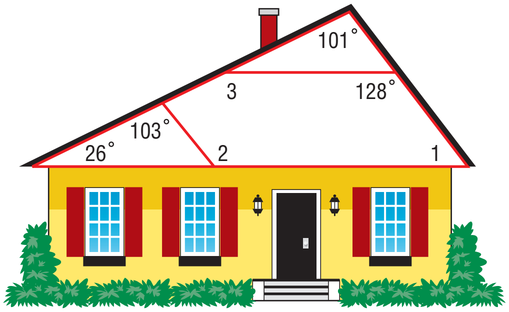


HOUSING **For Exercises 36–38, use the following information.**
The two braces for the roof of a house form triangles. Find each measure.

36. $m\angle 1$

37. $m\angle 2$

38. $m\angle 3$



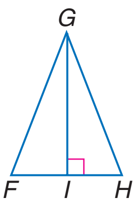
PROOF **For Exercises 39–44, write the specified type of proof.**

39. flow proof

Given: $\angle FGI \cong \angle IGH$

$\overline{GI} \perp \overline{FH}$

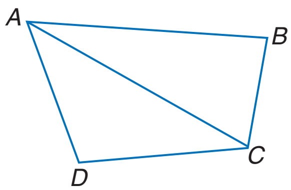
Prove: $\angle F \cong \angle H$



40. two-column

Given: $ABCD$ is a quadrilateral.

Prove: $m\angle DAB + m\angle B + m\angle BCD + m\angle D = 360$



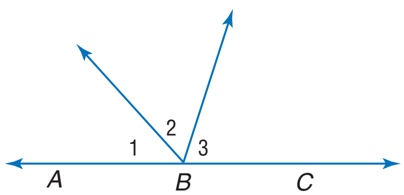
41. two-column proof of Theorem 4.3

42. flow proof of Corollary 4.1

43. paragraph proof of Corollary 4.2

44. two-column proof of Theorem 4.2

45. CRITICAL THINKING $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are in a 4:5:6 ratio. Find the measure of each angle.



Jed Jacobsohn/Getty Images

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

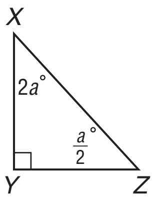
How are the angles of triangles used to make kites?

Include the following in your answer:

- if two angles of two triangles are congruent, how you can find the measure of the third angle, and
- if one angle measures 90, describe the other two angles.

Standardized Test Practice

47. In the triangle, what is the measure of $\angle Z$?



Ⓐ 18 Ⓑ 24

Ⓒ 72 Ⓓ 90

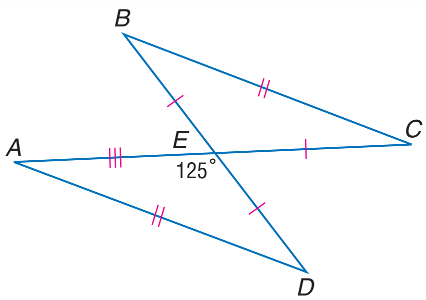
48. **ALGEBRA** The measure of the second angle of a triangle is three times the measure of the first, and the measure of the third angle is 25 more than the measure of the first. Find the measure of each angle.

Ⓐ 25, 85, 70 Ⓑ 31, 93, 56 Ⓒ 39, 87, 54 Ⓓ 42, 54, 84

Maintain Your Skills

Mixed Review

Identify the indicated type of triangle if $\overline{BC} \cong \overline{AD}$, $\overline{EB} \cong \overline{EC}$, $\overline{AC}$ bisects $\overline{BD}$, and $m\angle AED = 125$. *(Lesson 4-1)*



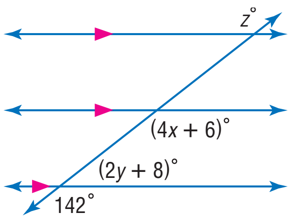
49. scalene triangles
50. obtuse triangles
51. isosceles triangles

Find the distance between each pair of parallel lines. *(Lesson 3-6)*

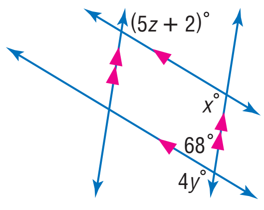
52. $y = x + 6$, $y = x - 10$
53. $y = -2x + 3$, $y = -2x - 7$
54. $4x - y = 20$, $4x - y = 3$
55. $2x - 3y = -9$, $2x - 3y = -6$

Find x, y, and z in each figure. *(Lesson 3-2)*

56.



57.



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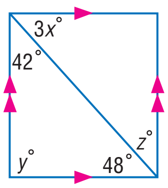


Getting Ready for the Next Lesson

PREREQUISITE SKILL **Name the property of congruence that justifies each statement.** *(To review* ***properties of congruence****, see Lessons 2-5 and 2-6.)*

59. $\angle 1 \cong \angle 1$ and $\overline{AB} \cong \overline{AB}$.
60. If $\overline{AB} \cong \overline{XY}$, then $\overline{XY} \cong \overline{AB}$.
61. If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
62. If $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$, then $\angle 2 \cong \angle 4$.
63. If $\overline{PQ} \cong \overline{XY}$ and $\overline{XY} \cong \overline{HK}$, then $\overline{PQ} \cong \overline{HK}$.
64. If $\overline{AB} \cong \overline{CD}$, $\overline{CD} \cong \overline{PQ}$, and $\overline{PQ} \cong \overline{XY}$, then $\overline{AB} \cong \overline{XY}$.

4-3 Congruent Triangles

What You'll Learn

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

Vocabulary

- congruent triangles
- congruence transformations

Why are triangles used in bridges?

In 1930, construction started on the West End Bridge in Pittsburgh, Pennsylvania. The arch of the bridge is trussed, not solid. Steel rods are arranged in a triangular web that lends structure and stability to the bridge.

CORRESPONDING PARTS OF CONGRUENT TRIANGLES Triangles that are the same size and shape are **congruent triangles**. Each triangle has three angles and three sides. If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.

If $\triangle ABC$ is congruent to $\triangle EFG$, the vertices of the two triangles correspond in the same order as the letters naming the triangles.

$$\triangle ABC \cong \triangle EFG$$

This correspondence of vertices can be used to name the corresponding congruent sides and angles of the two triangles.

$$\angle A \cong \angle E \qquad \angle B \cong \angle F \qquad \angle C \cong \angle G$$

$$\overline{AB} \cong \overline{EF} \qquad \overline{BC} \cong \overline{FG} \qquad \overline{AC} \cong \overline{EG}$$

The corresponding sides and angles can be determined from any congruence statement by following the order of the letters.

Study Tip

Congruent Parts
In congruent triangles, congruent sides are opposite congruent angles.

Key Concept ***Definition of Congruent Triangles (CPCTC)***

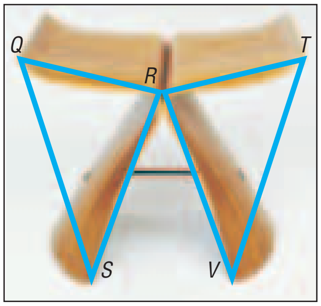
Two triangles are congruent if and only if their corresponding parts are congruent.

CPCTC stands for *corresponding parts of congruent triangles are congruent*. "If and only if" is used to show that both the conditional and its converse are true.

Aaron Haupt

Example 1 Corresponding Congruent Parts

FURNITURE DESIGN **The seat and legs of this stool form two triangles. Suppose the measures in inches are $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$, $TV = 24$, and $RV = 23$.**



a. Name the corresponding congruent angles and sides.

$\angle Q \cong \angle T$ $\angle QRS \cong \angle TRV$ $\angle S \cong \angle V$

$\overline{QR} \cong \overline{TR}$ $\overline{RS} \cong \overline{RV}$ $\overline{QS} \cong \overline{TV}$

b. Name the congruent triangles.

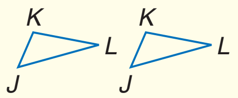
$\triangle QRS \cong \triangle TRV$

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

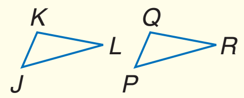
Theorem 4.4 Properties of Triangle Congruence

Congruence of triangles is reflexive, symmetric, and transitive.

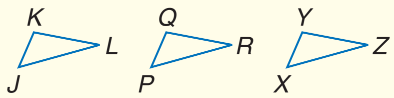
Reflexive
$\triangle JKL \cong \triangle JKL$



Symmetric
If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.



Transitive
If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$

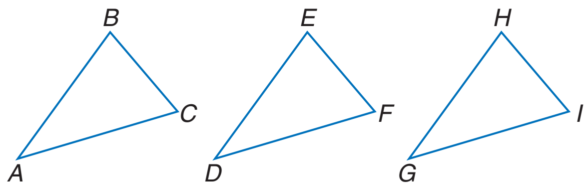


You will prove the symmetric and reflexive parts of Theorem 4.4 in Exercises 33 and 35, respectively.

Proof Theorem 4.4 (Transitive)

Given: $\triangle ABC \cong \triangle DEF$
$\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$



Proof:

Statements	Reasons
1. $\triangle ABC \cong \triangle DEF$	1. Given
2. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$	2. CPCTC
3. $\triangle DEF \cong \triangle GHI$	3. Given
4. $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$ $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, $\overline{DF} \cong \overline{GI}$	4. CPCTC
5. $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$	5. Congruence of angles is transitive.
6. $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, $\overline{AC} \cong \overline{GI}$	6. Congruence of segments is transitive.
7. $\triangle ABC \cong \triangle GHI$	7. Def. of $\cong$ $\triangle$s

Study Tip

Naming Congruent Triangles

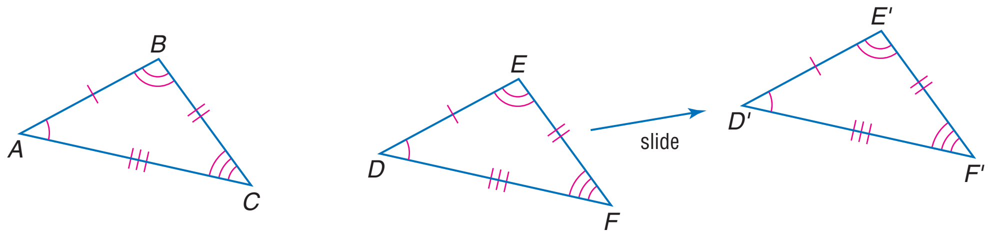
There are six ways to name each pair of congruent triangles.

Study Tip

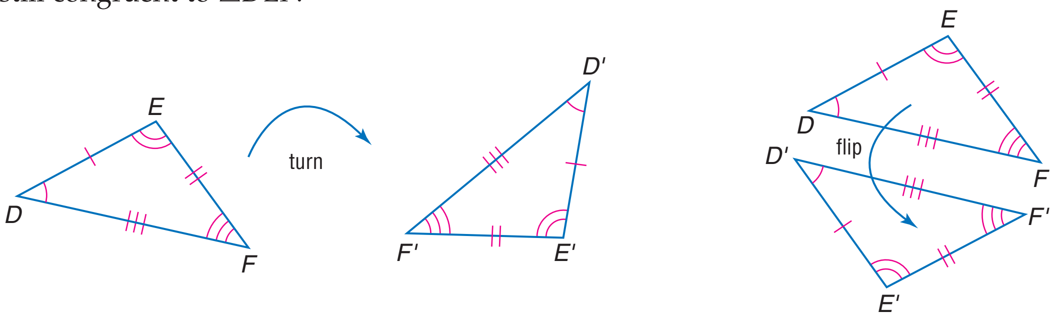
Transformations

Not all of the transformations preserve congruence. Only transformations that do not change the size or shape of the triangle are congruence transformations.

IDENTIFY CONGRUENCE TRANSFORMATIONS In the figures below, $\triangle ABC$ is congruent to $\triangle DEF$. If you *slide* $\triangle DEF$ up and to the right, $\triangle DEF$ is still congruent to $\triangle ABC$.



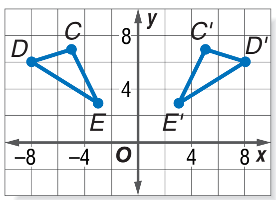
The congruency does not change whether you *turn* $\triangle DEF$ or *flip* $\triangle DEF$. $\triangle ABC$ is still congruent to $\triangle DEF$.



If you slide, flip, or turn a triangle, the size and shape do not change. These three transformations are called **congruence transformations**.

Example 2 Transformations in the Coordinate Plane

COORDINATE GEOMETRY The vertices of $\triangle CDE$ are $C(-5, 7)$, $D(-8, 6)$, and $E(-3, 3)$. The vertices of $\triangle C'D'E'$ are $C'(5, 7)$, $D'(8, 6)$, and $E'(3, 3)$.



a. Verify that $\triangle CDE \cong \triangle C'D'E'$.

Use the Distance Formula to find the length of each side in the triangles.

$$DC = \sqrt{[-8-(-5)]^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$D'C' = \sqrt{(8-5)^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$DE = \sqrt{[-8-(-3)]^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$D'E' = \sqrt{(8-3)^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$CE = \sqrt{[-5-(-3)]^2 + (7-3)^2} = \sqrt{4+16} \text{ or } \sqrt{20}$$

$$C'E' = \sqrt{(5-3)^2 + (7-3)^2} = \sqrt{4+16} \text{ or } \sqrt{20}$$

By the definition of congruence, $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$.

Use a protractor to measure the angles of the triangles. You will find that the measures are the same.

In conclusion, because $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$, $\angle D \cong \angle D'$, $\angle C \cong \angle C'$, and $\angle E \cong \angle E'$, $\triangle DCE \cong \triangle D'C'E'$.

b. Name the congruence transformation for $\triangle CDE$ and $\triangle C'D'E'$.

$\triangle C'D'E'$ is a flip of $\triangle CDE$.

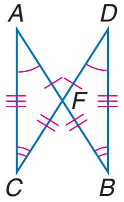
Check for Understanding

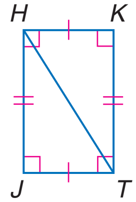
Concept Check

1. **Explain** how slides, flips, and turns preserve congruence.
2. **OPEN ENDED** Draw a pair of congruent triangles and label the congruent sides and angles.

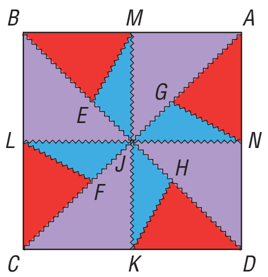
Guided Practice

Identify the congruent triangles in each figure.

3. 

4. 

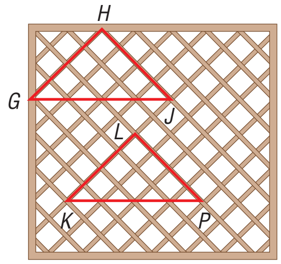
5. If $\triangle WXZ \cong \triangle STJ$, name the congruent angles and congruent sides.
6. **QUILTING** In the quilt design, assume that angles and segments that appear to be congruent are congruent. Indicate which triangles are congruent.



7. The coordinates of the vertices of $\triangle QRT$ and $\triangle Q'R'T'$ are $Q(-4, 3)$, $Q'(4, 3)$, $R(-4, -2)$, $R'(4, -2)$, $T(-1, -2)$, and $T'(1, -2)$. Verify that $\triangle QRT \cong \triangle Q'R'T'$. Then name the congruence transformation.

Application

8. **GARDENING** This garden lattice will be covered with morning glories in the summer. Wesley wants to save two triangular areas for artwork. If $\triangle GHJ \cong \triangle KLP$, name the corresponding congruent angles and sides.



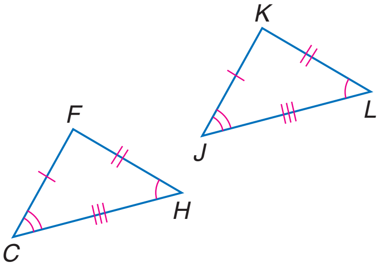
Practice and Apply

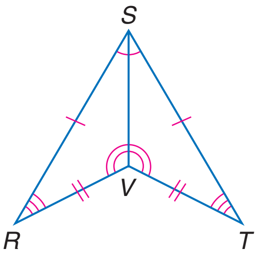
Homework Help

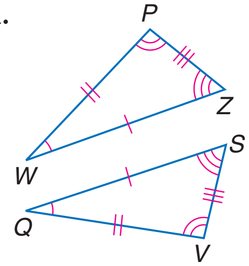
For Exercises	See Examples
9–22, 27–35	1
23–26	2

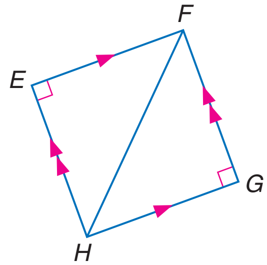
Extra Practice
See page 761.

Identify the congruent triangles in each figure.

9. 

10. 

11. 

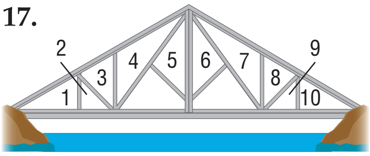
12. 

Name the congruent angles and sides for each pair of congruent triangles.

13. $\triangle TUV \cong \triangle XYZ$
14. $\triangle CDG \cong \triangle RSW$
15. $\triangle BCF \cong \triangle DGH$
16. $\triangle ADG \cong \triangle HKL$

Assume that segments and angles that appear to be congruent in the numbered triangles are congruent. Indicate which triangles are congruent.

17.



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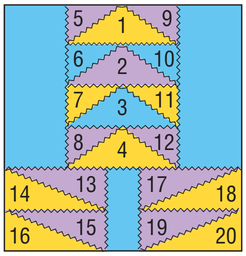

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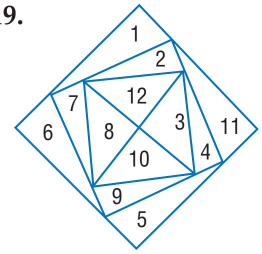
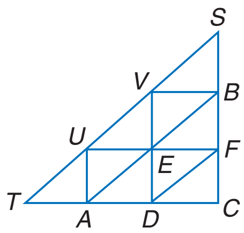
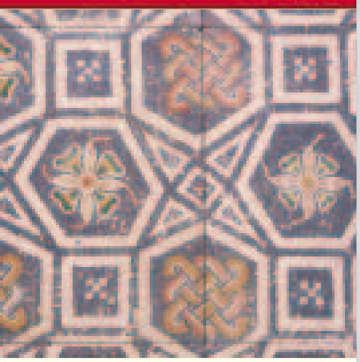


20. All of the small triangles in the figure at the right are congruent. Name three larger congruent triangles.



More About . . .



Mosaics

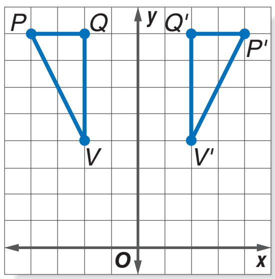
A mosaic is composed of glass, marble, or ceramic pieces often arranged in a pattern. The pieces, or *tesserae*, are set in cement. Mosaics are used to decorate walls, floors, and gardens.

Source: www.dimosaico.com

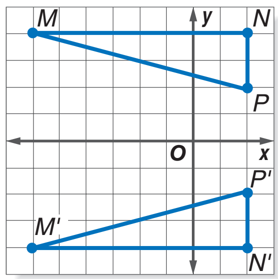
21. MOSAICS The picture at the left is the center of a Roman mosaic. Because the four triangles connect to a square, they have at least one side congruent to a side in another triangle. What else do you need to know to conclude that the four triangles are congruent?

Verify that each of the following preserves congruence and name the congruence transformation.

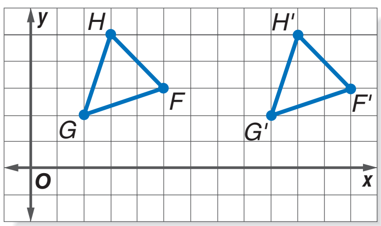
22. $\triangle PQV \cong \triangle P'Q'V'$



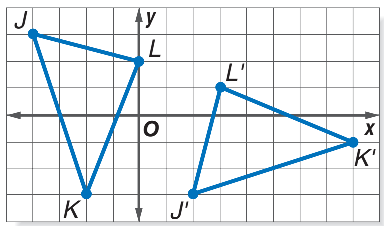
23. $\triangle MNP \cong \triangle M'N'P'$



24. $\triangle GHF \cong \triangle G'H'F'$



25. $\triangle JKL \cong \triangle J'K'L'$

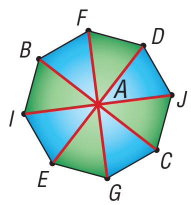


Determine whether each statement is *true* or *false*. Draw an example or counterexample for each.

26. Two triangles with corresponding congruent angles are congruent.

27. Two triangles with angles and sides congruent are congruent.

28. UMBRELLAS Umbrellas usually have eight congruent triangular sections with ribs of equal length. Are the statements $\triangle JAD \cong \triangle IAE$ and $\triangle JAD \cong \triangle EAI$ both correct? Explain.



North Carolina Museum of Art, Raleigh. Gift of Mr. & Mrs. Gordon Hanes

ALGEBRA **For Exercises 29 and 30, use the following information.**
$\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

29. Draw and label a figure to show the congruent triangles.

30. Find x.

ALGEBRA **For Exercises 31 and 32, use the following information.**
$\triangle JKL \cong \triangle DEF$, $m\angle J = 36$, $m\angle E = 64$, and $m\angle F = 3x + 52$.

31. Draw and label a figure to show the congruent triangles.

32. Find x.

33. **PROOF** The statements below can be used to prove that *congruence of triangles is symmetric*. Use the statements to construct a correct flow proof. Provide the reasons for each statement.

Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$

Flow Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$
___?___

$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$
___?___

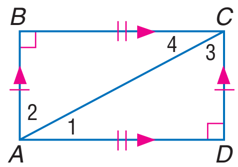
$\triangle RST \cong \triangle XYZ$
___?___

$\triangle XYZ \cong \triangle RST$
___?___

34. **PROOF** Copy the flow proof and provide the reasons for each statement.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \perp \overline{BC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle CAB$

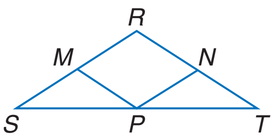


Flow Proof:

$\overline{AB} \cong \overline{CD}$ a. ?
$\overline{AD} \cong \overline{CB}$ b. ?
$\overline{AC} \cong \overline{CA}$ c. ?
$\overline{AD} \perp \overline{DC}$ d. ?
$\overline{AB} \perp \overline{BC}$ f. ?
$\overline{AD} \parallel \overline{BC}$ i. ?
$\overline{AB} \parallel \overline{CD}$ k. ?
$\angle D$ is a rt. $\angle$. e. ?
$\angle B$ is a rt. $\angle$. g. ?
$\angle 1 \cong \angle 4$ j. ?
$\angle 2 \cong \angle 3$ l. ?
$\angle D \cong \angle B$ h. ?
$\triangle ACD \cong \triangle CAB$ m. ?

35. **PROOF** Write a flow proof to prove *Congruence of triangles is reflexive*. (Theorem 4.4)

36. **CRITICAL THINKING** $\triangle RST$ is isosceles with $RS = RT$, M, N, and P are midpoints of their sides, $\angle S \cong \angle MPS$, and $\overline{NP} \cong \overline{MP}$. What else do you need to know to prove that $\triangle SMP \cong \triangle TNP$?



37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are triangles used in bridges?

Include the following in your answer:
- whether the shape of the triangle matters, and
- whether the triangles appear congruent.

Standardized Test Practice

38. Determine which statement is true given $\triangle ABC \cong \triangle XYZ$.

Ⓐ $\overline{BC} \cong \overline{ZX}$ Ⓑ $\overline{AC} \cong \overline{XZ}$ Ⓒ $\overline{AB} \cong \overline{YZ}$ Ⓓ cannot be determined

39. **ALGEBRA** Find the length of $\overline{DF}$ if $D(-5, 4)$ and $F(3, -7)$.

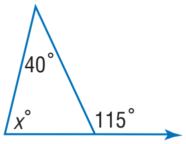
Ⓐ $\sqrt{5}$ Ⓑ $\sqrt{13}$ Ⓒ $\sqrt{57}$ Ⓓ $\sqrt{185}$

Maintain Your Skills

Mixed Review

Find x. *(Lesson 4-2)*

40.



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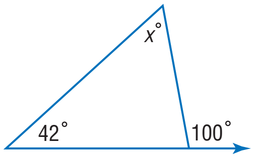
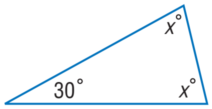


42.



Find x and the measure of each side of the triangle. *(Lesson 4-1)*

43. $\triangle BCD$ is isosceles with $\overline{BC} \cong \overline{CD}$, $BC = 2x + 4$, $BD = x + 2$, and $CD = 10$.

44. Triangle HKT is equilateral with $HK = x + 7$ and $HT = 4x - 8$.

Write an equation in slope-intercept form for the line that satisfies the given conditions. *(Lesson 3-4)*

45. contains $(0, 3)$ and $(4, -3)$

46. $m = \frac{3}{4}$, y-intercept $= 8$

47. parallel to $y = -4x + 1$; contains $(-3, 1)$

48. $m = -4$, contains $(-3, 2)$

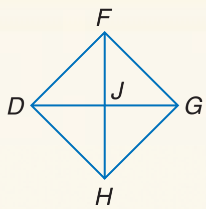
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.**
*(To review the **Distance Formula**, see Lesson 1-4.)*

49. $(-1, 7), (1, 6)$ 50. $(8, 2), (4, -2)$ 51. $(3, 5), (5, 2)$

Practice Quiz 1 — Lessons 4-1 through 4-3

1. Identify the isosceles triangles in the figure, if $\overline{FH}$ and $\overline{DG}$ are congruent perpendicular bisectors. *(Lesson 4-1)*

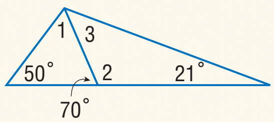


ALGEBRA **$\triangle ABC$ is equilateral with $AB = 2x$, $BC = 4x - 7$, and $AC = x + 3.5$.** *(Lesson 4-1)*

2. Find x.

3. Find the measure of each side.

4. Find the measure of each numbered angle. *(Lesson 4-2)*



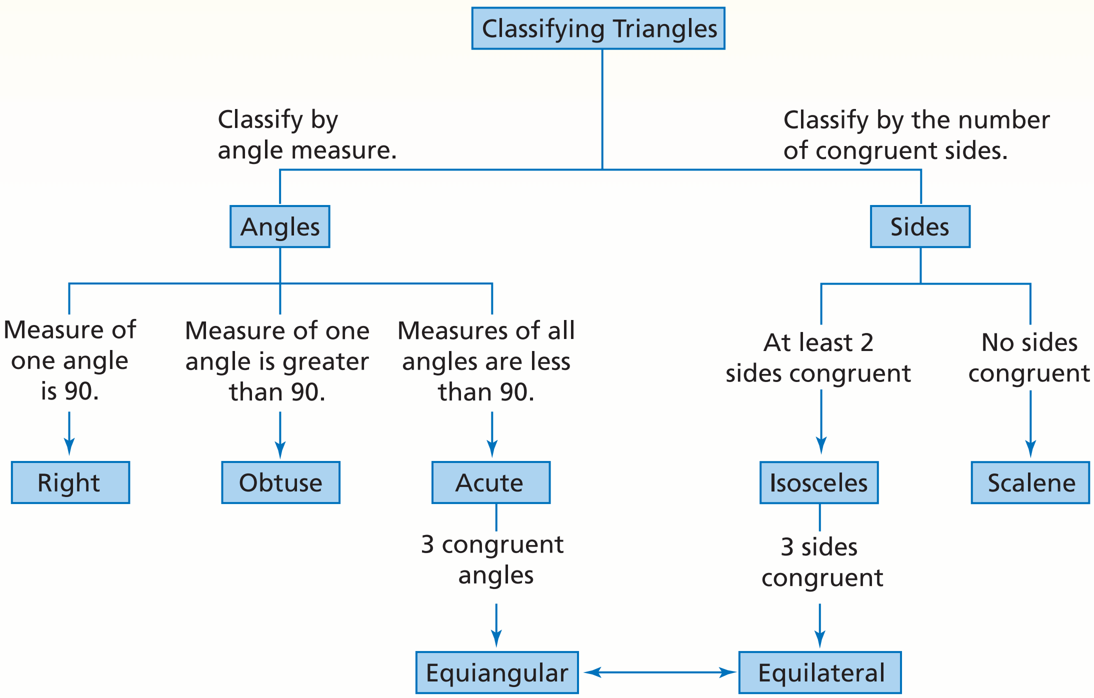
5. If $\triangle MNP \cong \triangle JKL$, name the corresponding congruent angles and sides. *(Lesson 4-3)*

Reading Mathematics

Making Concept Maps

When studying a chapter, it is wise to record the main topics and vocabulary you encounter. In this chapter, some of the new vocabulary words were *triangle*, *acute triangle*, *obtuse triangle*, *right triangle*, *equiangular triangle*, *scalene triangle*, *isosceles triangle*, and *equilateral triangle*. The triangles are all related by the size of the angles or the number of congruent sides.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the different types of triangles. The main ideas are in boxes. Any information that describes how to move from one box to the next is placed along the arrows.



Reading to Learn

1. Describe how to use the concept map to classify triangles by their side lengths.

2. In $\triangle ABC$, $m\angle A = 48$, $m\angle B = 41$, and $m\angle C = 91$. Use the concept map to classify $\triangle ABC$.

3. Identify the type of triangle that is linked to both classifications.

4-4 Proving Congruence—SSS, SAS

What You'll Learn

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

How do land surveyors use congruent triangles?

Land surveyors mark and establish property boundaries. To check a measurement, they mark out a right triangle and then mark a second triangle that is congruent to the first.

Vocabulary

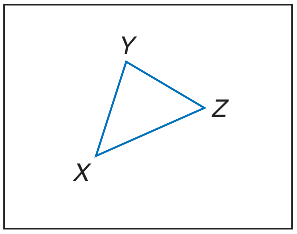
- included angle

SSS POSTULATE Is it always necessary to show that all of the corresponding parts of two triangles are congruent to prove that the triangles are congruent? In this lesson, we will explore two other methods to prove that triangles are congruent.

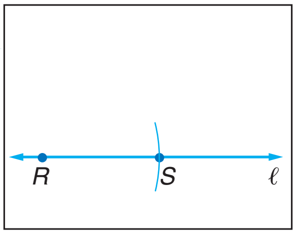
Construction

Congruent Triangles Using Sides

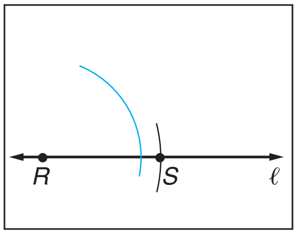
1. Draw a triangle and label the vertices X, Y, and Z.



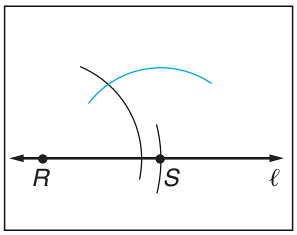
2. Use a straightedge to draw any line ℓ and select a point R. Use a compass to construct $\overline{RS}$ on ℓ such that $\overline{RS} \cong \overline{XZ}$.



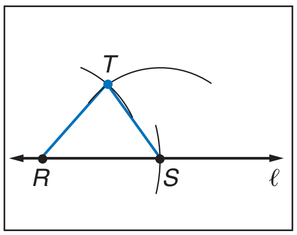
3. Using R as the center, draw an arc with radius equal to XY.



4. Using S as the center, draw an arc with radius equal to YZ.



5. Let T be the point of intersection of the two arcs. Draw $\overline{RT}$ and $\overline{ST}$ to form $\triangle RST$.



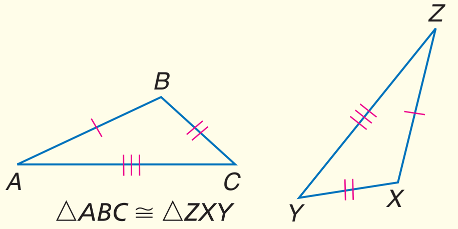
6. Cut out $\triangle RST$ and place it over $\triangle XYZ$. How does $\triangle RST$ compare to $\triangle XYZ$?

If the corresponding sides of two triangles are congruent, then the triangles are congruent. This is the Side-Side-Side Postulate, and is written as SSS.

Postulate 4.1

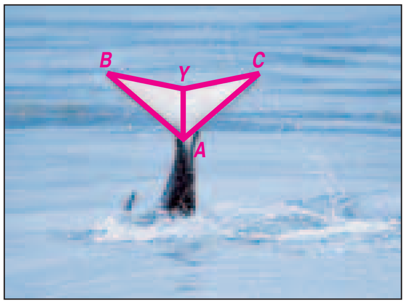
Side-Side-Side Congruence If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Abbreviation: *SSS*



$\triangle ABC \cong \triangle ZXY$

Example 1 Use SSS in Proofs



MARINE BIOLOGY **The tail of an orca whale can be viewed as two triangles that share a common side. Write a two-column proof to prove that $\triangle BYA \cong \triangle CYA$ if $\overline{AB} \cong \overline{AC}$ and $\overline{BY} \cong \overline{CY}$.**

Given: $\overline{AB} \cong \overline{AC}$; $\overline{BY} \cong \overline{CY}$

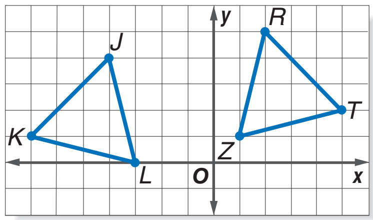
Prove: $\triangle BYA \cong \triangle CYA$

Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$; $\overline{BY} \cong \overline{CY}$	1. Given
2. $\overline{AY} \cong \overline{AY}$	2. Reflexive Property
3. $\triangle BYA \cong \triangle CYA$	3. SSS

Example 2 SSS on the Coordinate Plane

COORDINATE GEOMETRY **Determine whether $\triangle RTZ \cong \triangle JKL$ for $R(2, 5)$, $Z(1, 1)$, $T(5, 2)$, $L(-3, 0)$, $K(-7, 1)$, and $J(-4, 4)$. Explain.**



Use the Distance Formula to show that the corresponding sides are congruent.

$$RT = \sqrt{(2-5)^2 + (5-2)^2} = \sqrt{9+9} \text{ or } \sqrt{18}$$

$$JK = \sqrt{[-4-(-7)]^2 + (4-1)^2} = \sqrt{9+9} \text{ or } \sqrt{18}$$

$$TZ = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$KL = \sqrt{[-7-(-3)]^2 + (1-0)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$RZ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$JL = \sqrt{[-4-(-3)]^2 + (4-0)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$RT = JK$, $TZ = KL$, and $RZ = JL$. By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle RTZ \cong \triangle JKL$ by SSS.

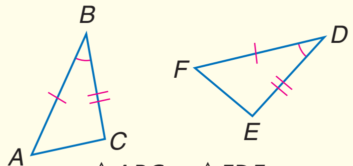
SAS POSTULATE Suppose you are given the measures of two sides and the angle they form, called the **included angle**. These conditions describe a unique triangle. Two triangles in which corresponding sides and the included pairs of angles are congruent provide another way to show that triangles are congruent.

www.geometryonline.com/extra_examples

Postulate 4.2

Side-Angle-Side Congruence If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Abbreviation: *SAS*



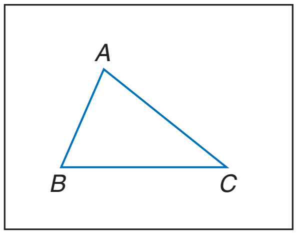
$\triangle ABC \cong \triangle FDE$

You can also construct congruent triangles given two sides and the included angle.

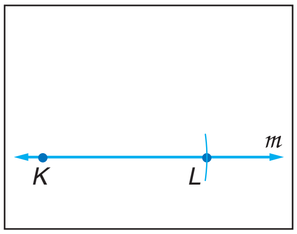
Construction

Congruent Triangles using Two Sides and the Included Angle

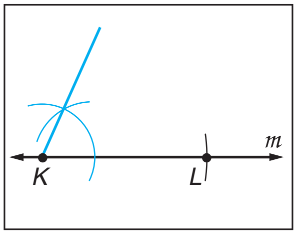
1. Draw a triangle and label its vertices *A*, *B*, and *C*.



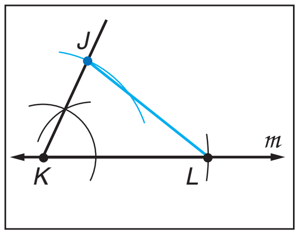
2. Select a point *K* on line m. Use a compass to construct $\overline{KL}$ on m such that $\overline{KL} \cong \overline{BC}$.



3. Construct an angle congruent to $\angle B$ using $\overrightarrow{KL}$ as a side of the angle and point *K* as the vertex.



4. Construct $\overline{JK}$ such that $\overline{JK} \cong \overline{AB}$. Draw $\overline{JL}$ to complete $\triangle JKL$.



5. Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

Study Tip

Flow Proofs

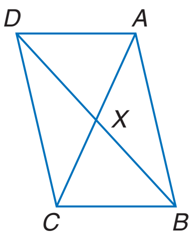
Flow proofs can be written vertically or horizontally.

Example 3 Use SAS in Proofs

Write a flow proof.

Given: X is the midpoint of $\overline{BD}$.
X is the midpoint of $\overline{AC}$.

Prove: $\triangle DXC \cong \triangle BXA$



Flow Proof:

X is the midpoint of $\overline{DB}$. (Given) → $\overline{DX} \cong \overline{BX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$

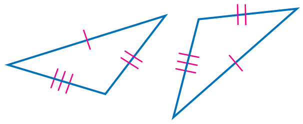
X is the midpoint of $\overline{AC}$. (Given) → $\overline{CX} \cong \overline{AX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$ (SAS)

$\angle DXC \cong \angle BXA$ (Vertical ∠s are ≅.) → $\triangle DXC \cong \triangle BXA$

Example 4 Identify Congruent Triangles

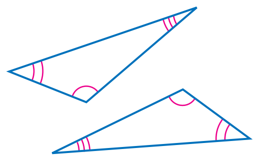
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

a.



Each pair of corresponding sides are congruent. The triangles are congruent by the SSS Postulate.

b.



The triangles have three pairs of corresponding angles congruent. This does not match the SSS Postulate or the SAS Postulate. It is *not possible* to prove the triangles congruent.

Check for Understanding

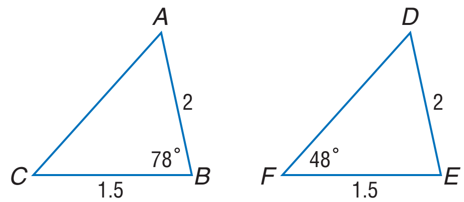
Concept Check

1. **OPEN ENDED** Draw a triangle and label the vertices. Name two sides and the included angle.

2. **FIND THE ERROR** Carmelita and Jonathan are trying to determine whether $\triangle ABC$ is congruent to $\triangle DEF$.

Carmelita
$\triangle ABC \cong \triangle DEF$ by SAS

Jonathan
Congruence cannot be determined.



Who is correct and why?

Guided Practice

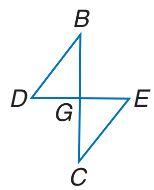
Determine whether $\triangle EFG \cong \triangle MNP$ given the coordinates of the vertices. Explain.

3. $E(-4, -3)$, $F(-2, 1)$, $G(-2, -3)$, $M(4, -3)$, $N(2, 1)$, $P(2, -3)$

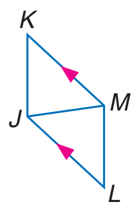
4. $E(-2, -2)$, $F(-4, 6)$, $G(-3, 1)$, $M(2, 2)$, $N(4, 6)$, $P(3, 1)$

5. Write a flow proof.
   Given: $\overline{DE}$ and $\overline{BC}$ bisect each other.
   Prove: $\triangle DGB \cong \triangle EGC$

6. Write a two-column proof.
   Given: $\overline{KM} \parallel \overline{JL}$, $\overline{KM} \cong \overline{JL}$
   Prove: $\triangle JKM \cong \triangle MLJ$



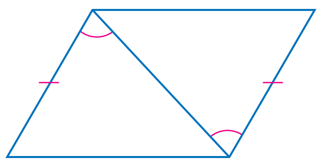
Exercise 5



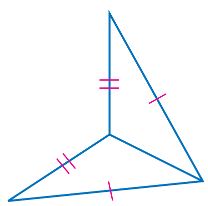
Exercise 6

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

7.

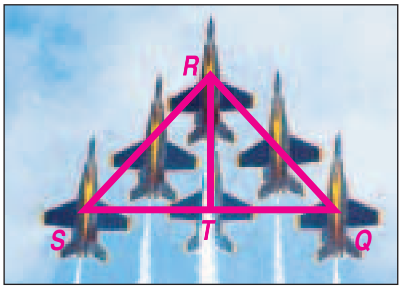


8.



Application

9. **PRECISION FLIGHT** The United States Navy Flight Demonstration Squadron, the Blue Angels, fly in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$.



Practice and Apply

Homework Help

For Exercises	See Examples
10–13	2
14–19	3
20–21, 28–29	1
22–27	4

Extra Practice
See page 761.

Determine whether $\triangle JKL \cong \triangle FGH$ given the coordinates of the vertices. Explain.

10. $J(-3, 2)$, $K(-7, 4)$, $L(-1, 9)$, $F(2, 3)$, $G(4, 7)$, $H(9, 1)$

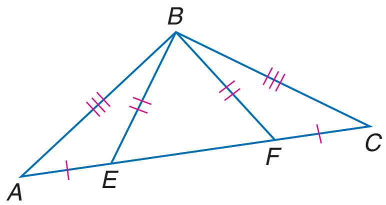
11. $J(-1, 1)$, $K(-2, -2)$, $L(-5, -1)$, $F(2, -1)$, $G(3, -2)$, $H(2, 5)$

12. $J(-1, -1)$, $K(0, 6)$, $L(2, 3)$, $F(3, 1)$, $G(5, 3)$, $H(8, 1)$

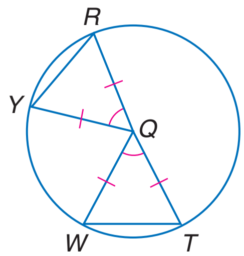
13. $J(3, 9)$, $K(4, 6)$, $L(1, 5)$, $F(1, 7)$, $G(2, 4)$, $H(-1, 3)$

Write a flow proof.

14. **Given:** $\overline{AE} \cong \overline{FC}$, $\overline{AB} \cong \overline{BC}$, $\overline{BE} \cong \overline{BF}$
Prove: $\triangle AFB \cong \triangle CEB$

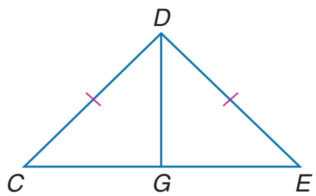


15. **Given:** $\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$
$\angle RQY \cong \angle WQT$
Prove: $\triangle QWT \cong \triangle QYR$

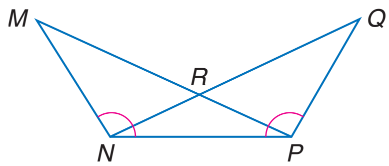


Write a two-column proof.

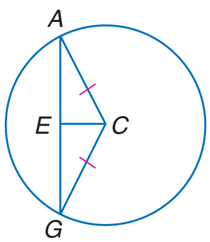
16. **Given:** $\triangle CDE$ is isosceles.
G is the midpoint of $\overline{CE}$.
Prove: $\triangle CDG \cong \triangle EDG$



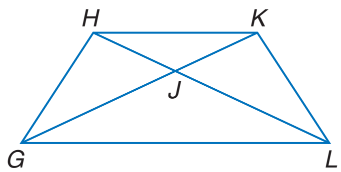
17. **Given:** $\triangle MRN \cong \triangle QRP$
$\angle MNP \cong \angle QPN$
Prove: $\triangle MNP \cong \triangle QPN$



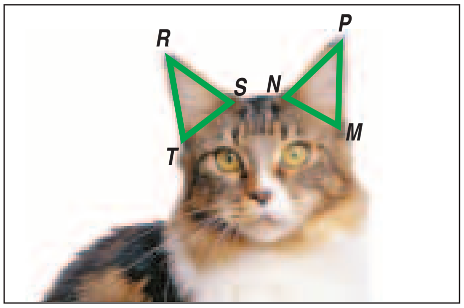
18. **Given:** $\overline{AC} \cong \overline{GC}$
$\overline{EC}$ bisects $\overline{AG}$.
Prove: $\triangle GEC \cong \triangle AEC$



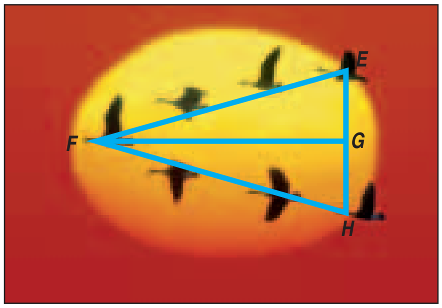
19. **Given:** $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



20. **CATS** A cat's ear is triangular in shape. Write a two-column proof to prove $\triangle RST \cong \triangle PNM$ if $\overline{RS} \cong \overline{PN}$, $\overline{RT} \cong \overline{MP}$, $\angle S \cong \angle N$, and $\angle T \cong \angle M$.

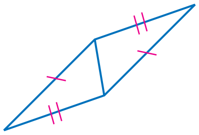


21. **GEESE** This photograph shows a flock of geese flying in formation. Write a two-column proof to prove that $\triangle EFG \cong \triangle HFG$, if $\overline{EF} \cong \overline{HF}$ and G is the midpoint of $\overline{EH}$.

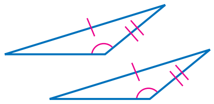


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

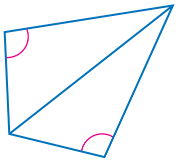
22.



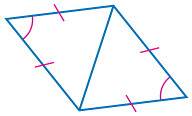
23.



24.



25.

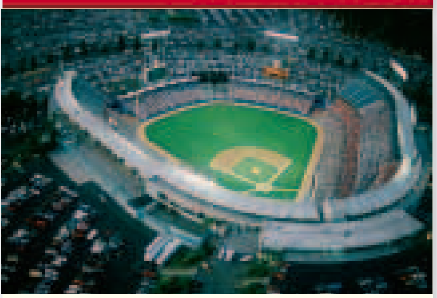


BASEBALL **For Exercises 26 and 27, use the following information.**
A baseball diamond is a square with four right angles and all sides congruent.

26. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

27. Write a two-column proof to prove that the angle formed by second base, home plate, and third base is the same as the angle formed by second base, home plate, and first base.

More About. . .



Baseball

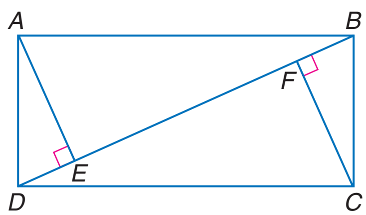
The infield is a square 90 feet on each side.

Source: www.mlb.com

28. **CRITICAL THINKING** Devise a plan and write a two-column proof for the following.

Given: $\overline{DE} \cong \overline{FB}$, $\overline{AE} \cong \overline{FC}$, $\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{DB}$

Prove: $\triangle ABD \cong \triangle CDB$



29. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do land surveyors use congruent triangles?

Include the following in your answer:

- description of three methods to prove triangles congruent, and
- another example of a career that uses properties of congruent triangles.

Standardized Test Practice

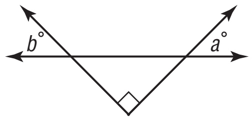
30. Which of the following statements about the figure is true?

Ⓐ $90 > a + b$

Ⓑ $a + b > 90$

Ⓒ $a + b = 90$

Ⓓ $a > b$



31. Classify the triangle with the measures of the angles in the ratio 3:6:7.

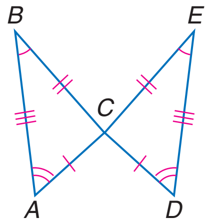
Ⓐ isosceles Ⓑ acute Ⓒ obtuse Ⓓ right

Maintain Your Skills

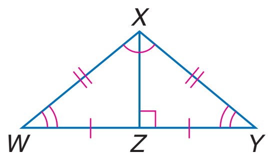
Mixed Review

Identify the congruent triangles in each figure. *(Lesson 4-3)*

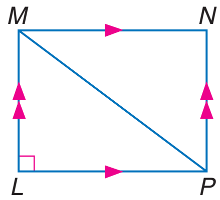
32.



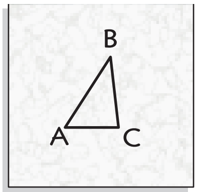
33.



34.



Find each measure if $\overline{PQ} \perp \overline{QR}$.
(Lesson 4-2)



35. $m\angle 2$ **36.** $m\angle 3$

37. $m\angle 5$ **38.** $m\angle 4$

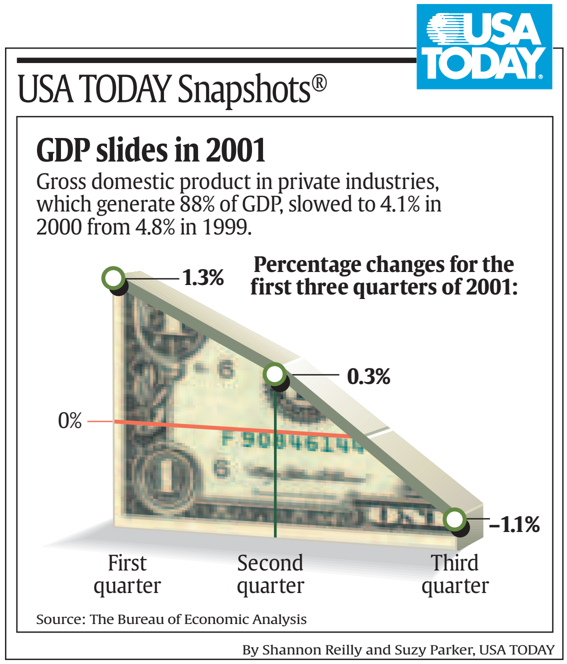
39. $m\angle 1$ **40.** $m\angle 6$

For Exercises 41–43, use the graphic at the right. *(Lesson 3-3)*

41. Find the rate of change from first quarter to the second quarter.

42. Find the rate of change from the second quarter to the third quarter.

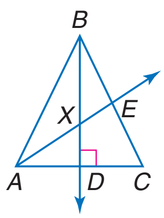
43. Compare the rate of change from the first quarter to the second, and the second quarter to the third. Which had the greater rate of change?



Getting Ready for the Next Lesson

PREREQUISITE SKILL $\overrightarrow{BD}$ **and** $\overrightarrow{AE}$ **are angle bisectors and segment bisectors. Name the indicated segments and angles.**

(To review ***bisectors of segments and angles****, see Lessons 1-5 and 1-6.)*



44. a segment congruent to $\overline{EC}$

45. an angle congruent to $\angle ABD$

46. an angle congruent to $\angle BDC$

47. a segment congruent to $\overline{AD}$

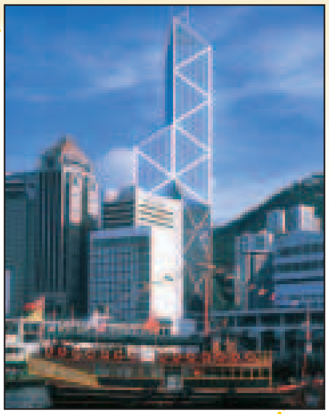
4-5 Proving Congruence—ASA, AAS

What You'll Learn

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.

How are congruent triangles used in construction?

The Bank of China Tower in Hong Kong has triangular trusses for structural support. These trusses form congruent triangles. In this lesson, we will explore two additional methods of proving triangles congruent.



Vocabulary

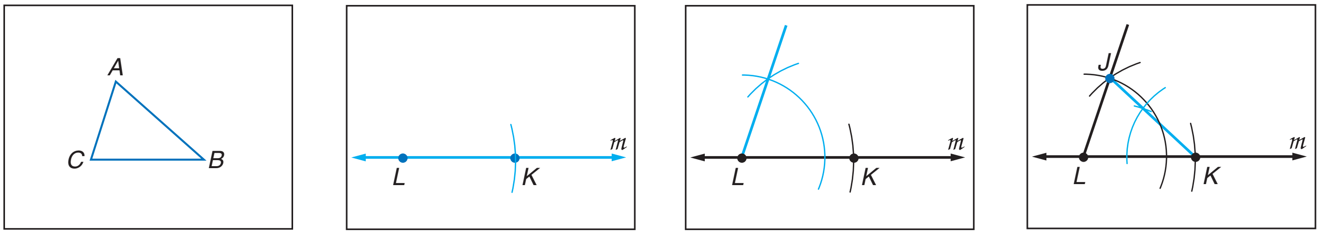
- included side

ASA POSTULATE Suppose you were given the measures of two angles of a triangle and the side between them, the **included side**. Do these measures form a unique triangle?

Construction

Congruent Triangles Using Two Angles and Included Side

1. Draw a triangle and label its vertices A, B, and C.
2. Draw any line m and select a point L. Construct $\overline{LK}$ such that $\overline{LK} \cong \overline{CB}$.
3. Construct an angle congruent to $\angle C$ at L using $\overrightarrow{LK}$ as a side of the angle.
4. Construct an angle congruent to $\angle B$ at K using $\overrightarrow{LK}$ as a side of the angle. Label the point where the new sides of the angles meet J.



5. Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

This construction leads to the Angle-Side-Angle Postulate, written as ASA.

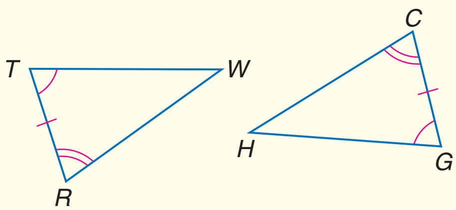
Study Tip

Reading Math
The included side refers to the side that each of the angles share.

Postulate 4.3

Angle-Side-Angle Congruence If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

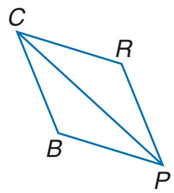
Abbreviation: *ASA*



$\triangle RTW \cong \triangle CGH$

Sylvain Grandadam/Photo Researchers

Example 1 Use ASA in Proofs



Write a paragraph proof.

Given: $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$.

Prove: $\triangle BCP \cong \triangle RCP$

Proof:

Since $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$, $\angle BCP \cong \angle RCP$ and $\angle BPC \cong \angle RPC$. $\overline{CP} \cong \overline{CP}$ by the Reflexive Property. By ASA, $\triangle BCP \cong \triangle RCP$.

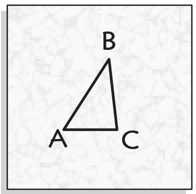
AAS THEOREM Suppose you are given the measures of two angles and a nonincluded side. Is this information sufficient to prove two triangles congruent?

Geometry Activity

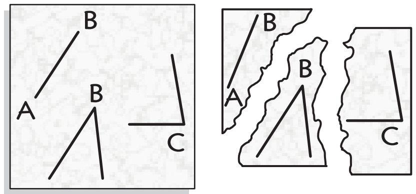
Angle-Angle-Side Congruence

Model

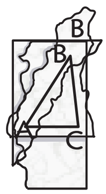
1. Draw a triangle on a piece of patty paper. Label the vertices A, B, and C.



2. Copy $\overline{AB}$, $\angle B$, and $\angle C$ on another piece of patty paper and cut them out.



3. Assemble them to form a triangle in which the side is not the included side of the angles.



Analyze

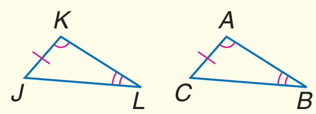
1. Place the original $\triangle ABC$ over the assembled figure. How do the two triangles compare?
2. **Make a conjecture** about two triangles with two angles and the nonincluded side of one triangle congruent to two angles and the nonincluded side of the other triangle.

This activity leads to the Angle-Angle-Side Theorem, written as AAS.

Theorem 4.5

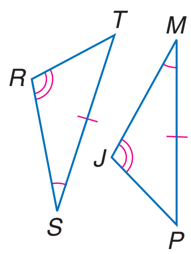
Angle-Angle-Side Congruence If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

Abbreviation: *AAS*



Example: $\triangle JKL \cong \triangle CAB$

Proof Theorem 4.5



Given: $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$

Prove: $\triangle JMP \cong \triangle RST$

Proof:

Statements	Reasons
1. $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$	1. Given
2. $\angle P \cong \angle T$	2. Third Angle Theorem
3. $\triangle JMP \cong \triangle RST$	3. ASA

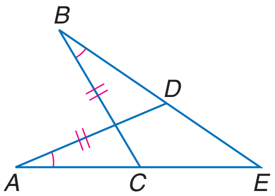
Study Tip

Overlapping Triangles

When triangles overlap, it is a good idea to draw each triangle separately and label the congruent parts.

Example 2 Use AAS in Proofs

Write a flow proof.



Given: $\angle EAD \cong \angle EBC$

$\overline{AD} \cong \overline{BC}$

Prove: $\overline{AE} \cong \overline{BE}$

Flow Proof:

$\angle EAD \cong \angle EBC$ — Given

$\overline{AD} \cong \overline{BC}$ — Given

$\angle E \cong \angle E$ — Reflexive Property

$\rightarrow$ $\triangle ADE \cong \triangle BCE$ — AAS $\rightarrow$ $\overline{AE} \cong \overline{BE}$ — CPCTC

You have learned several methods for proving triangle congruence. The Concept Summary lists ways to help you determine which method to use.

Concept Summary — Methods to Prove Triangle Congruence

Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle must be congruent to two angles and side of the other triangle.

Career Choices

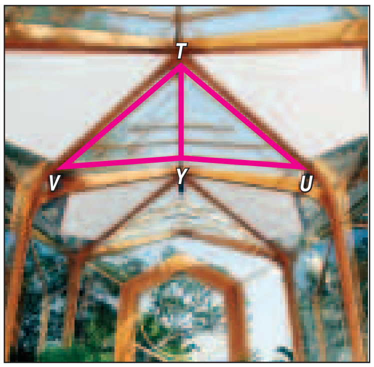
Architect

About 28% of architects are self-employed. Architects design a variety of buildings including offices, retail spaces, and schools.

Online Research

For information about a career as an architect, visit: www.geometryonline.com/careers

Example 3 Determine if Triangles Are Congruent



ARCHITECTURE **This glass chapel was designed by Frank Lloyd Wright's son, Lloyd Wright. Suppose the redwood supports, $\overline{TU}$ and $\overline{TV}$, measure 3 feet, $TY = 1.6$ feet, and $m\angle U$ and $m\angle V$ are 31. Determine whether $\triangle TYU \cong \triangle TYV$. Justify your answer.**

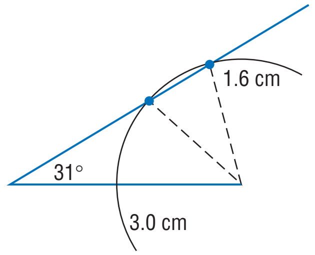
Explore We are given three measurements of each triangle. We need to determine whether the two triangles are congruent.

Plan Since $m\angle U = m\angle V$, $\angle U \cong \angle V$. Likewise, $TU = TV$ so $\overline{TU} \cong \overline{TV}$, and $TY = TY$ so $\overline{TY} \cong \overline{TY}$. Check each possibility using the five methods you know.

Solve We are given information about side-side-angle (SSA). This is not a method to prove two triangles congruent.

(continued on the next page)

Examine Use a compass, protractor, and ruler to draw a triangle with the given measurements. For simplicity of measurement, we will use centimeters instead of feet, so the measurements of the construction and those of the support beams will be proportional.



- Draw a segment 3.0 centimeters long.
- At one end, draw an angle of 31°. Extend the line longer than 3.0 centimeters.
- At the other end of the segment, draw an arc with a radius of 1.6 centimeters such that it intersects the line.

Notice that there are two possible segments that could determine the triangle. Since the given measurements do not lead to a unique triangle, we cannot show that the triangles are congruent.

Check for Understanding

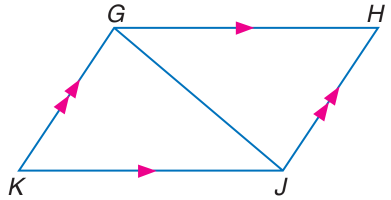
Concept Check

1. **Find a counterexample** to show why AAA (Angle-Angle-Angle) cannot be used to prove triangle congruence.
2. **OPEN ENDED** Draw a triangle and label the vertices. Name two angles and the included side.
3. **Explain** why AAS is a theorem, not a postulate.

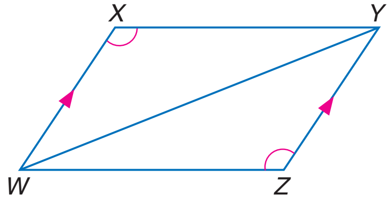
Guided Practice

Write a flow proof.

4. **Given:** $\overline{GH} \parallel \overline{KJ}$, $\overline{GK} \parallel \overline{HJ}$
   Prove: $\triangle GJK \cong \triangle JGH$

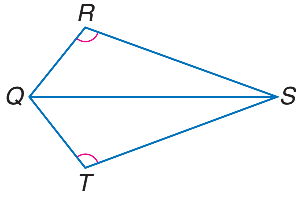


5. **Given:** $\overline{XW} \parallel \overline{YZ}$, $\angle X \cong \angle Z$
   Prove: $\triangle WXY \cong \triangle YZW$

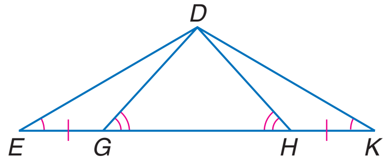


Write a paragraph proof.

6. **Given:** $\overline{QS}$ bisects $\angle RST$; $\angle R \cong \angle T$.
   Prove: $\triangle QRS \cong \triangle QTS$

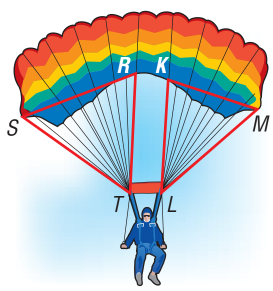


7. **Given:** $\angle E \cong \angle K$, $\angle DGH \cong \angle DHG$
   $\overline{EG} \cong \overline{KH}$
   Prove: $\triangle EGD \cong \triangle KHD$



Application

8. **PARACHUTES** Suppose $\overline{ST}$ and $\overline{ML}$ each measure 7 feet, $\overline{SR}$ and $\overline{MK}$ each measure 5.5 feet, and $m\angle T = m\angle L = 49$. Determine whether $\triangle SRT \cong \triangle MKL$. Justify your answer.



Practice and Apply

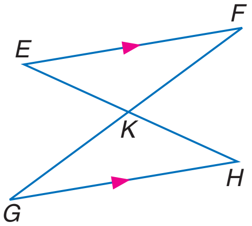
Homework Help

For Exercises	See Examples
9, 11, 14, 15–18	2
10, 12, 13, 19, 20	1
21–28	3

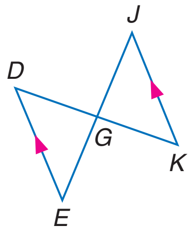
Extra Practice
See page 762.

Write a flow proof.

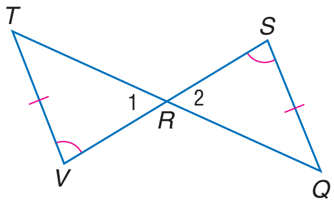
9. Given: $\overline{EF} \parallel \overline{GH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\overline{EK} \cong \overline{KH}$



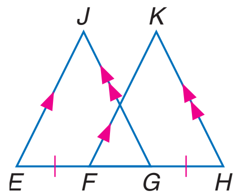
10. Given: $\overline{DE} \parallel \overline{JK}$, $\overline{DK}$ bisects $\overline{JE}$.
Prove: $\triangle EGD \cong \triangle JGK$



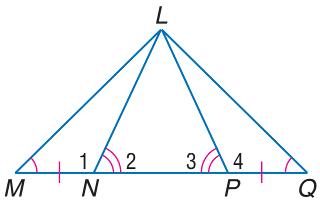
11. Given: $\angle V \cong \angle S$, $\overline{TV} \cong \overline{QS}$
Prove: $\overline{VR} \cong \overline{SR}$



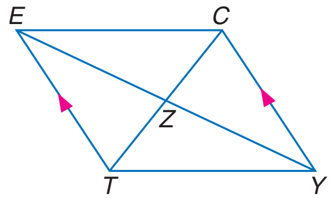
12. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\triangle EJG \cong \triangle FKH$



13. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$
$\angle 2 \cong \angle 3$
Prove: $\triangle MLP \cong \triangle QLN$

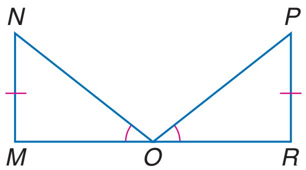


14. Given: Z is the midpoint of $\overline{CT}$.
$\overline{CY} \parallel \overline{TE}$
Prove: $\overline{YZ} \cong \overline{EZ}$

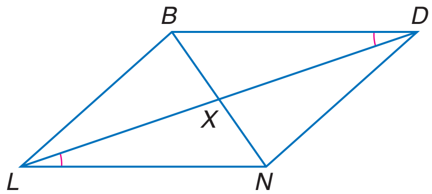


Write a paragraph proof.

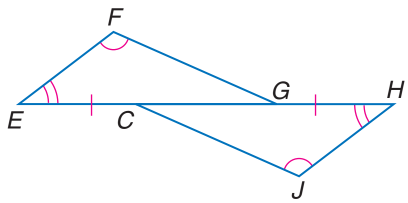
15. Given: $\angle NOM \cong \angle POR$,
$\overline{NM} \perp \overline{MR}$
$\overline{PR} \perp \overline{MR}$, $\overline{NM} \cong \overline{PR}$
Prove: $\overline{MO} \cong \overline{OR}$



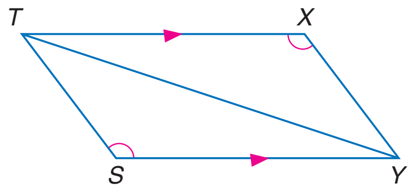
16. Given: $\overline{DL}$ bisects $\overline{BN}$,
$\angle XLN \cong \angle XDB$
Prove: $\overline{LN} \cong \overline{DB}$



17. Given: $\angle F \cong \angle J$, $\angle E \cong \angle H$
$\overline{EC} \cong \overline{GH}$
Prove: $\overline{EF} \cong \overline{HJ}$

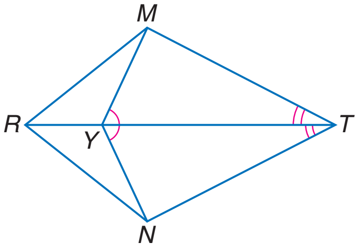


18. Given: $\overline{TX} \parallel \overline{SY}$
$\angle TXY \cong \angle TSY$
Prove: $\triangle TSY \cong \triangle YXT$

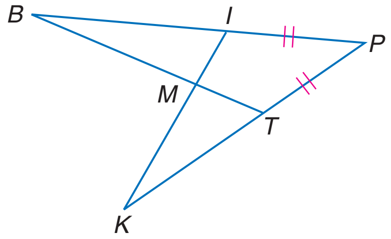


Write a two-column proof.

19. Given: $\angle MYT \cong \angle NYT$
$\angle MTY \cong \angle NTY$
Prove: $\triangle RYM \cong \triangle RYN$



20. Given: $\triangle BMI \cong \triangle KMT$
$\overline{IP} \cong \overline{PT}$
Prove: $\triangle IPK \cong \triangle TPB$



GARDENING **For Exercises 21 and 22, use the following information.**
Beth is planning a garden. She wants the triangular sections, $\triangle CFD$ and $\triangle HFG$, to be congruent. F is the midpoint of $\overline{DG}$, and $DG = 16$ feet.

21. Suppose $\overline{CD}$ and $\overline{GH}$ each measure 4 feet and the measure of $\angle CFD$ is 29. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.
22. Suppose F is the midpoint of $\overline{CH}$, and $\overline{CH} \cong \overline{DG}$. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

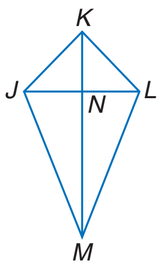
More About. . .

Kites

The largest kite ever flown was 210 feet long and 72 feet wide.

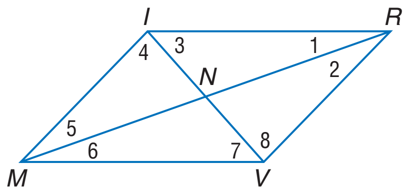
Source: *Guinness Book of World Records*

KITES **For Exercises 23 and 24, use the following information.**
Austin is building a kite. Suppose JL is 2 feet, JM is 2.7 feet, and the measure of $\angle NJM$ is 68.



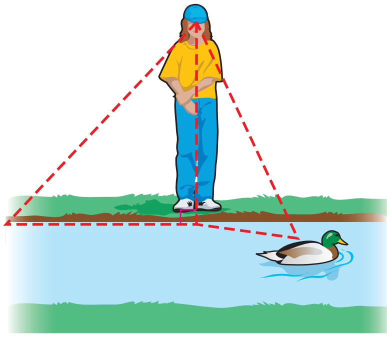
23. If N is the midpoint of $\overline{JL}$ and $\overline{KM} \perp \overline{JL}$, determine whether $\triangle JKN \cong \triangle LKN$. Justify your answer.
24. If $\overline{JM} \cong \overline{LM}$ and $\angle NJM \cong \angle NLM$, determine whether $\triangle JNM \cong \triangle LNM$. Justify your answer.

Complete each congruence statement and the postulate or theorem that applies.



25. If $\overline{IM} \cong \overline{RV}$ and $\angle 2 \cong \angle 5$, then $\triangle INM \cong \triangle$ __?__ by __?__.
26. If $\overline{IR} \parallel \overline{MV}$ and $\overline{IR} \cong \overline{MV}$, then $\triangle IRN \cong \triangle$ __?__ by __?__.
27. If $\overline{IV}$ and $\overline{RM}$ bisect each other, then $\triangle RVN \cong \triangle$ __?__ by __?__.
28. If $\angle MIR \cong \angle RVM$ and $\angle 1 \cong \angle 6$, then $\triangle MRV \cong \triangle$ __?__ by __?__.

29. **CRITICAL THINKING** Aiko wants to estimate the distance between herself and a duck. She adjusts the visor of her cap so that it is in line with her line of sight to the duck. She keeps her neck stiff and turns her body to establish a line of sight to a point on the ground. Then she paces out the distance to the new point. Is the distance from the duck the same as the distance she just paced out? Explain your reasoning.



30. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are congruent triangles used in construction?

Include the following in your answer:

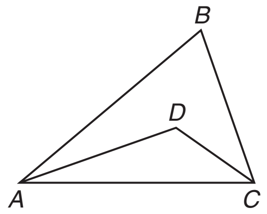
- explain how to determine whether the triangles are congruent, and
- why it is important that triangles used for structural support are congruent.

Standardized Test Practice

31. In $\triangle ABC$, $\overline{AD}$ and $\overline{DC}$ are angle bisectors and $m\angle B = 76$. What is the measure of $\angle ADC$?

Ⓐ 26 Ⓑ 52

Ⓒ 76 Ⓓ 128



32. **ALGEBRA** For a positive integer x, 1 percent of x percent of 10,000 equals

Ⓐ x. Ⓑ $10x$. Ⓒ $100x$. Ⓓ $1000x$.

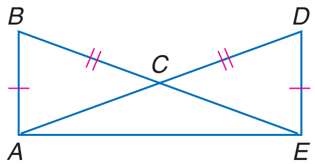
Maintain Your Skills

Mixed Review

Write a flow proof. *(Lesson 4-4)*

33. **Given:** $\overline{BA} \cong \overline{DE}$, $\overline{DA} \cong \overline{BE}$

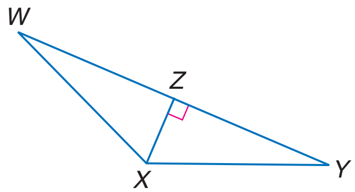
Prove: $\triangle BEA \cong \triangle DAE$



34. **Given:** $\overline{XZ} \perp \overline{WY}$

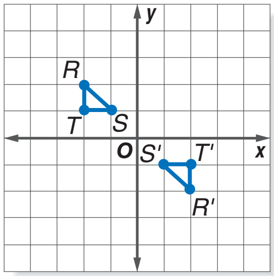
$\overline{XZ}$ bisects $\overline{WY}$.

Prove: $\triangle WZX \cong \triangle YZX$

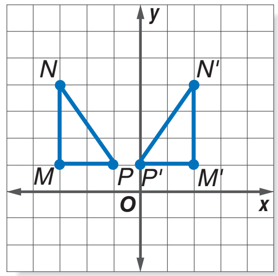


Verify that each of the following preserves congruence and name the congruence transformation. *(Lesson 4-3)*

35.



36.



Write each statement in if-then form. *(Lesson 2-3)*

37. Happy people rarely correct their faults.

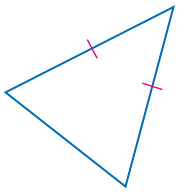
38. A champion is afraid of losing.

Getting Ready for the Next Lesson

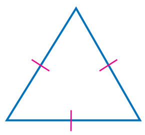
PREREQUISITE SKILL **Classify each triangle according to its sides.**

*(To review **classification by sides**, see Lesson 4-1.)*

39.



40.



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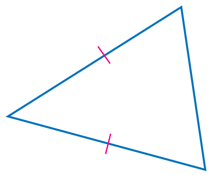


Geometry Activity

A Follow-Up of Lesson 4-5

Congruence in Right Triangles

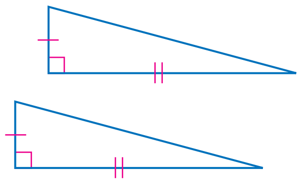
In Lessons 4-4 and 4-5, you learned theorems and postulates to prove triangles congruent. Do these theorems and postulates apply to right triangles?

Activity 1 Triangle Congruence

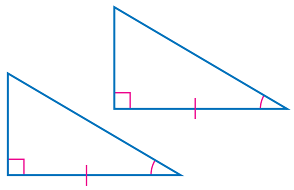
Model

Study each pair of right triangles.

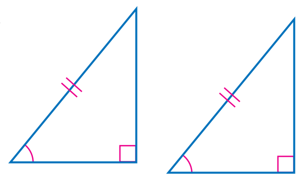
a.



b.



c.



Analyze

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?
2. Rewrite the congruence rules from Exercise 1 using *leg*, (L), or *hypotenuse*, (H), to replace *side*. Omit the *A* for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.
3. **Make a conjecture** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

In Lesson 4-5, you learned that SSA is not a valid test for determining triangle congruence. Can SSA be used to prove right triangles congruent?

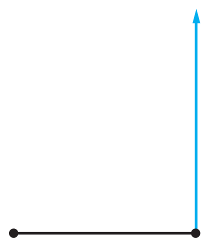
Activity 2 SSA and Right Triangles

Make a Model

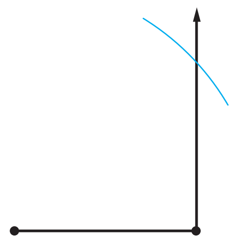
How many right triangles exist that have a hypotenuse of 10 centimeters and a leg of 7 centimeters?

Step 1 Draw $\overline{XY}$ so that $XY = 7$ centimeters.

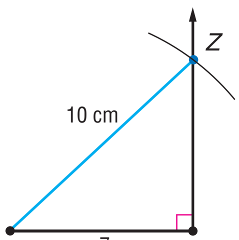
Step 2 Use a protractor to draw a ray from Y that is perpendicular to $\overline{XY}$.



Step 3 Open your compass to a width of 10 centimeters. Place the point at X and draw a long arc to intersect the ray.



Step 4 Label the intersection Z and draw $\overline{XZ}$ to complete $\triangle XYZ$.

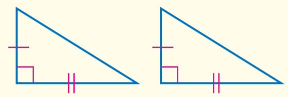
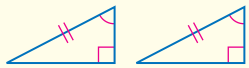
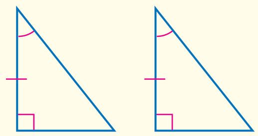
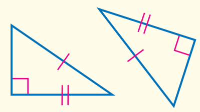


Analyze

4. Does the model yield a unique triangle?
5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?
6. **Make a conjecture** about the case of SSA that exists for right triangles.

The two activities provide evidence for four ways to prove right triangles congruent.

Key Concept — *Right Triangle Congruence*

Theorem	Abbreviation	Example
4.6 Leg-Leg Congruence If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.	LL	
4.7 Hypotenuse-Angle Congruence If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.	HA	
4.8 Leg-Angle Congruence If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.	LA	
Postulate		
4.4 Hypotenuse-Leg Congruence If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.	HL	

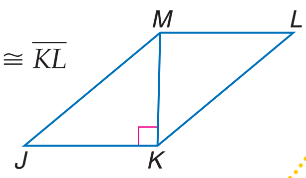
PROOF **Write a paragraph proof of each theorem.**

7. Theorem 4.6
8. Theorem 4.7
9. Theorem 4.8 (*Hint*: There are two possible cases.)

Use the figure to write a two-column proof.

10. **Given:** $\overline{ML} \perp \overline{MK}$, $\overline{JK} \perp \overline{KM}$, $\angle J \cong \angle L$
    Prove: $\overline{JM} \cong \overline{KL}$

11. **Given:** $\overline{JK} \perp \overline{KM}$, $\overline{JM} \cong \overline{KL}$, $\overline{ML} \parallel \overline{JK}$
    Prove: $\overline{ML} \cong \overline{JK}$



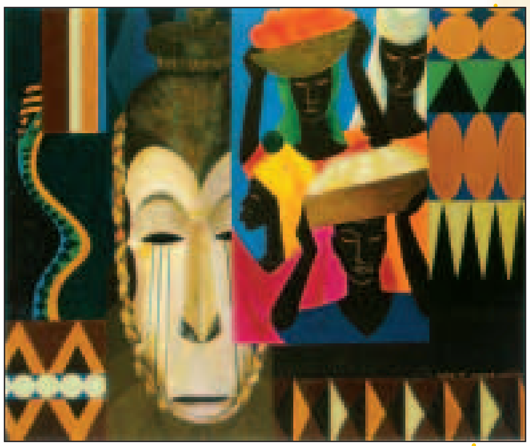
4-6 Isosceles Triangles

What You'll Learn

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

How are triangles used in art?

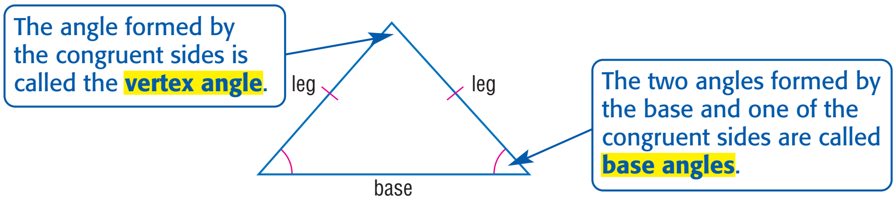
The art of Lois Mailou Jones, a twentieth-century artist, includes paintings and textile design, as well as book illustration. Notice the isosceles triangles in this painting, *Damballah*.



Vocabulary

- vertex angle
- base angles

PROPERTIES OF ISOSCELES TRIANGLES In Lesson 4-1, you learned that isosceles triangles have two congruent sides. Like the right triangle, the parts of an isosceles triangle have special names.



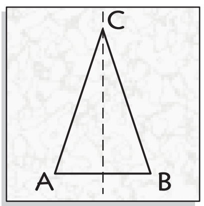
In this activity, you will investigate the relationship of the base angles and legs of an isosceles triangle.

Geometry Activity

Isosceles Triangles

Model

- Draw an acute triangle on patty paper with $\overline{AC} \cong \overline{BC}$.
- Fold the triangle through C so that A and B coincide.



Analyze

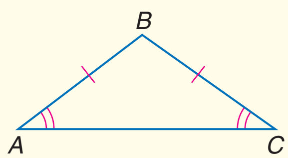
1. What do you observe about $\angle A$ and $\angle B$?
2. Draw an obtuse isosceles triangle. Compare the base angles.
3. Draw a right isosceles triangle. Compare the base angles.

The results of the Geometry Activity suggest Theorem 4.9.

Theorem 4.9

Isosceles Triangle Theorem If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

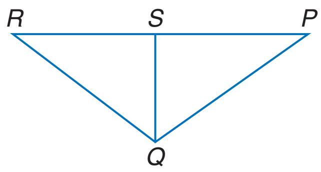
Example: If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.



Marvin T. Jones

Example 1 Proof of Theorem

Write a two-column proof of the Isosceles Triangle Theorem.



Given: $\triangle PQR$, $\overline{PQ} \cong \overline{RQ}$

Prove: $\angle P \cong \angle R$

Proof:

Statements	Reasons
1. Let S be the midpoint of $\overline{PR}$.	**1.** Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{QS}$.	**2.** Two points determine a line.
3. $\overline{PS} \cong \overline{RS}$	**3.** Midpoint Theorem
4. $\overline{QS} \cong \overline{QS}$	**4.** Congruence of segments is reflexive.
5. $\overline{PQ} \cong \overline{RQ}$	**5.** Given
6. $\triangle PQS \cong \triangle RQS$	**6.** SSS
7. $\angle P \cong \angle R$	**7.** CPCTC

Example 2 Find the Measure of a Missing Angle

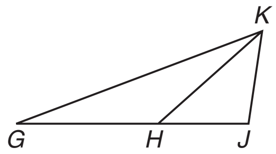
Standardized Test Practice

A B C D

Multiple-Choice Test Item

If $\overline{GH} \cong \overline{HK}$, $\overline{HJ} \cong \overline{JK}$, and $m\angle GJK = 100$, what is the measure of $\angle HGK$?

(A) 10 (B) 15 (C) 20 (D) 25



Test-Taking Tip

Diagrams Label the diagram with the given information. Use your drawing to plan the next step in solving the problem.

Read the Test Item

$\triangle GHK$ is isosceles with base $\overline{GK}$. Likewise, $\triangle HJK$ is isosceles with base $\overline{HK}$.

Solve the Test Item

Step 1 The base angles of $\triangle HJK$ are congruent. Let $x = m\angle KHJ = m\angle HKJ$.

$m\angle KHJ + m\angle HKJ + m\angle HJK = 180$ Angle Sum Theorem

$x + x + 100 = 180$ Substitution

$2x + 100 = 180$ Add.

$2x = 80$ Subtract 100 from each side.

$x = 40$ So, $m\angle KHJ = m\angle HKJ = 40$.

Step 2 $\angle GHK$ and $\angle KHJ$ form a linear pair. Solve for $m\angle GHK$.

$m\angle KHJ + m\angle GHK = 180$ Linear pairs are supplementary.

$40 + m\angle GHK = 180$ Substitution

$m\angle GHK = 140$ Subtract 40 from each side.

Step 3 The base angles of $\triangle GHK$ are congruent. Let y represent $m\angle HGK$ and $m\angle GKH$.

$m\angle GHK + m\angle HGK + m\angle GKH = 180$ Angle Sum Theorem

$140 + y + y = 180$ Substitution

$140 + 2y = 180$ Add.

$2y = 40$ Subtract 140 from each side.

$y = 20$ Divide each side by 2.

The measure of $\angle HGK$ is 20. Choice C is correct.

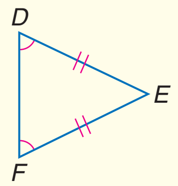
The converse of the Isosceles Triangle Theorem is also true.

Theorem 4.10

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Abbreviation: *Conv. of Isos. △ Th.*

Example: If $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{FE}$.

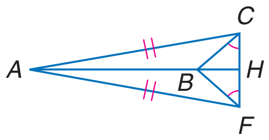


You will prove Theorem 4.10 in Exercise 33.

Example 3 Congruent Segments and Angles

a. Name two congruent angles.

$\angle AFC$ is opposite $\overline{AC}$ and $\angle ACF$ is opposite $\overline{AF}$, so $\angle AFC \cong \angle ACF$.



b. Name two congruent segments.

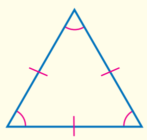
By the converse of the Isosceles Triangle Theorem, the sides opposite congruent angles are congruent. So, $\overline{BC} \cong \overline{BF}$.

You can use properties of triangles to prove Thales of Miletus' important geometric ideas. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

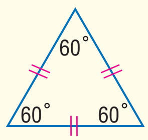
PROPERTIES OF EQUILATERAL TRIANGLES Recall that an equilateral triangle has three congruent sides. The Isosceles Triangle Theorem also applies to equilateral triangles. This leads to two corollaries about the angles of an equilateral triangle.

Corollaries

4.3 A triangle is equilateral if and only if it is equiangular.



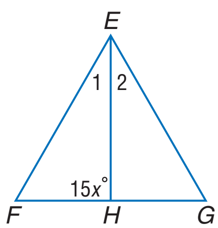
4.4 Each angle of an equilateral triangle measures 60°.



You will prove Corollaries 4.3 and 4.4 in Exercises 31 and 32.

Example 4 Use Properties of Equilateral Triangles

$\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$.



a. Find $m\angle 1$ and $m\angle 2$.

Each angle of an equilateral triangle measures 60°. So, $m\angle 1 + m\angle 2 = 60$. Since the angle was bisected, $m\angle 1 = m\angle 2$. Thus, $m\angle 1 = m\angle 2 = 30$.

b. ALGEBRA Find x.

$m\angle EFH + m\angle 1 + m\angle EHF = 180$	Angle Sum Theorem
$60 + 30 + 15x = 180$	$m\angle EFH = 60$, $m\angle 1 = 30$, $m\angle EFH = 15x$
$90 + 15x = 180$	Add.
$15x = 90$	Subtract 90 from each side.
$x = 6$	Divide each side by 15.

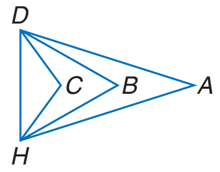
Check for Understanding

Concept Check

1. **Explain** how many angles in an isosceles triangle must be given to find the measures of the other angles.
2. **Name** the congruent sides and angles of isosceles $\triangle WXZ$ with base $\overline{WZ}$.
3. **OPEN ENDED** Describe a method to construct an equilateral triangle.

Guided Practice

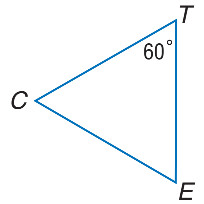
Refer to the figure.



4. If $\overline{AD} \cong \overline{AH}$, name two congruent angles.
5. If $\angle BDH \cong \angle BHD$, name two congruent segments.

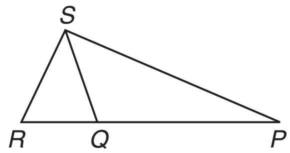
6. **ALGEBRA** Triangle GHF is equilateral with $m\angle F = 3x + 4$, $m\angle G = 6y$, and $m\angle H = 19z + 3$. Find x, y, and z.

Write a two-column proof.



7. **Given:** $\triangle CTE$ is isosceles with vertex $\angle C$.
   $m\angle T = 60$
   Prove: $\triangle CTE$ is equilateral.

Standardized Test Practice



8. If $\overline{PQ} \cong \overline{QS}$, $\overline{QR} \cong \overline{RS}$, and $m\angle PRS = 72$, what is the measure of $\angle QPS$?
   (A) 27 (B) 54 (C) 63 (D) 72

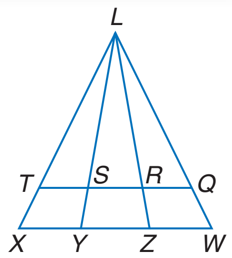
Practice and Apply

Homework Help

For Exercises	See Examples
9–14	3
15–22, 27–28, 34–37	4
23–26, 38–39	2
29–33	1

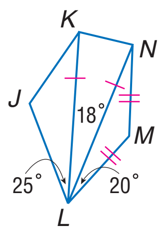
Extra Practice
See page 762.

Refer to the figure.



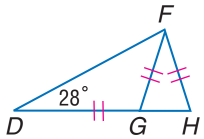
9. If $\overline{LT} \cong \overline{LR}$, name two congruent angles.
10. If $\overline{LX} \cong \overline{LW}$, name two congruent angles.
11. If $\overline{SL} \cong \overline{QL}$, name two congruent angles.
12. If $\angle LXY \cong \angle LYX$, name two congruent segments.
13. If $\angle LSR \cong \angle LRS$, name two congruent segments.
14. If $\angle LYW \cong \angle LWY$, name two congruent segments.

$\triangle KLN$ and $\triangle LMN$ are isosceles and $m\angle JKN = 130$. Find each measure.



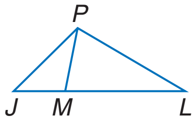
15. $m\angle LNM$
16. $m\angle M$
17. $m\angle LKN$
18. $m\angle J$

$\triangle DFG$ and $\triangle FGH$ are isosceles, $m\angle FDH = 28$ and $\overline{DG} \cong \overline{FG} \cong \overline{FH}$. Find each measure.



19. $m\angle DFG$
20. $m\angle DGF$
21. $m\angle FGH$
22. $m\angle GFH$

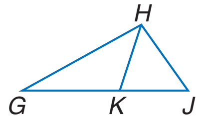
In the figure, $\overline{JM} \cong \overline{PM}$ and $\overline{ML} \cong \overline{PL}$.



23. If $m\angle PLJ = 34$, find $m\angle JPM$.

24. If $m\angle PLJ = 58$, find $m\angle PJL$.

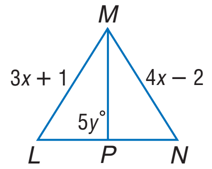
In the figure, $\overline{GK} \cong \overline{GH}$ and $\overline{HK} \cong \overline{KJ}$.



25. If $m\angle HGK = 28$, find $m\angle HJK$.

26. If $m\angle HGK = 42$, find $m\angle HJK$.

Triangle LMN is equilateral, and $\overline{MP}$ bisects $\overline{LN}$.

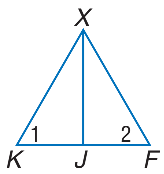


27. Find x and y.

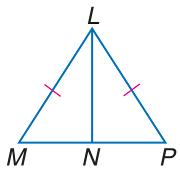
28. Find the measure of each side of $\triangle LMN$.

PROOF **Write a two-column proof.**

29. Given: $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle X$.
Prove: J is the midpoint of $\overline{KF}$.



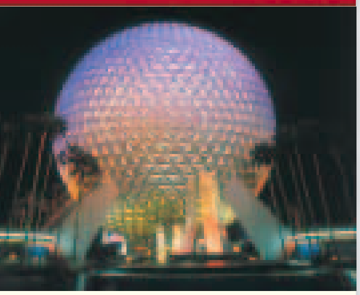
30. Given: $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$



31. Corollary 4.3 **32.** Corollary 4.4 **33.** Theorem 4.10

34. DESIGN The basic structure covering Spaceship Earth at the Epcot Center in Orlando, Florida, is a triangle. Describe the minimum requirement to show that these triangles are equilateral.

More About...



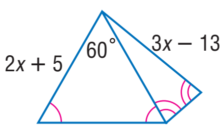
Design

Spaceship Earth is a completely spherical *geodesic dome* that is covered with 11,324 triangular aluminum and plastic alloy panels.

Source: disneyworld.disney.go.com

ALGEBRA **Find x.**

35.



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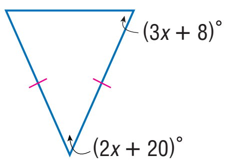
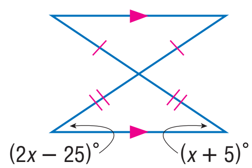
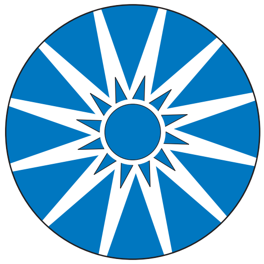


37.



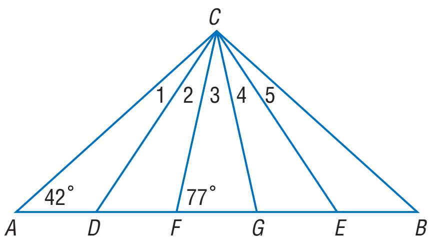
ARTISANS **For Exercises 38 and 39, use the following information.**
This geometric sign from the Grassfields area in Western Cameroon (Western Africa) uses approximations of isosceles triangles within and around two circles.



38. Trace the figure. Identify and draw one isosceles triangle from each set in the sign.

39. Describe the similarities between the different triangles.

40. CRITICAL THINKING In the figure, $\triangle ABC$ is isosceles, $\triangle DCE$ is equilateral, and $\triangle FCG$ is isosceles. Find the measures of the five numbered angles at vertex C.



Dallas & John Heaton/Stock Boston

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

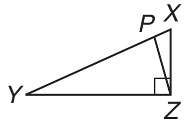
How are triangles used in art?

Include the following in your answer:

- at least three other geometric shapes frequently used in art, and
- a description of how isosceles triangles are used in the painting.

Standardized Test Practice

42. Given right triangle XYZ with hypotenuse $\overline{XY}$, YP is equal to YZ. If $m\angle PYZ = 26$, find $m\angle XZP$.



(A) 13 (B) 26 (C) 32 (D) 64

43. **ALGEBRA** A segment is drawn from (3, 5) to (9, 13). What are the coordinates of the midpoint of this segment?

(A) (3, 4) (B) (12, 18) (C) (6, 8) (D) (6, 9)

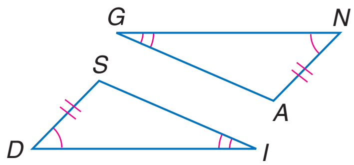
Maintain Your Skills

Mixed Review

Write a paragraph proof. *(Lesson 4-5)*

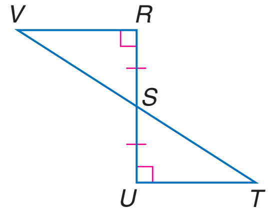
44. **Given:** $\angle N \cong \angle D$, $\angle G \cong \angle I$, $\overline{AN} \cong \overline{SD}$

Prove: $\triangle ANG \cong \triangle SDI$



45. **Given:** $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$

Prove: $\triangle VRS \cong \triangle TUS$



Determine whether $\triangle QRS \cong \triangle EGH$ given the coordinates of the vertices. Explain. *(Lesson 4-4)*

46. $Q(-3, 1)$, $R(1, 2)$, $S(-1, -2)$, $E(6, -2)$, $G(2, -3)$, $H(4, 1)$

47. $Q(1, -5)$, $R(5, 1)$, $S(4, 0)$, $E(-4, -3)$, $G(-1, 2)$, $H(2, 1)$

Construct a truth table for each compound statement. *(Lesson 2-2)*

48. a and b

49. $\sim p$ or $\sim q$

50. k and $\sim m$

51. $\sim y$ or z

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the coordinates of the midpoint of the segment with the given endpoints.** *(To review **finding midpoints**, see Lesson 1-5.)*

52. $A(2, 15)$, $B(7, 9)$

53. $C(-4, 6)$, $D(2, -12)$

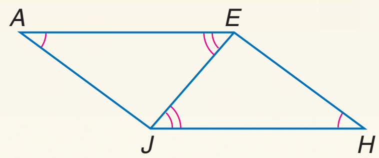
54. $E(3, 2.5)$, $F(7.5, 4)$

Practice Quiz 2

Lessons 4-4 through 4-6

1. Determine whether $\triangle JML \cong \triangle BDG$ given that $J(-4, 5)$, $M(-2, 6)$, $L(-1, 1)$, $B(-3, -4)$, $D(-4, -2)$, and $G(1, -1)$. *(Lesson 4-4)*

2. Write a two-column proof to prove that $\overline{AJ} \cong \overline{EH}$, given $\angle A \cong \angle H$, $\angle AEJ \cong \angle HJE$. *(Lesson 4-5)*

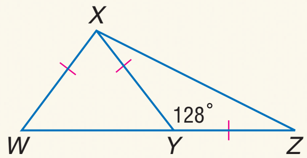


$\triangle WXY$ and $\triangle XYZ$ are isosceles and $m\angle XYZ = 128$. Find each measure. *(Lesson 4-6)*

3. $m\angle XWY$

4. $m\angle WXY$

5. $m\angle YZX$



4-7 Triangles and Coordinate Proof

What You'll Learn

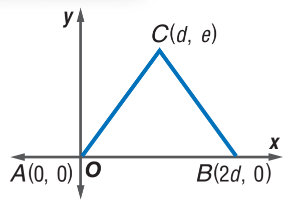
- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

Vocabulary

- coordinate proof

How can the coordinate plane be useful in proofs?

In this chapter, we have used several methods of proof. You have also used the coordinate plane to identify characteristics of a triangle. We can combine what we know about triangles in the coordinate plane with algebra in a new method of proof called *coordinate proof*.



POSITION AND LABEL TRIANGLES **Coordinate proof** uses figures in the coordinate plane and algebra to prove geometric concepts. The first step in writing a coordinate proof is the placement of the figure on the coordinate plane.

Study Tip

Placement of Figures
The guidelines apply to any polygon placed on the coordinate plane.

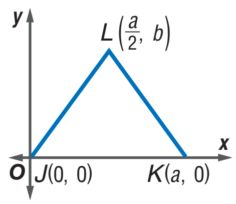
Key Concept *Placing Figures on the Coordinate Plane*

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Example 1 Position and Label a Triangle

Position and label isosceles triangle *JKL* on a coordinate plane so that base $\overline{JK}$ is *a* units long.

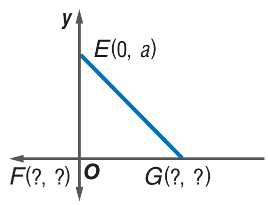
- Use the origin as vertex *J* of the triangle.
- Place the base of the triangle along the positive *x*-axis.
- Position the triangle in the first quadrant.
- Since *K* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is *a* because the base of the triangle is *a* units long.
- Since $\triangle JKL$ is isosceles, the *x*-coordinate of *L* is halfway between 0 and *a* or $\frac{a}{2}$. We cannot determine the *y*-coordinate in terms of *a*, so call it *b*.



Example 2 Find the Missing Coordinates

Name the missing coordinates of isosceles right $\triangle EFG$.

Vertex *F* is positioned at the origin; its coordinates are (0, 0). Vertex *E* is on the *y*-axis, and vertex *G* is on the *x*-axis. So $\angle EFG$ is a right angle. Since $\triangle EFG$ is isosceles, $\overline{EF} \cong \overline{GF}$. The distance from *E* to *F* is *a* units. The distance from *F* to *G* must be the same. So, the coordinates of *G* are (*a*, 0).

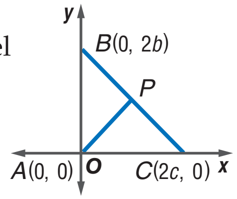


WRITE COORDINATE PROOFS After the figure has been placed on the coordinate plane and labeled, we can use coordinate proof to verify properties and to prove theorems. The Distance Formula, Slope Formula, and Midpoint Formula are often used in coordinate proof.

Example 3 Coordinate Proof

Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

The first step is to position and label a right triangle on the coordinate plane. Place the right angle at the origin and label it *A*. Use coordinates that are multiples of 2 because the Midpoint Formula takes half the sum of the coordinates.



Given: right $\triangle ABC$ with right $\angle BAC$

P is the midpoint of $\overline{BC}$.

Prove: $AP = \frac{1}{2}BC$

Proof:

By the Midpoint Formula, the coordinates of P are $\left(\frac{0 + 2c}{2}, \frac{2b + 0}{2}\right)$ or (c, b).

Use the Distance Formula to find AP and BC.

$$AP = \sqrt{(c-0)^2 + (b-0)^2}$$
$$= \sqrt{c^2 + b^2}$$

$$BC = \sqrt{(2c-0)^2 + (0-2b)^2}$$
$$BC = \sqrt{4c^2 + 4b^2} \text{ or } 2\sqrt{c^2 + b^2}$$
$$\frac{1}{2}BC = \sqrt{c^2 + b^2}$$

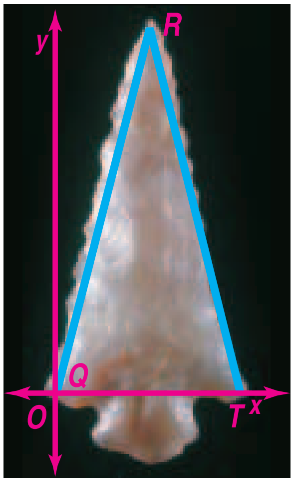
Therefore, $AP = \frac{1}{2}BC$.

Example 4 Classify Triangles

Study Tip

Vertex Angle
Remember from the Geometry Activity on page 216 that an isosceles triangle can be folded in half. Thus, the *x*-coordinate of the vertex angle is the same as the *x*-coordinate of the midpoint of the base.

ARROWHEADS **Write a coordinate proof to prove that this arrowhead is shaped like an isosceles triangle. The arrowhead is 3 inches long and 1.5 inches wide.**



The first step is to label the coordinates of each vertex. *Q* is at the origin, and *T* is at (1.5, 0). The *y*-coordinate of *R* is 3. The *x*-coordinate is halfway between 0 and 1.5 or 0.75. So, the coordinates of *R* are (0.75, 3).

If the legs of the triangle are the same length, the triangle is isosceles. Use the Distance Formula to determine the lengths of *QR* and *RT*.

$$QR = \sqrt{(0.75-0)^2 + (3-0)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

$$RT = \sqrt{(1.5-0.75)^2 + (0-3)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

Since each leg is the same length, $\triangle QRT$ is isosceles. The arrowhead is shaped like an isosceles triangle.

Check for Understanding

Concept Check

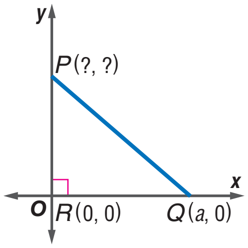
1. **Explain** how to position a triangle on the coordinate plane to simplify a proof.
2. **OPEN ENDED** Draw a scalene right triangle on the coordinate plane for use in a coordinate proof. Label the coordinates of each vertex.

Guided Practice

Position and label each triangle on the coordinate plane.

3. isosceles $\triangle FGH$ with base $\overline{FH}$ that is $2b$ units long
4. equilateral $\triangle CDE$ with sides a units long

Find the missing coordinates of each triangle.

5.


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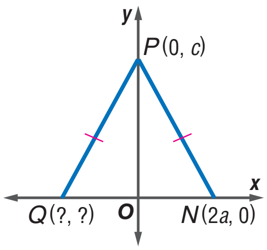

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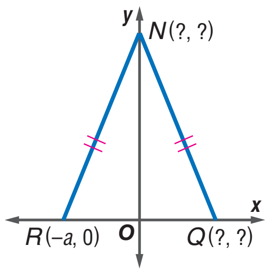


8. Write a coordinate proof for the following statement.
The midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices.

Application

9. **TEPEES** Write a coordinate proof to prove that the tepee is shaped like an isosceles triangle. Suppose the tepee is 8 feet tall and 4 feet wide.

Practice and Apply

Homework Help

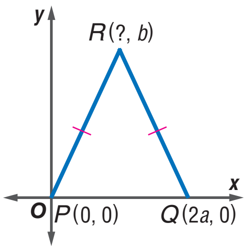
For Exercises	See Examples
10–15	1
16–24	2
25–29	3
30–33	4

Extra Practice
See page 762.

Position and label each triangle on the coordinate plane.

10. isosceles $\triangle QRT$ with base $\overline{QR}$ that is b units long
11. equilateral $\triangle MNP$ with sides $2a$ units long
12. isosceles right $\triangle JML$ with hypotenuse $\overline{JM}$ and legs c units long
13. equilateral $\triangle WXZ$ with sides $\frac{1}{2}b$ units long
14. isosceles $\triangle PWY$ with a base $\overline{PW}$ that is $(a + b)$ units long
15. right $\triangle XYZ$ with hypotenuse $\overline{XZ}$, $ZY = 2(XY)$, and $\overline{XY}$ b units long

Find the missing coordinates of each triangle.

16.


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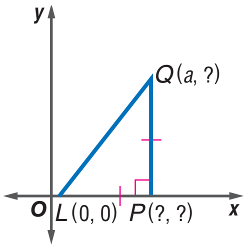

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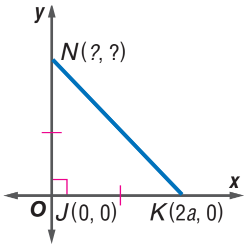

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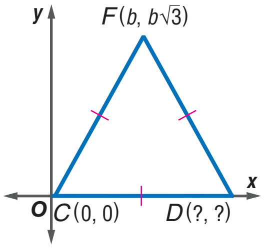

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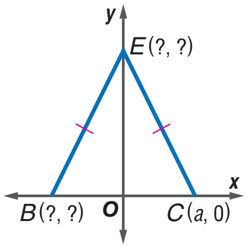

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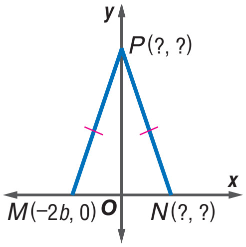

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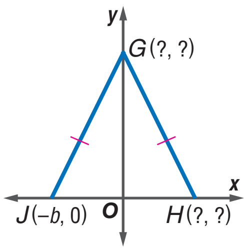

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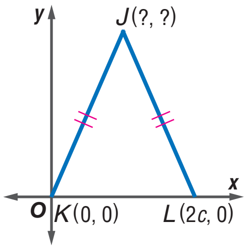

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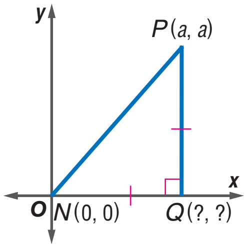


Write a coordinate proof for each statement.

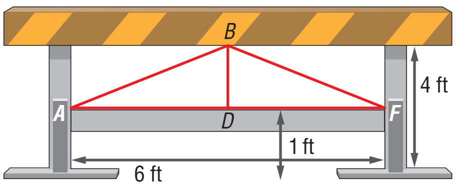
25. The segments joining the vertices to the midpoints of the legs of an isosceles triangle are congruent.

26. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

27. If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.

28. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one-half the length of the third side.

29. **STEEPLECHASE** Write a coordinate proof to prove that triangles ABD and FBD are congruent. $\overline{BD}$ is perpendicular to $\overline{AF}$, and B is the midpoint of the upper bar of the hurdle.



NAVIGATION **For Exercises 30 and 31, use the following information.**
A motor boat is located 800 yards east of the port. There is a ship 800 yards to the east, and another ship 800 yards to the north of the motor boat.

30. Write a coordinate proof to prove that the port, motor boat, and the ship to the north form an isosceles right triangle.

31. Write a coordinate proof to prove that the distance between the two ships is the same as the distance from the port to the northern ship.

HIKING **For Exercises 32 and 33, use the following information.**
Tami and Juan are hiking. Tami hikes 300 feet east of the camp and then hikes 500 feet north. Juan hikes 500 feet west of the camp and then 300 feet north.

32. Write a coordinate proof to prove that Juan, Tami, and the camp form a right triangle.

33. Find the distance between Tami and Juan.

More About. . .

Steeplechase

The Steeplechase is a horse race two to four miles long that focuses on jumping hurdles. The rails of the fences vary in height.

Source: www.steeplechasetimes.com

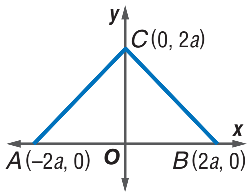
Find the coordinates of point Z so $\triangle XYZ$ is the indicated type of triangle. Point X has coordinates (0, 0) and Y has coordinates (a, b).

34. right triangle with right angle Z

35. isosceles triangle with base $\overline{XZ}$

36. scalene triangle

37. CRITICAL THINKING Classify $\triangle ABC$ by its angles and its sides. Explain.



38. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can the coordinate plane be useful in proofs?

Include the following in your answer:
- types of proof, and
- a theorem from this chapter that could be proved using a coordinate proof.

Standardized Test Practice

39. What is the length of the segment whose endpoints are at $(1, -2)$ and $(-3, 1)$?

(A) 3 (B) 4 (C) 5 (D) 6

40. ALGEBRA What are the coordinates of the midpoint of the line segment whose endpoints are $(-5, 4)$ and $(-2, -1)$?

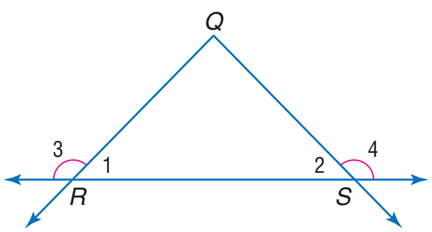
(A) (3, 3) (B) $(-3.5, 1.5)$ (C) $(-1.5, 2.5)$ (D) $(3.5, -2.5)$

Maintain Your Skills

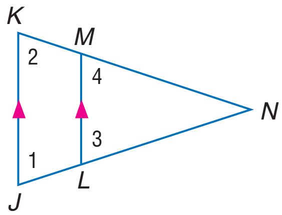
Mixed Review

Write a two-column proof. *(Lessons 4-5 and 4-6)*

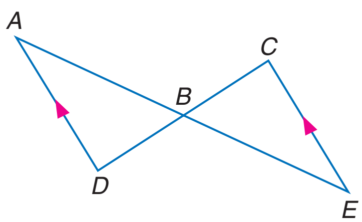
41. Given: $\angle 3 \cong \angle 4$
Prove: $\overline{QR} \cong \overline{QS}$



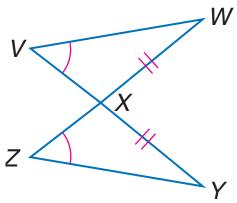
42. Given: isosceles triangle JKN with vertex $\angle N$, $\overline{JK} \parallel \overline{LM}$
Prove: $\triangle NML$ is isosceles.



43. Given: $\overline{AD} \cong \overline{CE}$, $\overline{AD} \parallel \overline{CE}$
Prove: $\triangle ABD \cong \triangle EBC$

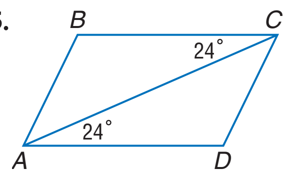


44. Given: $\overline{WX} \cong \overline{XY}$, $\angle V \cong \angle Z$
Prove: $\overline{WV} \cong \overline{YZ}$

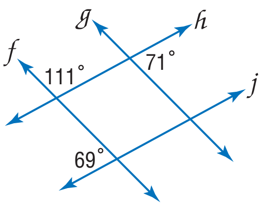


State which lines, if any, are parallel. State the postulate or theorem that justifies your answer. *(Lesson 3-5)*

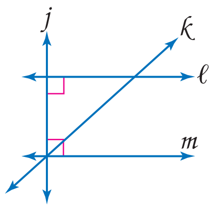
45.



46.



47.



CONTENTS

Chapter 4

Study Guide and Review

Vocabulary and Concept Check

acute triangle (p. 178)	equiangular triangle (p. 178)	isosceles triangle (p. 179)
base angles (p. 216)	equilateral triangle (p. 179)	obtuse triangle (p. 178)
congruence transformations (p. 194)	exterior angle (p. 186)	remote interior angles (p. 186)
congruent triangles (p. 192)	flow proof (p. 187)	right triangle (p. 178)
coordinate proof (p. 222)	included angle (p. 201)	scalene triangle (p. 179)
corollary (p. 188)	included side (p. 207)	vertex angle (p. 216)

A complete list of theorems and postulates can be found on pages R1–R8.

Exercises **Choose the letter of the word or phrase that best matches each statement.**

1. A triangle with an angle whose measure is greater than 90 is a(n) __?__ triangle.
2. A triangle with exactly two congruent sides is a(n) __?__ triangle.
3. The __?__ states that the sum of the measures of the angles of a triangle is 180.
4. If $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$ by __?__.
5. In an equiangular triangle, all angles are __?__ angles.
6. If two angles of a triangle and their included side are congruent to two angles and the included side of another triangle, this is called the __?__.
7. If $\angle A \cong \angle F$, $\angle B \cong \angle G$, and $\overline{AC} \cong \overline{FH}$, then $\triangle ABC \cong \triangle FGH$, by __?__.
8. A(n) __?__ angle of a triangle has a measure equal to the measures of the two remote interior angles of the triangle.

a. acute
b. AAS Theorem
c. ASA Theorem
d. Angle Sum Theorem
e. equilateral
f. exterior
g. isosceles
h. obtuse
i. right
j. SAS Theorem
k. SSS Theorem

Lesson-by-Lesson Review

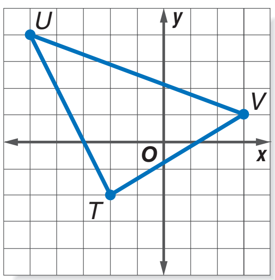
4-1 Classifying Triangles

See pages 178–183.

Concept Summary

- Triangles can be classified by their angles as acute, obtuse, or right.
- Triangles can be classified by their sides as scalene, isosceles, or equilateral.

Example **Find the measures of the sides of $\triangle TUV$. Classify the triangle by sides.**



Use the Distance Formula to find the measure of each side.

$$TU = \sqrt{[-5-(-2)]^2 + [4-(-2)]^2} = \sqrt{9+36} \text{ or } \sqrt{45}$$

$$UV = \sqrt{[3-(-5)]^2 + (1-4)^2} = \sqrt{64+9} \text{ or } \sqrt{73}$$

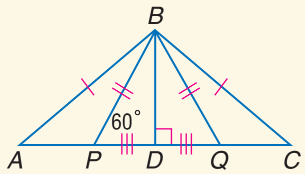
$$VT = \sqrt{(-2-3)^2 + (-2-1)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

Since none of the side measures are equal, $\triangle TUV$ is scalene.

www.geometryonline.com/vocabulary_review

Exercises **Classify each triangle by its angles and by its sides if $m\angle ABC = 100$.** *See Examples 1 and 2 on pages 178 and 179.*

9. $\triangle ABC$ **10.** $\triangle BDP$ **11.** $\triangle BPQ$



4-2 Angles of Triangles

See pages 185–191.

Concept Summary

- The sum of the measures of the angles of a triangle is 180.
- The measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

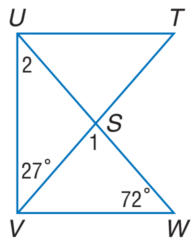
Example **If $\overline{TU} \perp \overline{UV}$ and $\overline{UV} \perp \overline{VW}$, find $m\angle 1$.**

$m\angle 1 + 72 + m\angle TVW = 180$ Angle Sum Theorem

$m\angle 1 + 72 + (90 - 27) = 180$ $m\angle TVW = 90 - 27$

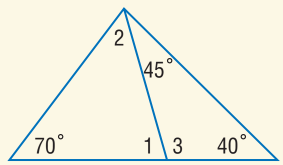
$m\angle 1 + 135 = 180$ Simplify.

$m\angle 1 = 45$ Subtract 135 from each side.



Exercises **Find each measure.** *See Example 1 on page 186.*

12. $m\angle 1$ **13.** $m\angle 2$ **14.** $m\angle 3$



4-3 Congruent Triangles

See pages 192–198.

Concept Summary

- Two triangles are congruent when all of their corresponding parts are congruent.

Example **If $\triangle EFG \cong \triangle JKL$, name the corresponding congruent angles and sides.**

$\angle E \cong \angle J$, $\angle F \cong \angle K$, $\angle G \cong \angle L$, $\overline{EF} \cong \overline{JK}$, $\overline{FG} \cong \overline{KL}$, and $\overline{EG} \cong \overline{JL}$.

Exercises **Name the corresponding angles and sides for each pair of congruent triangles.** *See Example 1 on page 193.*

15. $\triangle EFG \cong \triangle DCB$ **16.** $\triangle LCD \cong \triangle GCF$ **17.** $\triangle NCK \cong \triangle KER$

4-4 Proving Congruence—SSS, SAS

See pages 200–206.

Concept Summary

- If all of the corresponding sides of two triangles are congruent, then the triangles are congruent (SSS).
- If two corresponding sides of two triangles and the included angle are congruent, then the triangles are congruent (SAS).

Example Determine whether $\triangle ABC \cong \triangle TUV$. Explain.

$AB = \sqrt{[-1-(-2)]^2 + (1-0)^2}$
$= \sqrt{1+1}$ or $\sqrt{2}$

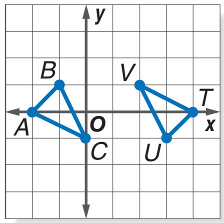
$BC = \sqrt{[0-(-1)]^2 + (-1-1)^2}$
$= \sqrt{1+4}$ or $\sqrt{5}$

$CA = \sqrt{(-2-0)^2 + [0-(-1)]^2}$
$= \sqrt{4+1}$ or $\sqrt{5}$

$TU = \sqrt{(3-4)^2 + (-1-0)^2}$
$= \sqrt{1+1}$ or $\sqrt{2}$

$UV = \sqrt{(2-3)^2 + [1-(-1)]^2}$
$= \sqrt{1+4}$ or $\sqrt{5}$

$VT = \sqrt{(4-2)^2 + (0-1)^2}$
$= \sqrt{4+1}$ or $\sqrt{5}$



By the definition of congruent segments, all corresponding sides are congruent. Therefore, $\triangle ABC \cong \triangle TUV$ by SSS.

Exercises Determine whether $\triangle MNP \cong \triangle QRS$ given the coordinates of the vertices. Explain. *See Example 2 on page 201.*

18. $M(0, 3)$, $N(-4, 3)$, $P(-4, 6)$, $Q(5, 6)$, $R(2, 6)$, $S(2, 2)$

19. $M(3, 2)$, $N(7, 4)$, $P(6, 6)$, $Q(-2, 3)$, $R(-4, 7)$, $S(-6, 6)$

4-5 Proving Congruence—ASA, AAS

See pages 207–213.

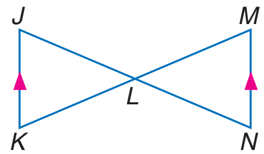
Concept Summary

- If two pairs of corresponding angles and the included sides of two triangles are congruent, then the triangles are congruent (ASA).
- If two pairs of corresponding angles and a pair of corresponding nonincluded sides of two triangles are congruent, then the triangles are congruent (AAS).

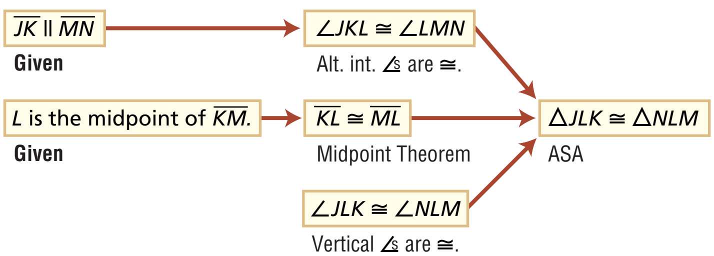
Example Write a proof.

Given: $\overline{JK} \parallel \overline{MN}$; L is the midpoint of $\overline{KM}$.

Prove: $\triangle JLK \cong \triangle NLM$



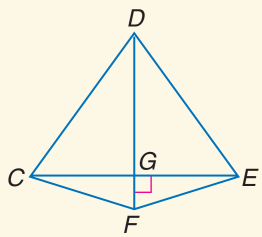
Flow proof:



Exercises For Exercises 20 and 21, use the figure. Write a two-column proof for each of the following. *See Example 2 on page 209.*

20. Given: $\overline{DF}$ bisects $\angle CDE$.
$\overline{CE} \perp \overline{DF}$
Prove: $\triangle DGC \cong \triangle DGE$

21. Given: $\triangle DGC \cong \triangle DGE$
$\triangle GCF \cong \triangle GEF$
Prove: $\triangle DFC \cong \triangle DFE$



• Extra Practice, see pages 760–762.
• Mixed Problem Solving, see page 785.

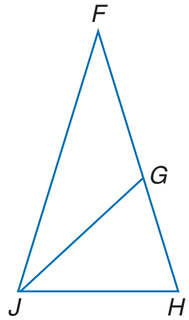
4-6 Isosceles Triangles

See pages 216–221.

Concept Summary

- Two sides of a triangle are congruent if and only if the angles opposite those sides are congruent.
- A triangle is equilateral if and only if it is equiangular.

Example **If $\overline{FG} \cong \overline{GJ}$, $\overline{GJ} \cong \overline{JH}$, $\overline{FJ} \cong \overline{FH}$, and $m\angle GJH = 40$, find $m\angle H$.**

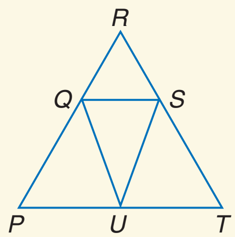


$\triangle GHJ$ is isosceles with base $\overline{GH}$, so $\angle JGH \cong \angle H$ by the Isosceles Triangle Theorem. Thus, $m\angle JGH = m\angle H$.

$m\angle GJH + m\angle JGH + m\angle H = 180$	Angle Sum Theorem
$40 + 2(m\angle H) = 180$	Substitution
$2(m\angle H) = 140$	Subtract 40 from each side.
$m\angle H = 70$	Divide each side by 2.

Exercises **For Exercises 22–25, refer to the figure at the right.**

See Example 2 on page 217.



22. If $\overline{PQ} \cong \overline{UQ}$ and $m\angle P = 32$, find $m\angle PUQ$.
23. If $\overline{PQ} \cong \overline{UQ}$, $\overline{PR} \cong \overline{RT}$, and $m\angle PQU = 40$, find $m\angle R$.
24. If $\overline{RQ} \cong \overline{RS}$ and $m\angle RQS = 75$, find $m\angle R$.
25. If $\overline{RQ} \cong \overline{RS}$, $\overline{RP} \cong \overline{RT}$, and $m\angle RQS = 80$, find $m\angle P$.

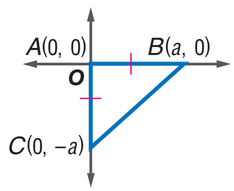
4-7 Triangles and Coordinate Proof

See pages 222–226.

Concept Summary

- Coordinate proofs use algebra to prove geometric concepts.
- The Distance Formula, Slope Formula, and Midpoint Formula are often used in coordinate proof.

Example **Position and label isosceles right triangle ABC with legs of length a units on the coordinate plane.**



- Use the origin as the vertex of $\triangle ABC$ that has the right angle.
- Place each base along an axis.
- Since B is on the x-axis, its y-coordinate is 0. Its x-coordinate is a because the leg $\overline{AB}$ of the triangle is a units long.
- Since $\triangle ABC$ is isosceles, C should also be a distance of a units from the origin. Its coordinates are $(0, -a)$.

Exercises **Position and label each triangle on the coordinate plane.**

See Example 1 on page 222.

26. isosceles $\triangle TRI$ with base $\overline{TI}$ $4a$ units long
27. equilateral $\triangle BCD$ with side length $6m$ units long
28. right $\triangle JKL$ with leg lengths of a units and b units

Chapter 4 Practice Test

Vocabulary and Concepts

Choose the letter of the type of triangle that best matches each phrase.

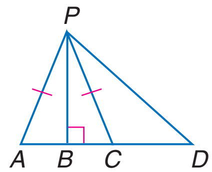
1. triangle with no sides congruent
2. triangle with at least two sides congruent
3. triangle with all sides congruent

a.	isosceles
b.	scalene
c.	equilateral

Skills and Applications

Identify the indicated triangles in the figure if $\overline{PB} \perp \overline{AD}$ and $\overline{PA} \cong \overline{PC}$.

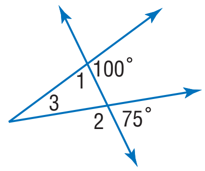
4. obtuse **5.** isosceles **6.** right



Questions 4–6

Find the measure of each angle in the figure.

7. $m\angle 1$ **8.** $m\angle 2$ **9.** $m\angle 3$



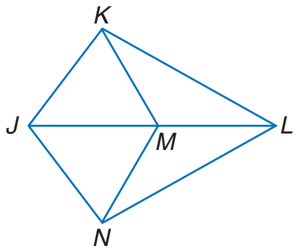
Questions 7–9

Name the corresponding angles and sides for each pair of congruent triangles.

10. $\triangle DEF \cong \triangle PQR$ **11.** $\triangle FMG \cong \triangle HNJ$ **12.** $\triangle XYZ \cong \triangle ZYX$

13. Determine whether $\triangle JKL \cong \triangle MNP$ given $J(-1, -2)$, $K(2, -3)$, $L(3, 1)$, $M(-6, -7)$, $N(-2, 1)$, and $P(5, 3)$. Explain.

14. Write a flow proof.
Given: $\triangle JKM \cong \triangle JNM$
Prove: $\triangle JKL \cong \triangle JNL$

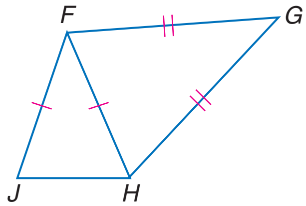


Question 14

In the figure, $\overline{FJ} \cong \overline{FH}$ and $\overline{GF} \cong \overline{GH}$.

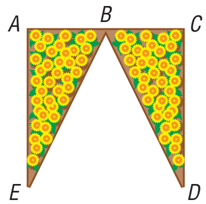
15. If $m\angle JFH = 34$, find $m\angle J$.

16. If $m\angle GHJ = 152$ and $m\angle G = 32$, find $m\angle JFH$.



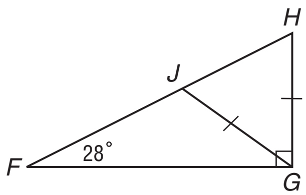
Questions 15–16

17. **LANDSCAPING** A landscaper designed a garden shaped as shown in the figure. The landscaper has decided to place point B 22 feet east of point A, point C 44 feet east of point A, point E 36 feet south of point A, and point D 36 feet south of point C. The angles at points A and C are right angles. Prove that $\triangle ABE \cong \triangle CBD$.



18. **STANDARDIZED TEST PRACTICE** In the figure, $\triangle FGH$ is a right triangle with hypotenuse $\overline{FH}$ and $GJ = GH$. What is the measure of $\angle JGH$?

(A) 104 (B) 62 (C) 56 (D) 28



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Chapter 4 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In 2002, Capitol City had a population of 2010, and Shelbyville had a population of 1040. If Capitol City grows at a rate of 150 people a year and Shelbyville grows at a rate of 340 people a year, when will the population of Shelbyville be greater than that of Capitol City? (Prerequisite Skill)

Ⓐ 2004 Ⓑ 2008

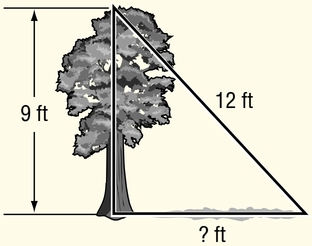
Ⓒ 2009 Ⓓ 2012

2. Which unit is most appropriate for measuring liquid in a bottle? (Lesson 1-2)

Ⓐ grams Ⓑ feet

Ⓒ liters Ⓓ meters

3. A 9-foot tree casts a shadow on the ground. The distance from the top of the tree to the end of the shadow is 12 feet. To the nearest foot, how long is the shadow? (Lesson 1-3)



Ⓐ 7 ft

Ⓑ 8 ft

Ⓒ 10 ft

Ⓓ 13 ft

4. Which of the following is the inverse of the statement *If it is raining, then Kamika carries an umbrella?* (Lesson 2-2)

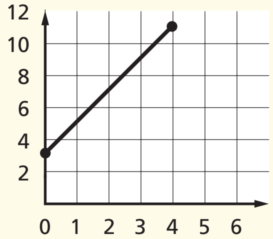
Ⓐ If Kamika carries an umbrella, then it is raining.

Ⓑ If Kamika does not carry an umbrella, then it is not raining.

Ⓒ If it is not raining, then Kamika carries an umbrella.

Ⓓ If it is not raining, then Kamika does not carry an umbrella.

5. Students in a math classroom simulated stock trading. Kris drew the graph below to model the value of his shares at closing. The graph that modeled the value of Mitzi's shares was parallel to the one Kris drew. Which equation might represent the line for Mitzi's graph? (Lesson 3-3)



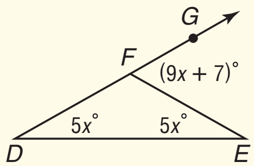
Ⓐ $-2x - y = 1$

Ⓑ $x - 2y = 1$

Ⓒ $x + 2y = 1$

Ⓓ $2x - y = 1$

6. What is $m\angle EFG$? (Lesson 4-2)



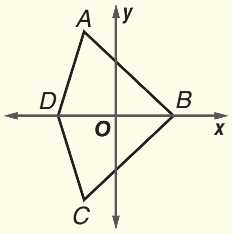
Ⓐ 35

Ⓑ 70

Ⓒ 90

Ⓓ 110

7. In the figure, $\triangle ABD \cong \triangle CBD$. If A has the coordinates $(-2, 4)$, what are the coordinates of C? (Lesson 4-3)



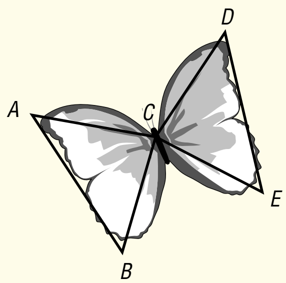
Ⓐ $(-4, -2)$

Ⓑ $(-4, 2)$

Ⓒ $(-2, -4)$

Ⓓ $(2, -4)$

8. The wings of some butterflies can be modeled by triangles as shown. If $\overline{AC} \cong \overline{DC}$ and $\angle ACB \cong \angle ECD$, which additional statements are needed to prove that $\triangle ACB \cong \triangle ECD$? (Lesson 4-4)



Ⓐ $\overline{BC} \cong \overline{CE}$

Ⓑ $\overline{AB} \cong \overline{ED}$

Ⓒ $\angle BAC \cong \angle CED$

Ⓓ $\angle ABC \cong \angle CDE$

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 795–810.

Part 2 Short Response/Grid In

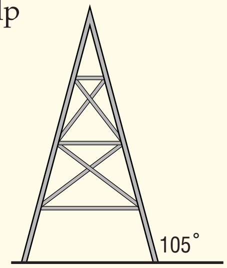
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Find the product $3s^2(2s^3 - 7)$. (Prerequisite Skill)

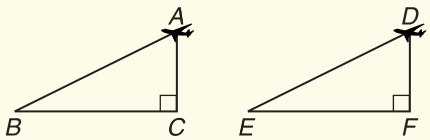
10. After a long workout, Brian noted, "If I do not drink enough water, then I will become dehydrated." He then made another statement, "If I become dehydrated, then I did not drink enough water." How is the second statement related to the original statement? (Lesson 2-2)

11. On a coordinate map, the towns of Creston and Milford are located at $(-1, -1)$ and $(1, 3)$, respectively. A third town, Dixville, is located at $(x, -1)$ so that Creston and Dixville are endpoints of the base of the isosceles triangle formed by the three locations. What is the value of x? (Lesson 4-1)

12. A watchtower, built to help prevent forest fires, was designed as an isosceles triangle. If the side of the tower meets the ground at a 105° angle, what is the measure of the angle at the top of the tower? (Lesson 4-2)



13. During a synchronized flying show, airplane A and airplane D are equidistant from the ground. They descend at the same angle to land at points B and E, respectively. Which postulate would prove that $\triangle ABC \cong \triangle DEF$? (Lesson 4-4)



14. $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$, and the measure of vertex angle B is three times $m\angle A$. What is $m\angle C$? (Lesson 4-6)

Test-Taking Tip

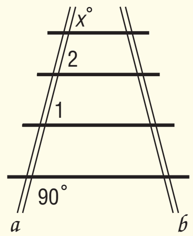
Question 8

- If you are not permitted to write in your test booklet, make a sketch of the figure on scrap paper.
- Mark the figure with all of the information you know so that you can determine the congruent triangles more easily.
- Make a list of postulates or theorems that you might use for this case.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

15. Train tracks a and b are parallel lines although they appear to come together to give the illusion of distance in a drawing. All of the railroad ties are parallel to each other.



a. What is the value of x? (Lesson 3-1)

b. What is the relationship between the tracks and the ties that run across the tracks? (Lesson 1-5)

c. What is the relationship between $\angle 1$ and $\angle 2$? Explain. (Lesson 3-2)

16. The measures of the angles of $\triangle ABC$ are $5x$, $4x - 1$, and $3x + 13$.

a. Draw a figure to illustrate $\triangle ABC$. (Lesson 4-1)

b. Find the measure of each angle of $\triangle ABC$. Explain. (Lesson 4-2)

c. Prove that $\triangle ABC$ is an isosceles triangle. (Lesson 4-6)