

2

Reasoning
and Proof**What** You'll Learn

- **Lessons 2-1 through 2-3** Make conjectures, determine whether a statement is true or false, and find counterexamples for statements.
- **Lesson 2-4** Use deductive reasoning to reach valid conclusions.
- **Lessons 2-5 and 2-6** Verify algebraic and geometric conjectures using informal and formal proof.
- **Lessons 2-7 and 2-8** Write proofs involving segment and angle theorems.

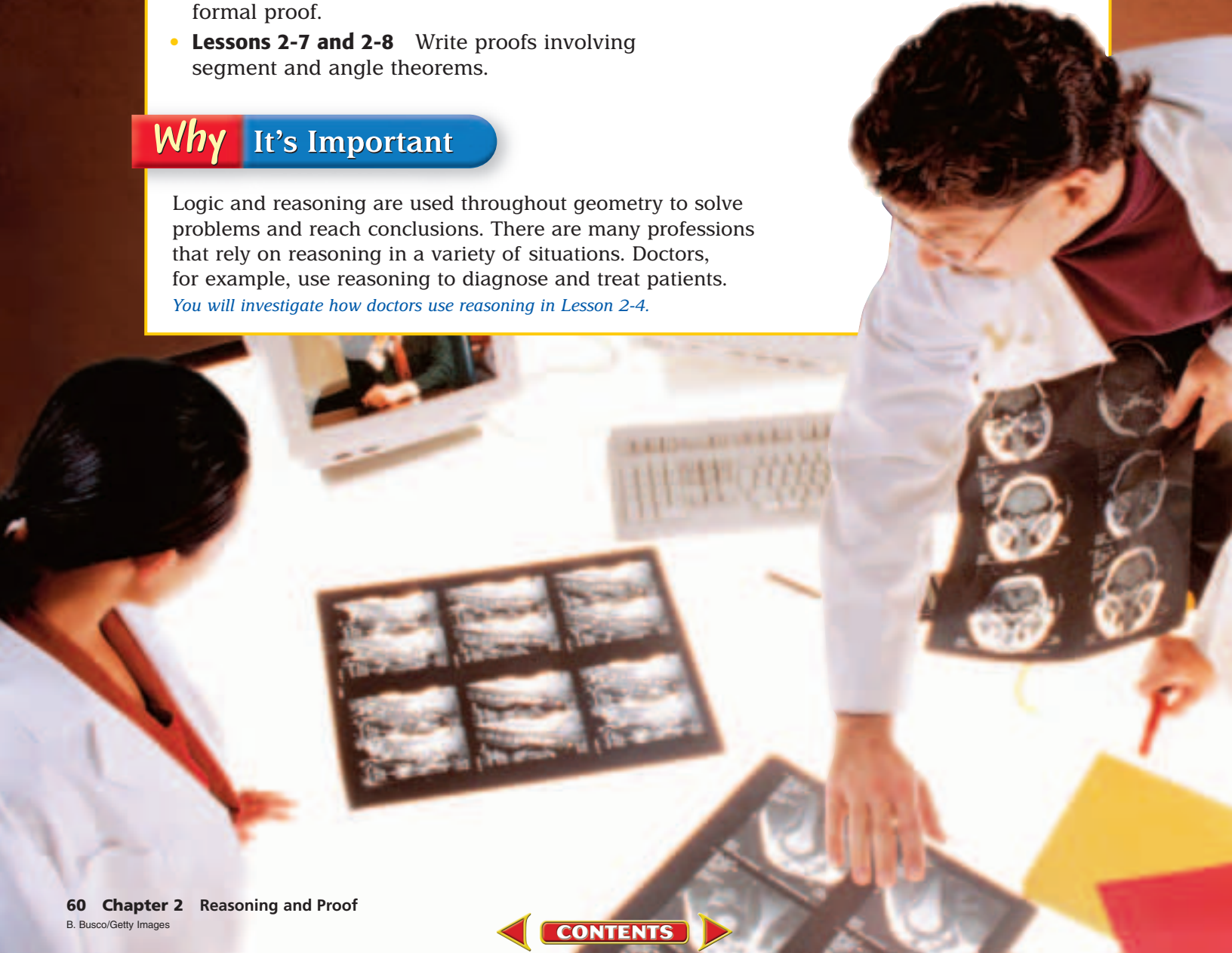
Key Vocabulary

- inductive reasoning (p. 62)
- deductive reasoning (p. 82)
- postulate (p. 89)
- theorem (p. 90)
- proof (p. 90)

Why It's Important

Logic and reasoning are used throughout geometry to solve problems and reach conclusions. There are many professions that rely on reasoning in a variety of situations. Doctors, for example, use reasoning to diagnose and treat patients.

You will investigate how doctors use reasoning in Lesson 2-4.



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

For Lesson 2-1

Evaluate Expressions

Evaluate each expression for the given value of n . (For review, see page 736.)

- | | | |
|------------------------|--|------------------------------|
| 1. $3n - 2; n = 4$ | 2. $(n + 1) + n; n = 6$ | 3. $n^2 - 3n; n = 3$ |
| 4. $180(n - 2); n = 5$ | 5. $n\left(\frac{n}{2}\right); n = 10$ | 6. $\frac{n(n-3)}{2}; n = 8$ |

For Lessons 2-6 through 2-8

Solve Equations

Solve each equation. (For review, see pages 737 and 738.)

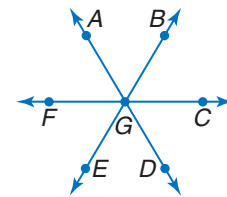
- | | | |
|------------------------|---------------------------------|---------------------------------|
| 7. $6x - 42 = 4x$ | 8. $8 - 3n = -2 + 2n$ | 9. $3(y + 2) = -12 + y$ |
| 10. $12 + 7x = x - 18$ | 11. $3x + 4 = \frac{1}{2}x - 5$ | 12. $2 - 2x = \frac{2}{3}x - 2$ |

For Lesson 2-8

Adjacent and Vertical Angles

For Exercises 13–14, refer to the figure at the right. (For review, see Lesson 1-5.)

13. If $m\angle AGB = 4x + 7$ and $m\angle EGD = 71$, find x .
14. If $m\angle BGC = 45$, $m\angle CGD = 8x + 4$, and $m\angle DGE = 15x - 7$, find x .

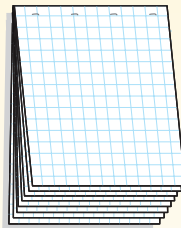


FOLDABLES™ Study Organizer

Reasoning and Proof Make this Foldable to help you organize your notes. Begin with eight sheets of $8\frac{1}{2}$ " by 11" grid paper.

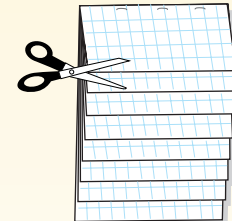
Step 1 Staple

Stack and staple the eight sheets together to form a booklet.



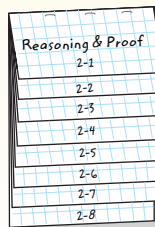
Step 2 Cut Tabs

Cut the bottom of each sheet to form a tabbed book.



Step 3 Label

Label each of the tabs with a lesson number. Add the chapter title to the first tab.



Reading and Writing As you read and study each lesson, use the corresponding page to write proofs and record examples of when you used logical reasoning in your daily life.

Inductive Reasoning and Conjecture

What You'll Learn

- Make conjectures based on inductive reasoning.
- Find counterexamples.

Vocabulary

- conjecture
- inductive reasoning
- counterexample

How can inductive reasoning help predict weather conditions?

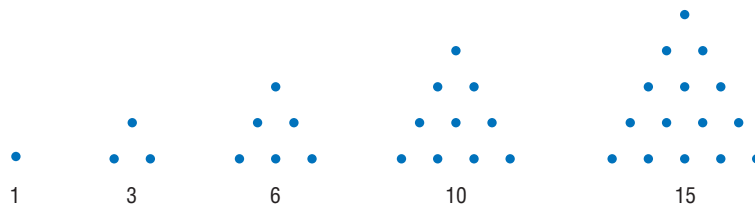
Meteorologists use science and weather patterns to make predictions about future weather conditions. They are able to make accurate educated guesses based on past weather patterns.



MAKE CONJECTURES A **conjecture** is an educated guess based on known information. Examining several specific situations to arrive at a conjecture is called inductive reasoning. **Inductive reasoning** is reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction.

Example 1 Patterns and Conjecture

The numbers represented below are called *triangular numbers*. Make a conjecture about the next triangular number based on the pattern.

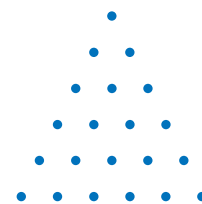


Observe: Each triangle is formed by adding another row of dots.

Find a Pattern: $1 \quad 3 \quad 6 \quad 10 \quad 15$
 $\quad \quad \quad +2 \quad +3 \quad +4 \quad +5$

The numbers increase by 2, 3, 4, and 5.

Conjecture: The next number will increase by 6. So, it will be $15 + 6$ or 21.



Study Tip

Conjectures

List your observations and identify patterns before you make a conjecture.

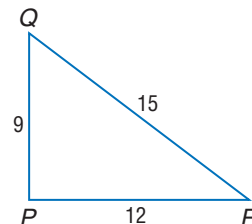
In Chapter 1, you learned some basic geometric concepts. These concepts can be used to make conjectures in geometry.

Example 2 Geometric Conjecture

For points P , Q , and R , $PQ = 9$, $QR = 15$, and $PR = 12$. Make a conjecture and draw a figure to illustrate your conjecture.

Given: points P , Q , and R ; $PQ = 9$, $QR = 15$, and $PR = 12$
 Examine the measures of the segments.
 Since $PQ + PR \neq QR$, the points cannot be collinear.

Conjecture: P , Q , and R are noncollinear.



FIND COUNTEREXAMPLES A conjecture based on several observations may be true in most circumstances, but false in others. It takes only one false example to show that a conjecture is not true. The false example is called a **counterexample**.

Example 3 Find a Counterexample

FINANCE Find a counterexample for the following statement based on the graph.

The rates for CDs are at least 1.5% less than the rates a year ago.

Examine the graph. The statement is true for 6-month, 1-year, and $2\frac{1}{2}$ -year CDs. However, the difference in the rate for a 5-year CD is 0.74% less, which is less than 1.5%. The statement is false for a 5-year certificate of deposit. Thus, the change in the 5-year rate is a counterexample to the original statement.



Log on for:

- Updated data
- More on finding counterexamples

www.geometryonline.com/usa_today



USA TODAY Snapshots®

Latest CD rates

Average certificate of deposit rates as of Wednesday:

6-month	This week	1.80%
	Last week	1.80%
	Year ago	4.55%
1-year	This week	2.12%
	Last week	2.11%
	Year ago	4.64%
2½-year	This week	2.96%
	Last week	2.96%
	Year ago	4.74%
5-year	This week	4.22%
	Last week	4.23%
	Year ago	4.96%

Source: Bank Rate Monitor, 800-327-7717, www.bankrate.com

USA TODAY

Check for Understanding

- Concept Check**
1. Write an example of a conjecture you have made outside of school.
 2. Determine whether the following conjecture is *always*, *sometimes*, or *never* true based on the given information.
Given: collinear points D , E , and F
Conjecture: $DE + EF = DF$
 3. **OPEN ENDED** Write a statement. Then find a counterexample for the statement.



www.geometryonline.com/extra_examples

CONTENTS

Guided Practice Make a conjecture about the next item in each sequence.

4. 
5. $-8, -5, -2, 1, 4$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

6. $PQ = RS$ and $RS = TU$ 7. \overline{AB} and \overline{CD} intersect at P .

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

8. **Given:** x is an integer.
Conjecture: $-x$ is negative.
9. **Given:** $WXYZ$ is a rectangle.
Conjecture: $WX = YZ$ and $WZ = XY$

Application 10. **HOUSES** Most homes in the northern United States have roofs made with steep angles. In the warmer areas of the southern states, homes often have flat roofs. Make a conjecture about why the roofs are different.

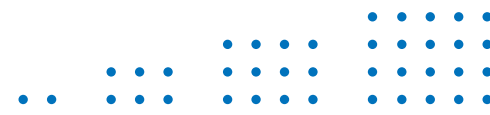

Practice and Apply

Homework Help

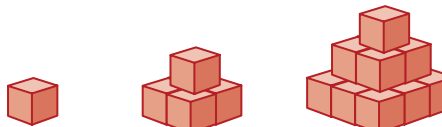
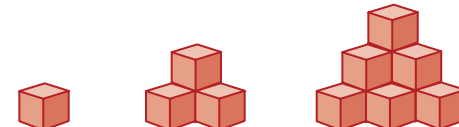
For Exercises	See Examples
11–20	1
21–28	2
29–36	3

Extra Practice
See page 756.

Make a conjecture about the next item in each sequence.

11. 
12. 
13. $1, 2, 4, 8, 16$ 14. $4, 6, 9, 13, 18$ 15. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3$
16. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ 17. $2, -6, 18, -54$ 18. $-5, 25, -125, 625$

Make a conjecture about the number of blocks in the next item of each sequence.

19.  20. 

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

21. Lines ℓ and m are perpendicular. 22. $A(-2, -11), B(2, 1), C(5, 10)$
23. $\angle 3$ and $\angle 4$ are a linear pair. 24. \overline{BD} is an angle bisector of $\angle ABC$.
25. $P(-1, 7), Q(6, -2), R(6, 5)$ 26. $H I J K$ is a square.
27. $P Q R S$ is a rectangle. 28. $\angle B$ is a right angle in $\triangle ABC$.

WebQuest

You can use scatter plots to make conjectures about the relationships between latitude, longitude, degree distance, and the monthly high temperature. Visit www.geometryonline.com/WebQuest to continue work on your WebQuest project.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

29. **Given:** $\angle 1$ and $\angle 2$ are complementary angles.

Conjecture: $\angle 1$ and $\angle 2$ form a right angle.

30. **Given:** $m + y \geq 10$, $y \geq 4$

Conjecture: $m \leq 6$

31. **Given:** points W , X , Y , and Z

Conjecture: W , X , Y , and Z are noncollinear.

32. **Given:** $A(-4, 8)$, $B(3, 8)$, $C(3, 5)$

Conjecture: $\triangle ABC$ is a right triangle.

33. **Given:** n is a real number.

Conjecture: n^2 is a nonnegative number.

34. **Given:** $DE = EF$

Conjecture: E is the midpoint of \overline{DF} .

35. **Given:** $JK = KL = LM = MJ$

Conjecture: $JKLM$ forms a square.

36. **Given:** noncollinear points R , S , and T

Conjecture: \overline{RS} , \overline{ST} , and \overline{RT} form a triangle.

- 37. **MUSIC** Many people learn to play the piano by ear. This means that they first learned how to play without reading music. What process did they use?

More About...



Music

The average medium-sized piano has about 230 strings. Each string has about 165 pounds of tension. That's a combined tension of about 18 tons.

Source: www.pianoworld.com

CHEMISTRY For Exercises 38–40, use the following information.

Hydrocarbons are molecules composed of only carbon (C) and hydrogen (H) atoms. The simplest hydrocarbons are called alkanes. The first three alkanes are shown below.

Alkanes			
Compound Name	Methane	Ethane	Propane
Chemical Formula	CH_4	C_2H_6	C_3H_8
Structural Formula	$\begin{array}{c} \text{H} \\ \\ \text{H}-\text{C}-\text{H} \\ \\ \text{H} \end{array}$	$\begin{array}{c} \text{H} \quad \text{H} \\ \quad \\ \text{H}-\text{C}-\text{C}-\text{H} \\ \quad \\ \text{H} \quad \text{H} \end{array}$	$\begin{array}{c} \text{H} \quad \text{H} \quad \text{H} \\ \quad \quad \\ \text{H}-\text{C}-\text{C}-\text{C}-\text{H} \\ \quad \quad \\ \text{H} \quad \text{H} \quad \text{H} \end{array}$

38. Make a conjecture about butane, which is the next compound in the group. Write its structural formula.
39. Write the chemical formula for the 7th compound in the group.
40. Develop a rule you could use to find the chemical formula of the n th substance in the alkane group.
41. **CRITICAL THINKING** The expression $n^2 - n + 41$ has a prime value for $n = 1$, $n = 2$, and $n = 3$. Based on this pattern, you might conjecture that this expression always generates a prime number for any positive integral value of n . Try different values of n to test the conjecture. Answer *true* if you think the conjecture is always true. Answer *false* and give a counterexample if you think the conjecture is false.



42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can inductive reasoning help predict weather conditions?

Include the following in your answer:

- an explanation as to how a conjecture about a weather pattern in the summer might be different from a similar weather pattern in the winter, and
- a conjecture about tomorrow's weather based on your local weather over the past several days.



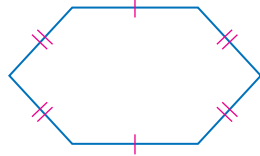
43. What is the next term in the sequence 1, 1, 2, 3, 5, 8?
 (A) 11 (B) 12 (C) 13 (D) 14
44. **ALGEBRA** If the average of six numbers is 18 and the average of three of the numbers is 15, then what is the sum of the remaining three numbers?
 (A) 21 (B) 45 (C) 53 (D) 63

Maintain Your Skills

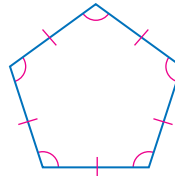
Mixed Review

Name each polygon by its number of sides and then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

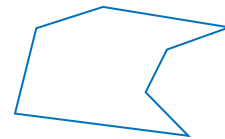
45.



46.

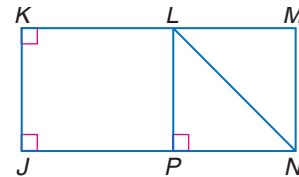


47.



Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)

48. $\angle KJN$ is a right angle.
 49. $\angle PLN \cong \angle NLM$
 50. $\angle PNL$ and $\angle MNL$ are complementary.
 51. $\angle KLN$ and $\angle MLN$ are supplementary.
 52. $\angle KLP$ is a right angle.



Find the coordinates of the midpoint of a segment having the given endpoints. (Lesson 1-3)

53. \overline{AB} for $A(-1, 3)$, $B(5, -5)$
 54. \overline{CD} for $C(4, 1)$, $D(-3, 7)$
 55. \overline{FG} for $F(4, -9)$, $G(-2, -15)$
 56. \overline{HJ} for $H(-5, -2)$, $J(7, 4)$
 57. \overline{KL} for $K(8, -1.8)$, $L(3, 6.2)$
 58. \overline{MN} for $M(-1.5, -6)$, $N(-4, 3)$

Find the value of the variable and MP , if P is between M and N . (Lesson 1-2)

59. $MP = 7x$, $PN = 3x$, $MN = 24$
 60. $MP = 2c$, $PN = 9c$, $MN = 63$
 61. $MP = 4x$, $PN = 5x$, $MN = 36$
 62. $MP = 6q$, $PN = 6q$, $MN = 60$
 63. $MP = 4y + 3$, $PN = 2y$, $MN = 63$
 64. $MP = 2b - 7$, $PN = 8b$, $MN = 43$

Getting Ready for the Next Lesson

BASIC SKILL Determine which values in the given replacement set make the inequality true.

65. $x + 2 > 5$
 {2, 3, 4, 5}
66. $12 - x < 0$
 {11, 12, 13, 14}
67. $5x + 1 > 25$
 {4, 5, 6, 7}

2-2 Logic

What You'll Learn

- Determine truth values of conjunctions and disjunctions.
- Construct truth tables.

Vocabulary

- statement
- truth value
- negation
- compound statement
- conjunction
- disjunction
- truth table

How does logic apply to school?

When you answer true-false questions on a test, you are using a basic principle of logic. For example, refer to the map, and answer *true* or *false*.

Raleigh is a city in North Carolina.

You know that there is only one correct answer, either true or false.



Study Tip

Statements

A mathematical statement with one or more variables is called an *open sentence*. The truth value of an open sentence cannot be determined until values are assigned to the variables. A statement with only numeric values is a *closed sentence*.

DETERMINE TRUTH VALUES A **statement**, like the true-false example above, is any sentence that is either true or false, but not both. Unlike a conjecture, we know that a statement is either true or false. The truth or falsity of a statement is called its **truth value**.

Statements are often represented using a letter such as p or q . The statement above can be represented by p .

p : Raleigh is a city in North Carolina. **This statement is true.**

The **negation** of a statement has the opposite meaning as well as an opposite truth value. For example, the negation of the statement above is *not* p .

not p : Raleigh is not a city in North Carolina. **In this case, the statement is false.**

Key Concept

Negation

- **Words** If a statement is represented by p , then *not* p is the negation of the statement.
- **Symbols** $\sim p$, read *not* p

Two or more statements can be joined to form a **compound statement**. Consider the following two statements.

p : Raleigh is a city in North Carolina.
 q : Raleigh is the capital of North Carolina.

The two statements can be joined by the word *and*.

p and q : Raleigh is a city in North Carolina, *and* Raleigh is the capital of North Carolina.

The statement formed by joining p and q is an example of a conjunction.

Key Concept

Conjunction

- **Words** A **conjunction** is a compound statement formed by joining two or more statements with the word *and*.
- **Symbols** $p \wedge q$, read *p and q*

A conjunction is true only when both statements in it are true. Since it is true that Raleigh is in North Carolina and it is the capital, the conjunction is also true.

Example 1 Truth Values of Conjunctions

Use the following statements to write a compound statement for each conjunction. Then find its truth value.

p : January 1 is the first day of the year.

q : $-5 + 11 = -6$

r : A triangle has three sides.

a. p and q

January is the first day of the year, and $-5 + 11 = -6$.
 p and q is false, because p is true and q is false.

b. $r \wedge p$

A triangle has three sides, and January 1 is the first day of the year.
 $r \wedge p$ is true, because r is true and p is true.

c. p and not r

January 1 is the first day of the year, and a triangle does not have three sides.
 p and not r is false, because p is true and not r is false.

d. $\sim q \wedge r$

$-5 + 11 \neq -6$, and a triangle has three sides
 $\sim q \wedge r$ is true because $\sim q$ is true and r is true.

Study Tip

Negations

The negation of a statement is not necessarily false. It has the opposite truth value of the original statement.

Statements can also be joined by the word *or*. This type of statement is a disjunction. Consider the following statements.

p : Ahmed studies chemistry.

q : Ahmed studies literature.

p or q : Ahmed studies chemistry, *or* Ahmed studies literature.

Key Concept

Disjunction

- **Words** A **disjunction** is a compound statement formed by joining two or more statements with the word *or*.
- **Symbols** $p \vee q$, read *p or q*

A disjunction is true if at least one of the statements is true. In the case of p or q above, the disjunction is true if Ahmed either studies chemistry or literature or both. The disjunction is false only if Ahmed studies neither chemistry nor literature.

Example 2 Truth Values of Disjunctions

Use the following statements to write a compound statement for each disjunction. Then find its truth value.

p : $100 \div 5 = 20$

q : The length of a radius of a circle is twice the length of its diameter.

r : The sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.

a. p or q

$100 \div 5 = 20$, or the length of a radius of a circle is twice the length of its diameter.

p or q is true because p is true. It does not matter that q is false.

b. $q \vee r$

The length of a radius of a circle is twice the length of its diameter, or the sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.

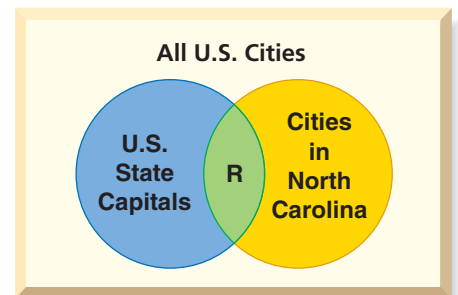
$q \vee r$ is false since neither statement is true.

Study Tip

Venn Diagrams

The size of the overlapping region in a Venn Diagram does not indicate how many items fall into that category.

Conjunctions can be illustrated with Venn diagrams. Refer to the statement at the beginning of the lesson. The Venn diagram at the right shows that Raleigh (R) is represented by the *intersection* of the set of cities in North Carolina and the set of state capitals. In other words, Raleigh must be in the set containing cities in North Carolina and in the set of state capitals.



A disjunction can also be illustrated with a Venn diagram. Consider the following statements.

p : Jerrica lives in a U.S. state capital.

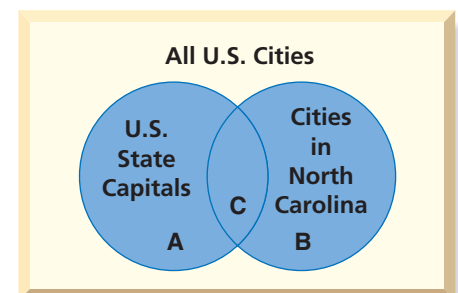
q : Jerrica lives in a North Carolina city.

$p \vee q$: Jerrica lives in a U.S. state capital, or Jerrica lives in a North Carolina city.

In the Venn diagrams, the disjunction is represented by the *union* of the two sets. The union includes all U.S. capitals and all cities in North Carolina. The city in which Jerrica lives could be located in any of the three regions of the union.

The three regions represent

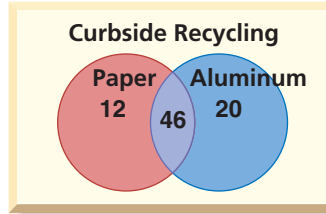
- A U.S. state capitals excluding the capital of North Carolina,
- B cities in North Carolina excluding the state capital, and
- C the capital of North Carolina, which is Raleigh.



Venn diagrams can be used to solve real-world problems involving conjunctions and disjunctions.

Example 3 Use Venn Diagrams

RECYCLING The Venn diagram shows the number of neighborhoods that have a curbside recycling program for paper or aluminum.



a. How many neighborhoods recycle both paper and aluminum?

The neighborhoods that have paper and aluminum recycling are represented by the intersection of the sets. There are 46 neighborhoods that have paper and aluminum recycling.

b. How many neighborhoods recycle paper or aluminum?

The neighborhoods that have paper or aluminum recycling are represented by the union of the sets. There are $12 + 46 + 20$ or 78 neighborhoods that have paper or aluminum recycling.

c. How many neighborhoods recycle paper and not aluminum?

The neighborhoods that have paper and not aluminum recycling are represented by the nonintersecting portion of the paper region. There are 12 neighborhoods that have paper and not aluminum recycling.

TRUTH TABLES A convenient method for organizing the truth values of statements is to use a **truth table**.

Negation	
p	$\sim p$
T	F
F	T

If p is a true statement, then $\sim p$ is a false statement.

If p is a false statement, then $\sim p$ is a true statement.

Truth tables can also be used to determine truth values of compound statements.

Study Tip

Tautology

A compound sentence is a *tautology* if its truth value is always true. For example, "It is snowing or it is not snowing" is a tautology.

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

A conjunction is true only when both statements are true.

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only when both statements are false.

You can use the truth values for negation, conjunction, and disjunction to construct truth tables for more complex compound statements.

Example 4 Construct Truth Tables

Construct a truth table for each compound statement.

a. $p \wedge \sim q$

Step 1 Make columns with the headings p , q , $\sim q$, and $p \wedge \sim q$.

Step 2 List the possible combinations of truth values for p and q .

Step 3 Use the truth values of q to determine the truth values of $\sim q$.

Step 4 Use the truth values for p and $\sim q$ to write the truth values for $p \wedge \sim q$.

Step 1 →

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Step 2 Step 3 Step 4

b. $\sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

c. $(p \wedge q) \vee r$

Make columns for p , q , $p \wedge q$, r , and $(p \wedge q) \vee r$.

p	q	$p \wedge q$	r	$(p \wedge q) \vee r$
T	T	T	T	T
T	F	F	T	T
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T
F	T	F	F	F
F	F	F	F	F

Study Tip

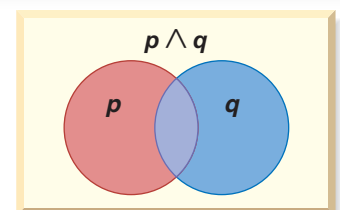
Truth Tables

Use the Fundamental Counting Principle to determine the number of rows necessary.

Check for Understanding

Concept Check

- Describe how to interpret the Venn diagram for $p \wedge q$.
- OPEN ENDED** Write a compound statement for each condition.
 - a true disjunction
 - a false conjunction
 - a true statement that includes a negation
- Explain** the difference between a conjunction and a disjunction.



Guided Practice Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

- p : $9 + 5 = 14$
 q : February has 30 days.
 r : A square has four sides.

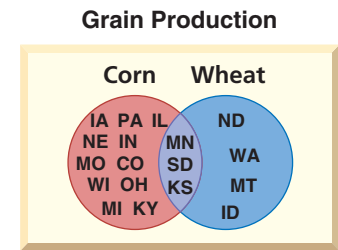
4. p and q 5. p and r 6. $q \wedge r$
 7. p or $\sim q$ 8. $q \vee r$ 9. $\sim p \vee \sim r$
10. Copy and complete the truth table.

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F		
F	T		
F	F		

Construct a truth table for each compound statement.

11. $p \wedge q$ 12. $q \vee r$ 13. $\sim p \wedge r$ 14. $(p \vee q) \vee r$

Application **AGRICULTURE** For Exercises 15–17, refer to the Venn diagram that represents the states producing more than 100 million bushels of corn or wheat per year.



Source: U.S. Department of Agriculture

15. How many states produce more than 100 million bushels of corn?
 16. How many states produce more than 100 million bushels of wheat?
 17. How many states produce more than 100 million bushels of corn and wheat?

Practice and Apply

Homework Help

For Exercises	See Examples
18–29	1, 2
30–41	4
42–48	3

Extra Practice
See page 756.

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

- p : $\sqrt{-64} = 8$
 q : An equilateral triangle has three congruent sides.
 r : $0 < 0$
 s : An obtuse angle measures greater than 90° and less than 180° .

18. p and q 19. p or q 20. p and r
 21. r and s 22. q or r 23. q and s
 24. $p \wedge s$ 25. $q \wedge r$ 26. $r \vee p$
 27. $s \vee q$ 28. $(p \wedge q) \vee s$ 29. $s \vee (q \text{ and } r)$

Copy and complete each truth table.

30.

p	q	$\sim p$	$\sim p \vee q$
T	T		
T	F		
F	T		
F	F		

31.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T		F	F	
T		F	T	
F		T	F	
F		T	T	

32. Copy and complete the truth table.

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

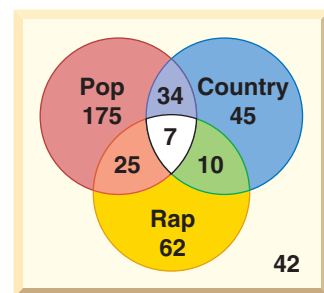
Construct a truth table for each compound statement.

33. q and r 34. p or q 35. p or r 36. p and q
 37. $q \wedge \sim r$ 38. $\sim p \wedge \sim q$ 39. $\sim p \vee (q \wedge \sim r)$ 40. $p \wedge (\sim q \vee \sim r)$

MUSIC For Exercises 41–44, use the following information.

A group of 400 teens were asked what type of music they listened to. They could choose among pop, rap, and country. The results are shown in the Venn diagram.

Music Preference



41. How many teens said that they listened to none of these types of music?
 42. How many said that they listened to all three types of music?
 43. How many said that they listened to only pop and rap music?
 44. How many teens said that they listened to pop, rap, or country music?

SCHOOL For Exercises 45–47, use the following information.

In a school of 310 students, 80 participate in academic clubs, 115 participate in sports, and 20 students participate in both.

45. Make a Venn diagram of the data.
 46. How many students participate in either clubs or sports?
 47. How many students do not participate in either clubs or sports?

RESEARCH For Exercises 48–50, use the Internet or another resource to determine whether each statement about cities in New York is *true* or *false*.

48. Albany is not located on the Hudson river.
 49. Either Rochester or Syracuse is located on Lake Ontario.
 50. It is false that Buffalo is located on Lake Erie.

CRITICAL THINKING For Exercises 51 and 52, use the following information.

All members of Team A also belong to Team B, but only some members of Team B also belong to Team C. Teams A and C have no members in common.

51. Draw a Venn diagram to illustrate the situation.
 52. Which of the following statements is true?
 a. If a person is a member of Team C, then the person is not a member of Team A.
 b. If a person is not a member of Team B, then the person is not a member of Team A.
 c. No person that is a member of Team A can be a member of Team C.



School

Nationwide, approximately 80% of high school seniors participate in extracurricular activities. Athletics, performing arts, and clubs are the most popular.

Source: National Center for Education Statistics



53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How does logic apply to school?

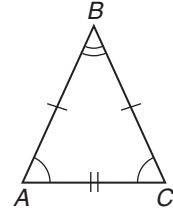
Include the following in your answer:

- an example of a conjunction using statements about your favorite subject and your favorite extracurricular activity, and
- a Venn diagram showing various characteristics of the members of your geometry class (for example, male/female, grade in school, and so on).



54. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?

- (A) $m\angle A = m\angle C$ (B) $m\angle A = m\angle B$
 (C) $AC = BC$ (D) $AB = AC$



55. **ALGEBRA** If the sum of two consecutive even integers is 78, which number is the greater of the two integers?

- (A) 36 (B) 38
 (C) 40 (D) 42

Maintain Your Skills

Mixed Review Make a conjecture about the next item in each sequence. (Lesson 2-1)

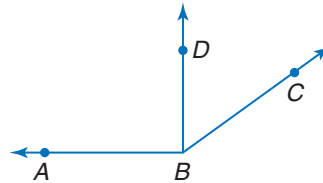
56. 3, 5, 7, 9 57. 1, 3, 9, 27 58. 6, 3, $\frac{3}{2}$, $\frac{3}{4}$
 59. 17, 13, 9, 5 60. 64, 16, 4, 1 61. 5, 15, 45, 135

COORDINATE GEOMETRY Find the perimeter of each polygon. Round answers to the nearest tenth. (Lesson 1-6)

62. triangle ABC with vertices $A(-6, 7)$, $B(1, 3)$, and $C(-2, -7)$
 63. square $DEFG$ with vertices $D(-10, -9)$, $E(-5, -2)$, $F(2, -7)$, and $G(-3, -14)$
 64. quadrilateral $HIIK$ with vertices $H(5, -10)$, $I(-8, -9)$, $J(-5, -5)$, and $K(-2, -4)$
 65. hexagon $LMNPQR$ with vertices $L(2, 1)$, $M(4, 5)$, $N(6, 4)$, $P(7, -4)$, $Q(5, -8)$, and $R(3, -7)$

Measure each angle and classify it as *right*, *acute*, or *obtuse*. (Lesson 1-4)

66. $\angle ABC$
 67. $\angle DBC$
 68. $\angle ABD$



69. **FENCING** Michelle wanted to put a fence around her rectangular garden. The front and back measured 35 feet each, and the sides measured 75 feet each. If she wanted to make sure that she had enough feet of fencing, how much should she buy? (Lesson 1-2)

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Evaluate each expression for the given values. (To review *evaluating algebraic expressions*, see page 736.)

70. $5a - 2b$ if $a = 4$ and $b = 3$ 71. $4cd + 2d$ if $c = 5$ and $d = 2$
 72. $4e + 3f$ if $e = -1$ and $f = -2$ 73. $3g^2 + h$ if $g = 8$ and $h = -8$



2-3

Conditional Statements

What You'll Learn

- Analyze statements in if-then form.
- Write the converse, inverse, and contrapositive of if-then statements.

Vocabulary

- conditional statement
- if-then statement
- hypothesis
- conclusion
- related conditionals
- converse
- inverse
- contrapositive
- logically equivalent

How are conditional statements used in advertisements?

Advertisers often lure consumers into purchasing expensive items by convincing them that they are getting something for free in addition to their purchase.

Sign Up for a Six-Month Fitness Plan and Get **Free** Six Months

Get \$1500 Cash Back When You Buy a New Car

Free Phone with Every One-Year Service Enrollment

IF-THEN STATEMENTS The statements above are examples of conditional statements. A **conditional statement** is a statement that can be written in *if-then form*. The first example above can be rewritten to illustrate this.

If you buy a car, *then* you get \$1500 cash back.

Key Concept

If-Then Statement

- **Words** An **if-then statement** is written in the form *if p, then q*. The phrase immediately following the word *if* is called the **hypothesis**, and the phrase immediately following the word *then* is called the **conclusion**.
- **Symbols** $p \rightarrow q$, read *if p then q*, or *p implies q*.

Study Tip

Reading Math

The word *if* is not part of the hypothesis. The word *then* is not part of the conclusion.

Example 1 Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

- a. If points A , B , and C lie on line ℓ , then they are collinear.

If $\underbrace{\text{points } A, B, \text{ and } C \text{ lie on line } \ell}_{\text{hypothesis}}$, then $\underbrace{\text{they are collinear}}_{\text{conclusion}}$.

Hypothesis: points A , B , and C lie on line ℓ

Conclusion: they are collinear

- b. The Tigers will play in the tournament if they win their next game.

Hypothesis: the Tigers win their next game

Conclusion: they will play in the tournament

Identifying the hypothesis and conclusion of a statement is helpful when writing statements in if-then form.

Study Tip

If-Then Statements

When you write a statement in if-then form, identify the condition that causes the result as the hypothesis. The result is the conclusion.

Example 2 Write a Conditional in If-Then Form

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

- a. An angle with a measure greater than 90 is an obtuse angle.

Hypothesis: an angle has a measure greater than 90

Conclusion: it is an obtuse angle

If an angle has a measure greater than 90, then it is an obtuse angle.

- b. Perpendicular lines intersect.

Sometimes you must add information to a statement. In this case, it is necessary to know that perpendicular lines come in pairs.

Hypothesis: two lines are perpendicular

Conclusion: they intersect

If two lines are perpendicular, then they intersect.

Recall that the truth value of a statement is either true or false. The hypothesis and conclusion of a conditional statement, as well as the conditional statement itself, can also be true or false.

Example 3 Truth Values of Conditionals

SCHOOL Determine the truth value of the following statement for each set of conditions.

If you get 100% on your test, then your teacher will give you an A.

- a. You get 100%; your teacher gives you an A.

The hypothesis is true since you got 100%, and the conclusion is true because the teacher gave you an A. Since what the teacher promised is true, the conditional statement is true.

- b. You get 100%; your teacher gives you a B.

The hypothesis is true, but the conclusion is false. Because the result is not what was promised, the conditional statement is false.

- c. You get 98%; your teacher gives you an A.

The hypothesis is false, and the conclusion is true. The statement does not say what happens if you do not get 100% on the test. You could still get an A. It is also possible that you get a B. In this case, we cannot say that the statement is false. Thus, the statement is true.

- d. You get 85%; your teacher gives you a B.

As in part c, we cannot say that the statement is false. Therefore, the conditional statement is true.

The resulting truth values in Example 3 can be used to create a truth table for conditional statements. Notice that a conditional statement is true in all cases except where the hypothesis is true and the conclusion is false.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Study Tip

Common Misconception

A true hypothesis does not necessarily mean that a conditional is true. Likewise, a false conclusion does not guarantee that a conditional is false.

CONVERSE, INVERSE, AND CONTRAPOSITIVE Other statements based on a given conditional statement are known as **related conditionals**.

Key Concept		Related Conditionals	
Statement	Formed by	Symbols	Examples
Conditional	given hypothesis and conclusion	$p \rightarrow q$	If two angles have the same measure, then they are congruent.
Converse	exchanging the hypothesis and conclusion of the conditional	$q \rightarrow p$	If two angles are congruent, then they have the same measure.
Inverse	negating both the hypothesis and conclusion of the conditional	$\sim p \rightarrow \sim q$	If two angles do not have the same measure, then they are not congruent.
Contrapositive	negating both the hypothesis and conclusion of the converse statement	$\sim q \rightarrow \sim p$	If two angles are not congruent, then they do not have the same measure.

Study Tip

Contrapositive

The relationship of the truth values of a conditional and its contrapositive is known as the Law of Contrapositive.

If a given conditional is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false.

Statements with the same truth values are said to be **logically equivalent**. So, a conditional and its contrapositive are logically equivalent as are the converse and inverse of a conditional. These relationships are summarized below.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Example 4 Related Conditionals

Write the converse, inverse, and contrapositive of the statement *Linear pairs of angles are supplementary*. Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.

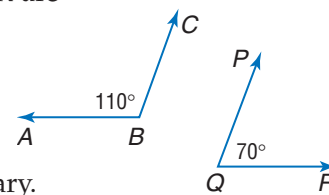
First, write the conditional in if-then form.

Conditional: If two angles form a linear pair, then they are supplementary.
The conditional statement is true.

Write the converse by switching the hypothesis and conclusion of the conditional.

Converse: If two angles are supplementary, then they form a linear pair. The converse is false. $\angle ABC$ and $\angle PQR$ are supplementary, but are not a linear pair.

Inverse: If two angles do not form a linear pair, then they are not supplementary. The inverse is false. $\angle ABC$ and $\angle PQR$ do not form a linear pair, but they are supplementary.



The contrapositive is the negation of the hypothesis and conclusion of the converse.

Contrapositive: If two angles are not supplementary, then they do not form a linear pair. The contrapositive is true.



Check for Understanding

- Concept Check**
1. **Explain** why writing a conditional statement in if-then form is helpful.
 2. **OPEN ENDED** Write an example of a conditional statement.
 3. **Compare and contrast** the inverse and contrapositive of a conditional.

Guided Practice Identify the hypothesis and conclusion of each statement.

4. If it rains on Monday, then I will stay home.
5. If $x - 3 = 7$, then $x = 10$.
6. If a polygon has six sides, then it is a hexagon.

Write each statement in if-then form.

7. A 32-ounce pitcher holds a quart of liquid.
8. The sum of the measures of supplementary angles is 180.
9. An angle formed by perpendicular lines is a right angle.

Determine the truth value of the following statement for each set of conditions.

If you drive faster than 65 miles per hour on the interstate, then you will receive a speeding ticket.

10. You drive 70 miles per hour, and you receive a speeding ticket.
11. You drive 62 miles per hour, and you do not receive a speeding ticket.
12. You drive 68 miles per hour, and you do not receive a speeding ticket.

Write the converse, inverse, and contrapositive of each conditional statement.

Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

13. If plants have water, then they will grow.
14. Flying in an airplane is safer than riding in a car.

- Application**
15. **FORESTRY** In different regions of the country, different variations of trees dominate the landscape. In Colorado, aspen trees cover high areas of the mountains. In Florida, cypress trees rise from swamps. In Vermont, maple trees are prevalent. Write these conditionals in if-then form.

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–27	2
28–39	3
40–45	4

Extra Practice
See page 756.

Identify the hypothesis and conclusion of each statement.

16. If $2x + 6 = 10$, then $x = 2$.
17. If you are a teenager, then you are at least 13 years old.
18. If you have a driver's license, then you are at least 16 years old.
19. If three points lie on a line, then they are collinear.
20. "If a man hasn't discovered something that he will die for, he isn't fit to live."
(Martin Luther King, Jr., 1963)
21. If the measure of an angle is between 0 and 90, then the angle is acute.

Write each statement in if-then form.

22. Get a free visit with a one-year fitness plan.
23. Math teachers love to solve problems.
24. "I think, therefore I am." (Descartes)
25. Adjacent angles have a common side.
26. Vertical angles are congruent.
27. Equiangular triangles are equilateral.

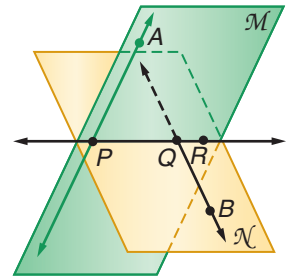
Determine the truth value of the following statement for each set of conditions.

If you are over 18 years old, then you vote in all elections.

28. You are 19 years old and you vote.
29. You are 16 years old and you vote.
30. You are 21 years old and do not vote.
31. You are 17 years old and do not vote.
32. Your sister is 21 years old and votes.
33. Your dad is 45 years old and does not vote.

In the figure, P , Q , and R are collinear, P and A lie in plane \mathcal{M} , and Q and B lie in plane \mathcal{N} . Determine the truth value of each statement.

34. P , Q , and R lie in plane \mathcal{M} .
35. \overline{QB} lies in plane \mathcal{N} .
36. Q lies in plane \mathcal{M} .
37. P , Q , A , and B are coplanar.
38. \overline{AP} contains Q .
39. Planes \mathcal{M} and \mathcal{N} intersect at \overline{RQ} .



Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

40. If you live in Dallas, then you live in Texas.
41. If you exercise regularly, then you are in good shape.
42. The sum of two complementary angles is 90.
43. All rectangles are quadrilaterals.
44. All right angles measure 90.
45. Acute angles have measures less than 90.

••• **SEASONS** For Exercises 46 and 47, use the following information.

Due to the movement of Earth around the sun, summer days in Alaska have more hours of daylight than darkness, and winter days have more hours of darkness than daylight.

46. Write two true conditional statements in if-then form for summer days and winter days in Alaska.
47. Write the converse of the two true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.
48. **CRITICAL THINKING** Write a false conditional statement. Is it possible to insert the word *not* into your conditional to make it true? If so, write the true conditional.
49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are conditional statements used in advertisements?

Include the following in your answer:

- an example of a conditional statement in if-then form, and
- an example of a conditional statement that is not in if-then form.



More About . . .

Seasons

At the poles, sunlight may shine continuously for six months during spring and summer, but never rises more than 23.5° above the horizon. During the other six months of the year, the poles are in darkness.

Source: U.S. Geological Survey



50. Which statement has the same truth value as the following statement?
If Ava and Willow are classmates, then they go to the same school.
- (A) If Ava and Willow go to the same school, then they are classmates.
(B) If Ava and Willow are not classmates, then they do not go to the same school.
(C) If Ava and Willow do not go to the same school, then they are not classmates.
(D) If Ava and Willow go to the same school, then they are not classmates.
51. **ALGEBRA** In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?
- (A) 2 (B) 8 (C) 12 (D) 20

Maintain Your Skills

Mixed Review Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value. *(Lesson 2-2)*

p: George Washington was the first president of the United States.

q: A hexagon has five sides.

r: $60 \times 3 = 18$

52. $p \wedge q$

53. $q \vee r$

54. $p \vee q$

55. $\sim q \vee r$

56. $p \wedge \sim q$

57. $\sim p \wedge \sim r$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. *(Lesson 2-1)*

58. $ABCD$ is a rectangle.

59. In $\triangle FGH$, $m\angle F = 45$, $m\angle G = 67$, $m\angle H = 68$.

60. $J(-3, 2)$, $K(1, 8)$, $L(5, 2)$

61. In $\triangle PQR$, $m\angle PQR = 90$

Use the Distance Formula to find the distance between each pair of points.

(Lesson 1-3)

62. $C(-2, -1)$, $D(0, 3)$

63. $J(-3, 5)$, $K(1, 0)$

64. $P(-3, -1)$, $Q(2, -3)$

65. $R(1, -7)$, $S(-4, 3)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Identify the operation used to change Equation (1) to Equation (2). *(To review solving equations, see pages 737 and 738.)*

66. (1) $3x + 4 = 5x - 8$

67. (1) $\frac{1}{2}(a - 5) = 12$

68. (1) $8p = 24$

(2) $3x = 5x - 12$

(2) $\bar{a} - 5 = 24$

(2) $p = 3$

Practice Quiz 1

Lessons 2-1 through 2-3

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

(Lesson 2-1)

1. **Given:** $WX = XY$

Conjecture: W , X , and Y are collinear.

2. **Given:** $\angle 1$ and $\angle 2$ are complementary.

$\angle 2$ and $\angle 3$ are complementary.

Conjecture: $m\angle 1 = m\angle 3$

Construct a truth table for each compound statement. *(Lesson 2-2)*

3. $\sim p \wedge q$

4. $p \vee (q \wedge r)$

5. Write the converse, inverse, and contrapositive of the following conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample. *(Lesson 2-3)*

If two angles are adjacent, then the angles have a common vertex.



Biconditional Statements

Ashley began a new summer job, earning \$10 an hour. If she works over 40 hours a week, she earns time and a half, or \$15 an hour. If she earns \$15 an hour, she has worked over 40 hours a week.

p : Ashley earns \$15 an hour

q : Ashley works over 40 hours a week

$p \rightarrow q$: If Ashley earns \$15 an hour, she has worked over 40 hours a week.

$q \rightarrow p$: If Ashley works over 40 hours a week, she earns \$15 an hour.

In this case, both the conditional and its converse are true. The conjunction of the two statements is called a **biconditional**.

Key Concept

Biconditional Statement

- **Words** A biconditional statement is the conjunction of a conditional and its converse.
- **Symbols** $(p \rightarrow q) \wedge (q \rightarrow p)$ is written $(p \leftrightarrow q)$ and read p if and only if q .

If and only if can be abbreviated iff.

So, the biconditional statement is as follows.

$p \leftrightarrow q$: Ashley earns \$15 an hour *if and only if* she works over 40 hours a week.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

- a. Two angle measures are complements if and only if their sum is 90.

Conditional: If two angle measures are complements, then their sum is 90.

Converse: If the sum of two angle measures is 90, then they are complements.

Both the conditional and the converse are true, so the biconditional is true.

- b. $x > 9$ iff $x > 0$

Conditional: If $x > 9$, then $x > 0$.

Converse: If $x > 0$, then $x > 9$.

The conditional is true, but the converse is not. Let $x = 2$. Then $2 > 0$ but $2 \not> 9$. So, the biconditional is false.

Reading to Learn

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. A calculator will run if and only if it has batteries.
2. Two lines intersect if and only if they are not vertical.
3. Two angles are congruent if and only if they have the same measure.
4. $3x - 4 = 20$ iff $x = 7$.
5. A line is a segment bisector if and only if it intersects the segment at its midpoint.

2-4

Deductive Reasoning

What You'll Learn

- Use the Law of Detachment.
- Use the Law of Syllogism.

How does deductive reasoning apply to health?

When you are ill, your doctor may prescribe an antibiotic to help you get better. Doctors may use a dose chart like the one shown to determine the correct amount of medicine you should take.

Weight (kg)	Dose (mg)
10-20	150
20-30	200
30-40	250
40-50	300
50-60	350
60-70	400

Vocabulary

- deductive reasoning
- Law of Detachment
- Law of Syllogism

LAW OF DETACHMENT The process that doctors use to determine the amount of medicine a patient should take is called **deductive reasoning**. Unlike inductive reasoning, which uses examples to make a conjecture, deductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions.

A form of deductive reasoning that is used to draw conclusions from true conditional statements is called the **Law of Detachment**.

Key Concept

Law of Detachment

- **Words** If $p \rightarrow q$ is true and p is true, then q is also true.
- **Symbols** $[(p \rightarrow q) \wedge p] \rightarrow q$

Study Tip

Validity

When you apply the Law of Detachment, make sure that the conditional is true before you test the validity of the conclusion.

Example 1 Determine Valid Conclusions

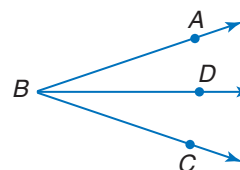
The following is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.

If a ray is an angle bisector, then it divides the angle into two congruent angles.

a. Given: \overrightarrow{BD} bisects $\angle ABC$.

Conclusion: $\angle ABD \cong \angle CBD$

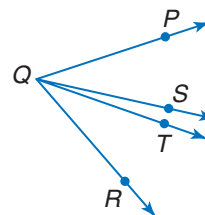
The hypothesis states that \overrightarrow{BD} is the bisector of $\angle ABC$. Since the conditional is true and the hypothesis is true, the conclusion is valid.



b. Given: $\angle PQT \cong \angle RQS$

Conclusion: \overrightarrow{QS} and \overrightarrow{QT} are angle bisectors.

Knowing that a conditional statement and its conclusion are true does not make the hypothesis true. An angle bisector divides an angle into two separate congruent angles. In this case, the given angles are not separated by one ray. Instead, they overlap. The conclusion is not valid.



LAW OF SYLLOGISM Another law of logic is the **Law of Syllogism**. It is similar to the Transitive Property of Equality.

Key Concept

Law of Syllogism

- **Words** If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.
- **Symbols** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Study Tip

Conditional Statements

Label the hypotheses and conclusions of a series of statements before applying the Law of Syllogism.

Example 2 Determine Valid Conclusions From Two Conditionals

CHEMISTRY Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements.

- a. (1) If the symbol of a substance is Pb, then it is lead.
(2) The atomic number of lead is 82.

Let p , q , and r represent the parts of the statement.

p : the symbol of a substance is Pb

q : it is lead

r : the atomic number is 82

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If the symbol of a substance is Pb, then its atomic number is 82.*

- b. (1) Water can be represented by H_2O .
(2) Hydrogen (H) and oxygen (O) are in the atmosphere.

There is no valid conclusion. While both statements are true, the conclusion of each statement is not used as the hypothesis of the other.

Example 3 Analyze Conclusions

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

- a. (1) Vertical angles are congruent.
(2) If two angles are congruent, then their measures are equal.
(3) If two angles are vertical, then their measures are equal.

p : two angles are vertical

q : they are congruent

r : their measures are equal

Statement (3) is a valid conclusion by the Law of Syllogism.

- b. (1) If a figure is a square, then it is a polygon.
(2) Figure A is a polygon.
(3) Figure A is a square.

Statement (1) is true, but statement (3) does not follow from statement (2).

Not all polygons are squares.

Statement (3) is invalid.



Check for Understanding

- Concept Check**
- OPEN ENDED** Write an example to illustrate the correct use of the Law of Detachment.
 - Explain** how the Transitive Property of Equality is similar to the Law of Syllogism.
 - FIND THE ERROR** An article in a magazine states that if you get seasick, then you will get dizzy. It also says that if you get seasick, you will get an upset stomach. Suzanne says that this means that if you get dizzy, then you will get an upset stomach. Lakeisha says that she is wrong. Who is correct? Explain.

Guided Practice Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If two angles are vertical angles, then they are congruent.

- Given:** $\angle A$ and $\angle B$ are vertical angles.
Conclusion: $\angle A \cong \angle B$
- Given:** $\angle C \cong \angle D$
Conclusion: $\angle C$ and $\angle D$ are vertical angles.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

- If you are 18 years old, you are in college.
You are in college.
- The midpoint divides a segment into two congruent segments.
If two segments are congruent, then their measures are equal.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

- (1) If Molly arrives at school at 7:30 A.M., she will get help in math.
(2) If Molly gets help in math, then she will pass her math test.
(3) If Molly arrives at school at 7:30 A.M., then she will pass her math test.
- (1) Right angles are congruent.
(2) $\angle X \cong \angle Y$
(3) $\angle X$ and $\angle Y$ are right angles.

Application **INSURANCE** For Exercises 10 and 11, use the following information. An insurance company advertised the following monthly rates for life insurance.

If you are a:	Premium for \$30,000 Coverage	Premium for \$50,000 Coverage
Female, age 35.....	\$14.35.....	\$19.00
Male, age 35.....	\$16.50.....	\$21.63
Female, age 45.....	\$21.63.....	\$25.85
Male, age 45.....	\$23.75.....	\$28.90

- If Ann is 35 years old and she wants to purchase \$30,000 of insurance from this company, then what is her premium?
- If Terry paid \$21.63 for life insurance, can you conclude that Terry is 35? Explain.

Practice and Apply

Homework Help

For Exercises	See Examples
12–19	1
20–23	2
24–29	3

Extra Practice
See page 757.

For Exercises 12–19, determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If two numbers are odd, then their sum is even.

12. **Given:** The sum of two numbers is 22.
Conclusion: The two numbers are odd.
13. **Given:** The numbers are 5 and 7.
Conclusion: The sum is even.
14. **Given:** 11 and 23 are added together.
Conclusion: The sum of 11 and 23 is even.
15. **Given:** The numbers are 2 and 6.
Conclusion: The sum is odd.

If three points are noncollinear, then they determine a plane.

16. **Given:** A , B , and C are noncollinear.
Conclusion: A , B , and C determine a plane.
17. **Given:** E , F , and G lie in plane M .
Conclusion: E , F , and G are noncollinear.
18. **Given:** P and Q lie on a line.
Conclusion: P and Q determine a plane.
19. **Given:** $\triangle XYZ$
Conclusion: X , Y , and Z determine a plane.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

20. If you spend money on it, then it is a business.
If you spend money on it, then it is fun.
21. If the measure of an angle is less than 90 , then it is acute.
If an angle is acute, then it is not obtuse.
22. If X is the midpoint of segment YZ , then $YX = XZ$.
If the measures of two segments are equal, then they are congruent.
23. If two lines intersect to form a right angle, then they are perpendicular.
Lines ℓ and m are perpendicular.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

24. (1) In-line skaters live dangerously.
(2) If you live dangerously, then you like to dance.
(3) If you are an in-line skater, then you like to dance.
25. (1) If the measure of an angle is greater than 90 , then it is obtuse.
(2) $m\angle ABC > 90$
(3) $\angle ABC$ is obtuse.
26. (1) Vertical angles are congruent.
(2) $\angle 3 \cong \angle 4$
(3) $\angle 3$ and $\angle 4$ are vertical angles.
27. (1) If an angle is obtuse, then it cannot be acute.
(2) $\angle A$ is obtuse.
(3) $\angle A$ cannot be acute.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

28. (1) If you drive safely, then you can avoid accidents.
(2) Tika drives safely.
(3) Tika can avoid accidents.
29. (1) If you are a customer, then you are always right.
(2) If you are a teenager, then you are always right.
(3) If you are a teenager, then you are a customer.

More About . . .



Literature

The Pulitzer Prize is awarded annually for outstanding contributions in the fields of journalism, literature, drama, and music.

Source: www.pulitzer.org

30. **LITERATURE** John Steinbeck, a Pulitzer Prize winning author, lived in Monterey, California, for part of his life. In 1945, he published the book, *Cannery Row*, about many of his local working-class heroes from Monterey. If you visited Cannery Row in Monterey during the 1940s, then you could hear the grating noise of the fish canneries. Write a valid conclusion to the following hypothesis. *If John Steinbeck lived in Monterey in 1941, . . .*

31. **SPORTS** In the 2002 Winter Olympics, Canadian speed skater Catriona Le May Doan won her second Olympic title in 500-meter speed skating. Ms. Doan was in the last heat for the second round of that race. Use the two true conditional statements to reach a valid conclusion about Ms. Doan's 2002 competition.
- (1) If Catriona Le May Doan skated her second 500 meters in 37.45 seconds, then she would beat the time of Germany's Monique Garbrecht-Enfeldt.
(2) If Ms. Doan beat the time of Monique Garbrecht-Enfeldt, then she would win the race.



Online Research Data Update Use the Internet or another resource to find the winning times for other Olympic events. Write statements using these times that can lead to a valid conclusion. Visit www.geometryonline.com/data_update to learn more.

32. **CRITICAL THINKING** An advertisement states that "If you like to ski, then you'll love Snow Mountain Resort." Stacey likes to ski, but when she went to Snow Mountain Resort, she did not like it very much. If you know that Stacey saw the ad, explain how her reasoning was flawed.
33. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How does deductive reasoning apply to health?

Include the following in your answer:

- an explanation of how doctors may use deductive reasoning to prescribe medicine, and
- an example of a doctor's uses of deductive reasoning to diagnose an illness, such as strep throat or chickenpox.

Standardized Test Practice

A B C D

34. Based on the following statements, which statement must be true?
- I If Yasahiro is an athlete and he gets paid, then he is a professional athlete.
II Yasahiro is not a professional athlete.
III Yasahiro is an athlete.
- (A) Yasahiro is an athlete and he gets paid.
(B) Yasahiro is a professional athlete or he gets paid.
(C) Yasahiro does not get paid.
(D) Yasahiro is not an athlete.

35. **ALGEBRA** At a restaurant, a diner uses a coupon for 15% off the cost of one meal. If the diner orders a meal regularly priced at \$16 and leaves a tip of 20% of the discounted meal, how much does she pay in total?
 (A) \$15.64 (B) \$16.32 (C) \$16.80 (D) \$18.72

Maintain Your Skills

Mixed Review **ADVERTISING** For Exercises 36–38, use the following information. (Lesson 2-3)

Advertising writers frequently use if-then statements to relay a message and promote their product. An ad for a type of Mexican food reads, *If you're looking for a fast, easy way to add some fun to your family's menu, try Casa Fiesta.*

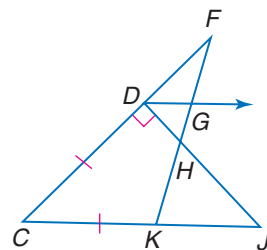
36. Write the converse of the conditional.
 37. What do you think the advertiser wants people to conclude about Casa Fiesta products?
 38. Does the advertisement say that Casa Fiesta adds fun to your family's menu?

Construct a truth table for each compound statement. (Lesson 2-2)

39. $q \wedge r$ 40. $\sim p \vee r$ 41. $p \wedge (q \vee r)$ 42. $p \vee (\sim q \wedge r)$

For Exercises 43–47, refer to the figure at the right. (Lesson 1-5)

43. Which angle is complementary to $\angle FDG$?
 44. Name a pair of vertical angles.
 45. Name a pair of angles that are noncongruent and supplementary.
 46. Identify $\angle FDH$ and $\angle CDH$ as *congruent*, *adjacent*, *vertical*, *complementary*, *supplementary*, and/or a *linear pair*.
 47. Can you assume that $\overline{DC} \cong \overline{CK}$? Explain.



Use the Pythagorean Theorem to find the distance between each pair of points. (Lesson 1-3)

48. $A(1, 5), B(-2, 9)$ 49. $C(-4, -2), D(2, 6)$
 50. $F(7, 4), G(1, 0)$ 51. $M(-5, 0), N(4, 7)$

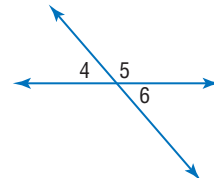
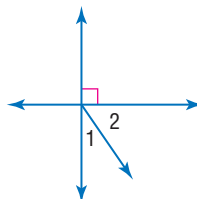
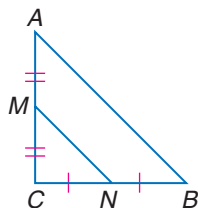
For Exercises 52–55, draw and label a figure for each relationship. (Lesson 1-1)

52. \overline{FG} lies in plane \mathcal{M} and contains point H .
 53. Lines r and s intersect at point W .
 54. Line ℓ contains P and Q , but does not contain R .
 55. Planes \mathcal{A} and \mathcal{B} intersect in line n .

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write what you can assume about the segments or angles listed for each figure. (To review information from figures, see Lesson 1-5.)

56. $\overline{AM}, \overline{CM}, \overline{CN}, \overline{BN}$ 57. $\angle 1, \angle 2$ 58. $\angle 4, \angle 5, \angle 6$





Geometry Activity

A Follow-Up of Lesson 2-4

Matrix Logic

Deductive reasoning can be used in problem-solving situations. One method of solving problems uses a table. This method is called **matrix logic**.

Example

GEOLOGY On a recent test, Rashaun was given five different mineral samples to identify, along with the chart at right. Rashaun observed the following.

- Sample C is brown.
- Samples B and E are harder than glass.
- Samples D and E are red.

Identify each of the samples.

Mineral	Color	Hardness
Biotite	brown or black	softer than glass
Halite	white	softer than glass
Hematite	red	softer than glass
Feldspar	white, pink, or green	harder than glass
Jaspar	red	harder than glass

Make a table to organize the information. Mark each false condition with an X and each true condition with a \checkmark . The first observation is that Sample C is brown. Only one of the minerals, biotite, is brown, so place a check in the box that corresponds to biotite and Sample C. Then place an X in each of the other boxes in the same column and row.

Sample	A	B	C	D	E
Biotite	X	X	\checkmark	X	X
Halite		X	X	X	X
Hematite		X	X		X
Feldspar			X	X	X
Jaspar	X	X	X	X	\checkmark

The second observation is that Samples B and E are harder than glass. Place an X in each box for minerals that are softer than glass. The third observation is that Samples D and E are red. Mark the boxes accordingly. Notice that Sample E has an X in all but one box. Place a check mark in the remaining box, and an X in all other boxes in that row.

Sample	A	B	C	D	E
Biotite	X	X	\checkmark	X	X
Halite	\checkmark	X	X	X	X
Hematite	X	X	X	\checkmark	X
Feldspar	X	\checkmark	X	X	X
Jaspar	X	X	X	X	\checkmark

Then complete the table. Sample A is Halite, Sample B is Feldspar, Sample C is Biotite, Sample D is Hematite, and Sample E is Jaspar.

Exercises

1. Nate, John, and Nick just began after-school jobs. One works at a veterinarian's office, one at a computer store, and one at a restaurant. Nate buys computer games on the way to work. Nick is allergic to cat hair. John receives free meals at his job. Who works at which job?
2. Six friends live in consecutive apartments on the same side of their apartment building. Anita lives in apartment C. Kelli's apartment is just past Scott's. Anita's closest neighbors are Eric and Ava. Scott's apartment is not A through D. Eric's apartment is before Ava's. If Roberto lives in one of the apartments, who lives in which apartment?

Postulates and Paragraph Proofs

What You'll Learn

- Identify and use basic postulates about points, lines, and planes.
- Write paragraph proofs.

Vocabulary

- postulate
- axiom
- theorem
- proof
- paragraph proof
- informal proof

How were postulates used by the founding fathers of the United States?

U.S. Supreme Court Justice William Douglas stated “The First Amendment makes confidence in the common sense of our people and in the maturity of their judgment the great postulate of our democracy.” The writers of the constitution assumed that citizens would act and speak with common sense and maturity. Some statements in geometry also must be assumed or accepted as true.



POINTS, LINES, AND PLANES In geometry, a **postulate**, or **axiom**, is a statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true. The basic ideas about points, lines, and planes can be stated as postulates.

Postulates

2.1 Through any two points, there is exactly one line.

2.2 Through any three points not on the same line, there is exactly one plane.

Example 1 Points and Lines

COMPUTERS Jessica is setting up a network for her father’s business. There are five computers in his office. Each computer needs to be connected to every other computer. How many connections does Jessica need to make?

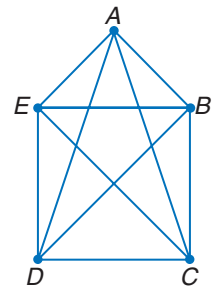
Explore There are five computers, and each is connected to four others.

Plan Draw a diagram to illustrate the solution.

Solve Let noncollinear points A , B , C , D , and E represent the five computers. Connect each point with every other point. Then, count the number of segments.

Between every two points there is exactly one segment. So, the connection between computer A and computer B is the same as the connection between computer B and computer A . For the five points, ten segments can be drawn.

Examine \overline{AB} , \overline{AC} , \overline{AD} , \overline{AE} , \overline{BC} , \overline{BD} , \overline{BE} , \overline{CD} , \overline{CE} , and \overline{DE} each represent a connection between two computers. So there will be ten connections among the five computers.



Study Tip

Drawing Diagrams

When listing segments, start with one vertex and draw all of the segments from that vertex. Then move on to the other vertices until all possible segments have been drawn.

There are other postulates that are based on relationships among points, lines, and planes.

Postulates

- 2.3** A line contains at least two points.
- 2.4** A plane contains at least three points not on the same line.
- 2.5** If two points lie in a plane, then the entire line containing those points lies in that plane.
- 2.6** If two lines intersect, then their intersection is exactly one point.
- 2.7** If two planes intersect, then their intersection is a line.

Example 2 Use Postulates

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

a. If points A , B , and C lie in plane \mathcal{M} , then they are collinear.

Sometimes; A , B , and C do not necessarily have to be collinear to lie in plane \mathcal{M} .

b. There is exactly one plane that contains noncollinear points P , Q , and R .

Always; Postulate 2.2 states that through any three noncollinear points, there is exactly one plane.

c. There are at least two lines through points M and N .

Never; Postulate 2.1 states that through any two points, there is exactly one line.

PARAGRAPH PROOFS Undefined terms, definitions, postulates, and algebraic properties of equality are used to prove that other statements or conjectures are true. Once a statement or conjecture has been shown to be true, it is called a **theorem**, and it can be used like a definition or postulate to justify that other statements are true.

You will study and use various methods to verify or prove statements and conjectures in geometry. A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true. One type of proof is called a **paragraph proof** or **informal proof**. In this type of proof, you write a paragraph to explain why a conjecture for a given situation is true.

Study Tip

Proofs

Before writing a proof, you should have a plan. One strategy is to *work backward*. Start with what you want to prove, and work backward step by step until you reach the given information.

Key Concept

Proofs

Five essential parts of a good proof:

- State the theorem or conjecture to be proven.
- List the given information.
- If possible, draw a diagram to illustrate the given information.
- State what is to be proved.
- Develop a system of deductive reasoning.

In Lesson 1-2, you learned the relationship between segments formed by the midpoint of a segment. This statement can be proven, and the result stated as a theorem.

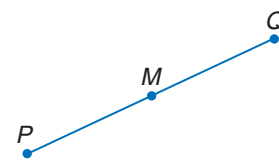
Example 3 Write a Paragraph Proof

Given that M is the midpoint of \overline{PQ} , write a paragraph proof to show that $\overline{PM} \cong \overline{MQ}$.

Given: M is the midpoint of \overline{PQ} .

Prove: $\overline{PM} \cong \overline{MQ}$.

From the definition of midpoint of a segment, $PM = MQ$. This means that \overline{PM} and \overline{MQ} have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent. Thus, $\overline{PM} \cong \overline{MQ}$.



Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is known as the Midpoint Theorem.

Theorem 2.8

Midpoint Theorem If M is the midpoint of \overline{AB} , then $\overline{AM} \cong \overline{MB}$.

Check for Understanding

Concept Check

1. Explain how deductive reasoning is used in a proof.
2. **OPEN ENDED** Draw figures to illustrate Postulates 2.6 and 2.7.
3. List the types of reasons that can be used for justification in a proof.

Guided Practice

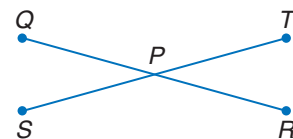
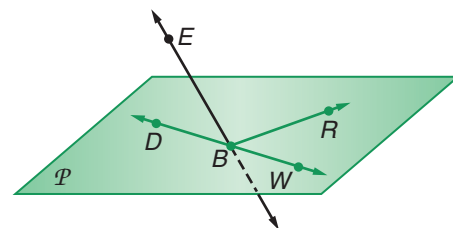
Determine the number of segments that can be drawn connecting each pair of points.



6. Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.
The intersection of three planes is two lines.

In the figure, \overleftrightarrow{BD} and \overleftrightarrow{BR} are in plane \mathcal{P} , and W is on \overleftrightarrow{BD} . State the postulate or definition that can be used to show each statement is true.

7. B , D , and W are collinear.
8. E , B , and R are coplanar.
9. R and W are collinear.
10. **PROOF** In the figure at the right, P is the midpoint of \overline{QR} and \overline{ST} , and $\overline{QR} \cong \overline{ST}$. Write a paragraph proof to show that $PQ = PT$.



Application

11. **DANCING** Six students are participating in a dance to celebrate the opening of a new community center. The students, each connected to each of the other students with wide colored ribbons, will move in a circular motion. How many ribbons are needed?



Practice and Apply

Homework Help

For Exercises	See Examples
12–15	1
16–21	2
22–28	3

Extra Practice
See page 757.

Determine the number of segments that can be drawn connecting each pair of points.

12. •



13. • •



14. •



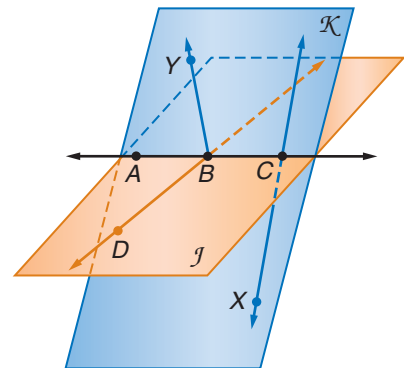
15. • •



Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

- Three points determine a plane.
- Points G and H are in plane X . Any point collinear with G and H is in plane X .
- The intersection of two planes can be a point.
- Points S , T , and U determine three lines.
- Points A and B lie in at least one plane.
- If line ℓ lies in plane P and line m lies in plane Q , then lines ℓ and m lie in plane R .

In the figure at the right, \overleftrightarrow{AC} and \overleftrightarrow{BD} lie in plane J , and \overleftrightarrow{BY} and \overleftrightarrow{CX} lie in plane K . State the postulate that can be used to show each statement is true.



- C and D are collinear.
- \overleftrightarrow{XB} lies in plane K .
- Points A , C , and X are coplanar.
- \overleftrightarrow{AD} lies in plane J .
- X and Y are collinear.
- Points Y , D , and C are coplanar.
- PROOF** Point C is the midpoint of \overline{AB} and B is the midpoint of \overline{CD} . Prove that $\overline{AC} \cong \overline{BD}$.
- MODELS** Faith's teacher asked her to make a figure showing the number of lines and planes formed from four points that are noncollinear and noncoplanar. Faith decided to make a mobile of straws, pipe cleaners, and colored sheets of tissue paper. She plans to glue the paper to the straws and connect the straws together to form a group of connected planes. How many planes and lines will she have?
- CAREERS** Many professions use deductive reasoning and paragraph proofs. For example, a police officer uses deductive reasoning investigating a traffic accident and then writes the findings in a report. List a profession, and describe how it can use paragraph proofs.

Career Choices



Detective

A police detective gathers facts and collects evidence for use in criminal cases. The facts and evidence are used together to prove a suspect's guilt in court.

Online Research

For information about a career as a detective, visit:

www.geometryonline.com/careers

31. **CRITICAL THINKING** You know that three noncollinear points lie in a single plane. In Exercise 29, you found the number of planes defined by four noncollinear points. What are the least and greatest number of planes defined by five noncollinear points?

32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are postulates used in literature?

Include the following in your answer:

- an example of a postulate in historic United States' documents, and
- an example of a postulate in mathematics.



33. Which statement cannot be true?

- (A) A plane can be determined using three noncollinear points.
- (B) Two lines intersect at exactly one point.
- (C) At least two lines can contain the same two points.
- (D) A midpoint divides a segment into two congruent segments.

34. **ALGEBRA** For all values of x , $(8x^4 - 2x^2 + 3x - 5) - (2x^4 + x^3 + 3x + 5) =$

- (A) $6x^4 - x^3 - 2x^2 - 10$.
- (B) $6x^4 - 3x^2 + 6x - 10$.
- (C) $6x^4 + x^3 - 2x^2 + 6x$.
- (D) $6x^4 - 3x^2$.

Maintain Your Skills

Mixed Review

35. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)

- (1) Part-time jobs require 20 hours of work per week.
- (2) Jamie has a part-time job.
- (3) Jamie works 20 hours per week.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample. (Lesson 2-3)

- 36. If you have access to the Internet at your house, then you have a computer.
- 37. If $\triangle ABC$ is a right triangle, one of its angle measures is greater than 90.
- 38. **BIOLOGY** Use a Venn diagram to illustrate the following statement.
If an animal is a butterfly, then it is an arthropod. (Lesson 2-2)

Use the Distance Formula to find the distance between each pair of points.
(Lesson 1-3)

- 39. $D(3, 3), F(4, -1)$
- 40. $M(0, 2), N(-5, 5)$
- 41. $P(-8, 2), Q(1, -3)$
- 42. $R(-5, 12), S(2, 1)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.
(To review *solving equations*, see pages 737 and 738.)

- 43. $m - 17 = 8$
- 44. $3y = 57$
- 45. $\frac{y}{6} + 12 = 14$
- 46. $-t + 3 = 27$
- 47. $8n - 39 = 41$
- 48. $-6x + 33 = 0$



2-6 Algebraic Proof

What You'll Learn

- Use algebra to write two-column proofs.
- Use properties of equality in geometry proofs.

How is mathematical evidence similar to evidence in law?

Lawyers develop their cases using logical arguments based on evidence to lead a jury to a conclusion favorable to their case. At the end of a trial, a lawyer will make closing remarks summarizing the evidence and testimony that they feel proves their case. These closing arguments are similar to a proof in mathematics.



Vocabulary

- deductive argument
- two-column proof
- formal proof

ALGEBRAIC PROOF Algebra is a system with sets of numbers, operations, and properties that allow you to perform algebraic operations.

Concept Summary Properties of Equality for Real Numbers

Reflexive Property	For every number a , $a = a$.
Symmetric Property	For all numbers a and b , if $a = b$, then $b = a$.
Transitive Property	For all numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	For all numbers a , b , and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	For all numbers a , b , and c , if $a = b$, then $a \cdot c = b \cdot c$ and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	For all numbers a and b , if $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	For all numbers a , b , and c , $a(b + c) = ab + ac$.

Study Tip

Commutative and Associative Properties

Throughout this text, we shall assume the Commutative and Associative Properties for addition and multiplication.

The properties of equality can be used to justify each step when solving an equation. A group of algebraic steps used to solve problems form a **deductive argument**.

Example 1 Verify Algebraic Relationships

Solve $3(x - 2) = 42$.

Algebraic Steps	Properties
$3(x - 2) = 42$	Original equation
$3x - 6 = 42$	Distributive Property
$3x - 6 + 6 = 42 + 6$	Addition Property
$3x = 48$	Substitution Property
$\frac{3x}{3} = \frac{48}{3}$	Division Property
$x = 16$	Substitution Property

Example 1 is a proof of the conditional statement *If $5x + 3(x - 2) = 42$, then $x = 6$.* Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reason for each statement.

In geometry, a similar format is used to prove conjectures and theorems. A **two-column proof**, or **formal proof**, contains statements and reasons organized in two columns. In a two-column proof, each step is called a *statement*, and the properties that justify each step are called *reasons*.

Example 2 Write a Two-Column Proof

Write a two-column proof.

a. If $3\left(x - \frac{5}{3}\right) = 1$, then $x = 2$

Statements	Reasons
1. $3\left(x - \frac{5}{3}\right) = 1$	1. Given
2. $3x - 3\left(\frac{5}{3}\right) = 1$	2. Distributive Property
3. $3x - 5 = 1$	3. Substitution
4. $3x - 5 + 5 = 1 + 5$	4. Addition Property
5. $3x = 6$	5. Substitution
6. $\frac{3x}{3} = \frac{6}{3}$	6. Division Property
7. $x = 2$	7. Substitution

b. **Given:** $\frac{7}{2} - n = 4 - \frac{1}{2}n$

Prove: $n = -1$

Proof:

Statements	Reasons
1. $\frac{7}{2} - n = 4 - \frac{1}{2}n$	1. Given
2. $2\left(\frac{7}{2} - n\right) = 2\left(4 - \frac{1}{2}n\right)$	2. Multiplication Property
3. $7 - 2n = 8 - n$	3. Distributive Property
4. $7 - 2n + n = 8 - n + n$	4. Addition Property
5. $7 - n = 8$	5. Substitution
6. $7 - n - 7 = 8 - 7$	6. Subtraction Property
7. $-n = 1$	7. Substitution
8. $\frac{-n}{-1} = \frac{1}{-1}$	8. Division Property
9. $n = -1$	9. Substitution

Study Tip

Mental Math

If your teacher permits you to do so, some steps may be eliminated by performing mental calculations. For example, in part a of Example 2, statements 4 and 6 could be omitted. Then the reason for statements 5 would be Addition Property and Division Property for statement 7.

GEOMETRIC PROOF Since geometry also uses variables, numbers, and operations, many of the properties of equality used in algebra are also true in geometry. For example, segment measures and angle measures are real numbers, so properties from algebra can be used to discuss their relationships. Some examples of these applications are shown below.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

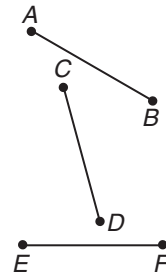


Example 3 Justify Geometric Relationships

Multiple-Choice Test Item

If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then which of the following is a valid conclusion?

- I $AB = CD$ and $CD = EF$
 - II $\overline{AB} \cong \overline{EF}$
 - III $AB = EF$
- (A) I only (B) I and II
(C) I and III (D) I, II, and III



Test-Taking Tip

More than one statement may be correct. Work through each problem completely before indicating your answer.

Read the Test Item

Determine whether the statements are true based on the given information.

Solve the Test Item

Statement I:

Examine the given information, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$. From the definition of congruent segments, if $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $AB = CD$ and $CD = EF$. Thus, Statement I is true.

Statement II:

By the definition of congruent segments, if $AB = EF$, then $\overline{AB} \cong \overline{EF}$. Statement II is true also.

Statement III:

If $AB = CD$ and $CD = EF$, then $AB = EF$ by the Transitive Property. Thus, Statement III is true.

Because Statements I, II, and III are true, choice D is correct.

In Example 3, each conclusion was justified using a definition or property. This process is used in geometry to verify and prove statements.

Example 4 Geometric Proof

TIME On a clock, the angle formed by the hands at 2:00 is a 60° angle. If the angle formed at 2:00 is congruent to the angle formed at 10:00, prove that the angle at 10:00 is a 60° angle.

Given: $m\angle 2 = 60$
 $\angle 2 \cong \angle 10$

Prove: $m\angle 10 = 60$



Proof:

Statements	Reasons
1. $m\angle 2 = 60$ $\angle 2 \cong \angle 10$	1. Given
2. $m\angle 2 = m\angle 10$	2. Definition of congruent angles
3. $60 = m\angle 10$	3. Substitution
4. $m\angle 10 = 60$	4. Symmetric Property

Check for Understanding

- Concept Check**
- OPEN ENDED** Write a statement that illustrates the Substitution Property of Equality.
 - Describe** the parts of a two-column proof.
 - State** the part of a conditional that is related to the *Given* statement of a proof. What part is related to the *Prove* statement?

Guided Practice State the property that justifies each statement.

- If $2x = 5$, then $x = \frac{5}{2}$.
- If $\frac{x}{2} = 7$, then $x = 14$.
- If $x = 5$ and $b = 5$, then $x = b$.
- If $XY - AB = WZ - AB$, then $XY = WZ$.
- Solve $\frac{x}{2} + 4x - 7 = 11$. List the property that justifies each step.
- Complete the following proof.

Given: $5 - \frac{2}{3}x = 1$

Prove: $x = 6$

Proof:

Statements	Reasons
a. $\underline{\quad ? \quad}$	a. Given
b. $3\left(5 - \frac{2}{3}x\right) = 3(1)$	b. $\underline{\quad ? \quad}$
c. $15 - 2x = 3$	c. $\underline{\quad ? \quad}$
d. $\underline{\quad ? \quad}$	d. Subtraction Prop.
e. $x = 6$	e. $\underline{\quad ? \quad}$

PROOF Write a two-column proof.

- Prove that if $25 = -7(y - 3) + 5y$, then $-2 = y$.
- If rectangle $ABCD$ has side lengths $AD = 3$ and $AB = 10$, then $AC = BD$.
- The Pythagorean Theorem states that in a right triangle ABC , $c^2 = a^2 + b^2$. Prove that $a = \sqrt{c^2 - b^2}$.

Standardized Test Practice

- ALGEBRA** If $8 + x = 12$, then $4 - x = \underline{\quad ? \quad}$.
 (A) 28 (B) 24 (C) 0 (D) 4

Practice and Apply

Homework Help

For Exercises	See Examples
15, 16, 20	1
14, 17–19, 21	2
22–27	3
28, 29	4

Extra Practice
See page 757.

State the property that justifies each statement.

- If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, $m\angle A = m\angle C$.
- If $HJ + 5 = 20$, then $HJ = 15$.
- If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.
- If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.
- If $\frac{1}{2}AB = \frac{1}{2}EF$, then $AB = EF$.
- $AB = AB$

20. If $2\left(x - \frac{3}{2}\right) = 5$, which property can be used to support the statement $2x - 3 = 5$?
21. Which property allows you to state $m\angle 4 = m\angle 5$, if $m\angle 4 = 35$ and $m\angle 5 = 35$?
22. If $\frac{1}{2}AB = \frac{1}{2}CD$, which property can be used to justify the statement $AB = CD$?
23. Which property could be used to support the statement $EF = JK$, given that $EF = GH$ and $GH = JK$?

Complete each proof.

24. Given: $\frac{3x + 5}{2} = 7$

Prove: $x = 3$

Proof:

Statements	Reasons
a. $\frac{3x + 5}{2} = 7$	a. ?
b. ?	b. Mult. Prop.
c. $3x + 5 = 14$	c. ?
d. $3x = 9$	d. ?
e. ?	e. Div. Prop.

25. Given: $2x - 7 = \frac{1}{3}x - 2$

Prove: $x = 3$

Proof:

Statements	Reasons
a. ?	a. Given
b. ?	b. Mult. Prop.
c. $6x - 21 = x - 6$	c. ?
d. ?	d. Subt. Prop.
e. $5x = 15$	e. ?
f. ?	f. Div. Prop.

PROOF Write a two-column proof.

26. If $4 - \frac{1}{2}a = \frac{7}{2} - a$, then $a = -1$.

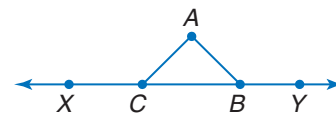
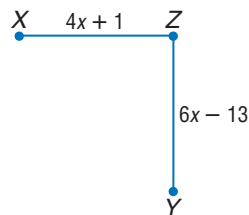
27. If $-2y + \frac{3}{2} = 8$, then $y = -\frac{13}{4}$.

28. If $-\frac{1}{2}m = 9$, then $m = -18$.

29. If $5 - \frac{2}{3}z = 1$, then $z = 6$.

30. If $XZ = ZY$, $XZ = 4x + 1$, and $ZY = 6x - 13$, then $x = 7$.

31. If $m\angle ACB = m\angle ABC$, then $m\angle XCA = m\angle YBA$.



- 32. **PHYSICS** Kinetic energy is the energy of motion. The formula for kinetic energy is $E_k = h \cdot f + W$, where h represents Planck's Constant, f represents the frequency of its photon, and W represents the work function of the material being used. Solve this formula for f and justify each step.



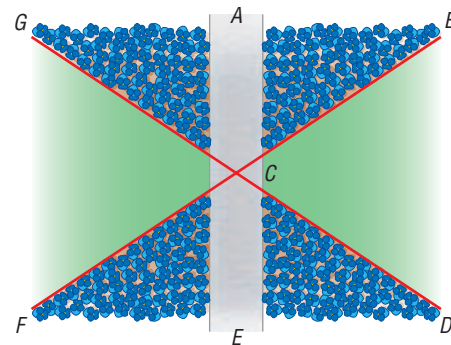
More About . . .

Physics

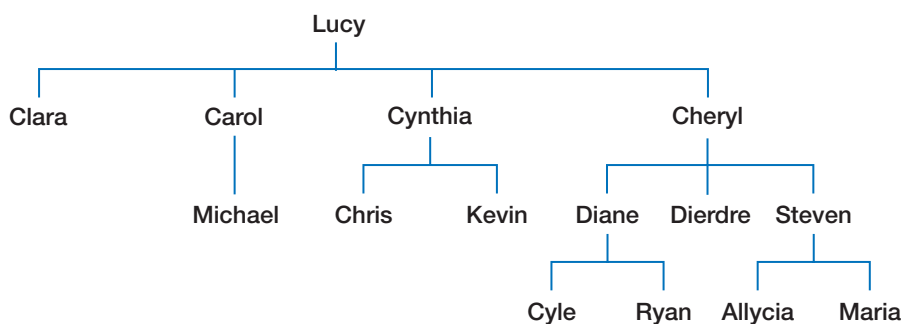
A gymnast exhibits kinetic energy when performing on the balance beam. The movements and flips show the energy that is being displayed while the gymnast is moving.

Source: www.infoplease.com

33. **GARDENING** Areas in the southwest and southeast have cool but mild winters. In these areas, many people plant pansies in October so that they have flowers outside year-round. In the arrangement of pansies shown, the walkway divides the two sections of pansies into four beds that are the same size. If $m\angle ACB = m\angle DCE$, what could you conclude about the relationship among $\angle ACB$, $\angle DCE$, $\angle ECF$, and $\angle ACG$?



- CRITICAL THINKING** For Exercises 34 and 35, use the following information. Below is a family tree of the Gibbs family. Clara, Carol, Cynthia, and Cheryl are all daughters of Lucy. Because they are sisters, they have a transitive and symmetric relationship. That is, Clara is a sister of Carol, Carol is a sister of Cynthia, so Clara is a sister of Cynthia.



34. What other relationships in a family have reflexive, symmetric, or transitive relationships? Explain why. Remember that the child or children of each person are listed beneath that person's name. Consider relationships such as first cousin, ancestor or descendent, aunt or uncle, sibling, or any other relationship.
35. Construct your family tree on one or both sides of your family and identify the reflexive, symmetric, or transitive relationships.

36. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

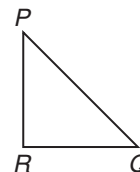
How is mathematical evidence similar to evidence in law?

Include the following in your answer:

- a description of how evidence is used to influence jurors' conclusions in court, and
- a description of the evidence used to make conclusions in mathematics.



37. In $\triangle PQR$, $m\angle P = m\angle Q$ and $m\angle R = 2(m\angle Q)$. Find $m\angle P$ if $m\angle P + m\angle Q + m\angle R = 180$.
- (A) 30 (B) 45
(C) 60 (D) 90



38. **ALGEBRA** If 4 more than x is 5 less than y , what is x in terms of y ?

- (A) $y - 1$ (B) $y - 9$ (C) $y + 9$ (D) $y - 5$



Maintain Your Skills

- Mixed Review** 39. **CONSTRUCTION** There are four buildings on the Medfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 2-5)

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning. A number is divisible by 3 if it is divisible by 6. (Lesson 2-4)

40. **Given:** 24 is divisible by 6. **Conclusion:** 24 is divisible by 3.
 41. **Given:** 27 is divisible by 3. **Conclusion:** 27 is divisible by 6.
 42. **Given:** 85 is not divisible by 3. **Conclusion:** 85 is not divisible by 6.

Write each statement in if-then form. (Lesson 2-3)

43. "Happy people rarely correct their faults." (*La Rochefoucauld*)
 44. "If you don't know where you are going, you will probably end up somewhere else." (*Laurence Peters*)
 45. "A champion is afraid of losing." (*Billie Jean King*)
 46. "If we would have new knowledge, we must get a whole new world of questions." (*Susanne K. Langer*)

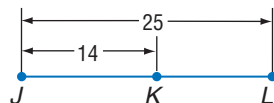
Find the precision for each measurement. (Lesson 1-2)

47. 13 feet 48. 5.9 meters 49. 74 inches 50. 3.1 kilometers

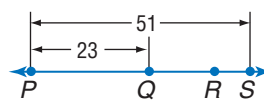
Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the measure of each segment. (To review segment measures, see Lesson 1-2.)

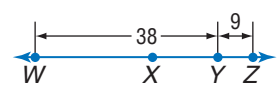
51. \overline{KL}



52. \overline{QS}



53. \overline{WZ}



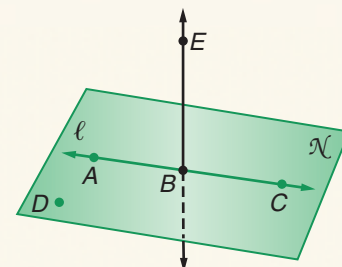
Practice Quiz 2

Lessons 2-4 through 2-6

- Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)
 - If n is an integer, then n is a real number.
 - n is a real number.
 - n is an integer.

In the figure at the right, A , B , and C are collinear. Points A , B , C , and D lie in plane N . State the postulate or theorem that can be used to show each statement is true. (Lesson 2-5)

- A , B , and D determine plane N .
- \overrightarrow{BE} intersects \overline{AC} at B .
- ℓ lies in plane N .



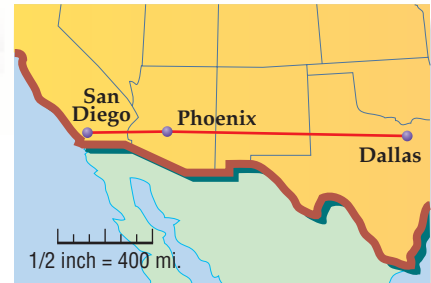
- PROOF** If $2(n - 3) + 5 = 3(n - 1)$, prove that $n = 2$. (Lesson 2-6)

What You'll Learn

- Write proofs involving segment addition.
- Write proofs involving segment congruence.

How can segment relationships be used for travel?

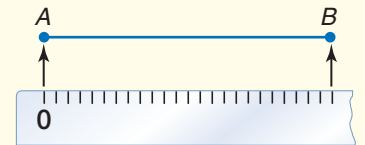
When leaving San Diego, the pilot said that the flight would be about 360 miles to Phoenix before continuing on to Dallas. When the plane left Phoenix, the pilot said that the flight would be flying about 1070 miles to Dallas.



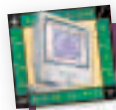
SEGMENT ADDITION In Lesson 1-2, you measured segments with a ruler by placing the mark for zero on one endpoint, then finding the distance to the other endpoint. This illustrates the **Ruler Postulate**.

Postulate 2.8

Ruler Postulate The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero, and B corresponds to a positive real number.



The Ruler Postulate can be used to further investigate line segments.



Geometry Software Investigation

Adding Segment Measures

Construct a Figure

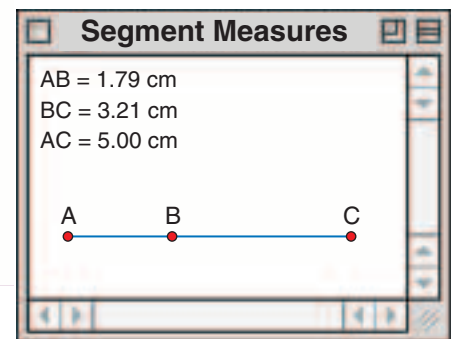
- Use The Geometer's Sketchpad to construct \overline{AC} .
- Place point B on \overline{AC} .
- Find AB , BC , and AC .

Analyze the Model

1. What is the sum $AB + BC$?
2. Move B . Find AB , BC and AC . What is the sum of $AB + BC$?
3. Repeat moving B , measuring the segments, and finding the sum $AB + BC$ three times. Record your results.

Make a Conjecture

4. What is true about the relationship of AB , BC , and AC ?
5. Is it possible to place B on \overline{AC} so that this relationship is not true?



Study Tip

Betweenness

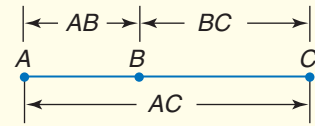
In general, the definition of *between* is that B is between A and C if A , B , and C are collinear and $AB + BC = AC$.

Examine the measures AB , BC , and AC in the Geometry Activity. Notice that wherever B is placed between A and C , $AB + BC = AC$. This suggests the following postulate.

Postulate 2.9

Segment Addition Postulate If B is between A and C , then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C .



Example 1 Proof With Segment Addition

Prove the following.

Given: $PQ = RS$

Prove: $PR = QS$

Proof:



Statements

1. $PQ = RS$
2. $PQ + QR = QR + RS$
3. $PQ + QR = PR$
 $QR + RS = QS$
4. $PR = QS$

Reasons

1. Given
2. Addition Property
3. Segment Addition Postulate
4. Substitution

SEGMENT CONGRUENCE In Lesson 2-5, you learned that once a theorem is proved, it can be used in proofs of other theorems. One theorem we can prove is similar to properties of equality from algebra.

Theorem 2.2

Segment Congruence

Congruence of segments is reflexive, symmetric, and transitive.

Reflexive Property $\overline{AB} \cong \overline{AB}$

Symmetric Property If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive Property If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

You will prove the first two properties in Exercises 10 and 24.

Proof Transitive Property of Congruence

Given: $\overline{MN} \cong \overline{PQ}$

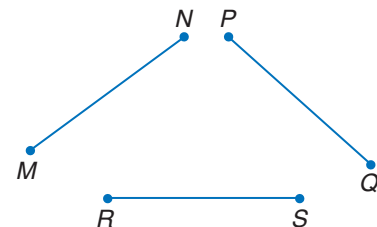
$\overline{PQ} \cong \overline{RS}$

Prove: $\overline{MN} \cong \overline{RS}$

Proof:

Method 1 Paragraph Proof

Since $\overline{MN} \cong \overline{PQ}$ and $\overline{PQ} \cong \overline{RS}$, $MN = PQ$ and $PQ = RS$ by the definition of congruent segments. By the Transitive Property of Equality, $MN = RS$. Thus, $\overline{MN} \cong \overline{RS}$ by the definition of congruent segments.



Method 2 Two-Column Proof

Statements	Reasons
1. $\overline{MN} \cong \overline{PQ}, \overline{PQ} \cong \overline{RS}$	1. Given
2. $MN = PQ, PQ = RS$	2. Definition of congruent segments
3. $MN = RS$	3. Transitive Property
4. $\overline{MN} \cong \overline{RS}$	4. Definition of congruent segments

The theorems about segment congruence can be used to prove segment relationships.

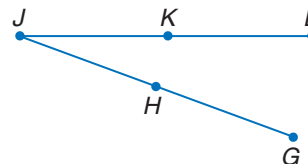
Example 2 Proof With Segment Congruence

Prove the following.

Given: $\overline{JK} \cong \overline{KL}, \overline{HJ} \cong \overline{GH}, \overline{KL} \cong \overline{HJ}$

Prove: $\overline{GH} \cong \overline{JK}$

Proof:



Statements	Reasons
1. $\overline{JK} \cong \overline{KL}, \overline{KL} \cong \overline{HJ}$	1. Given
2. $\overline{JK} \cong \overline{HJ}$	2. Transitive Property
3. $\overline{HJ} \cong \overline{GH}$	3. Given
4. $\overline{JK} \cong \overline{GH}$	4. Transitive Property
5. $\overline{GH} \cong \overline{JK}$	5. Symmetric Property

Check for Understanding

- Concept Check**
- Choose two cities from a United States road map. Describe the distance between the cities using the Reflexive Property.
 - OPEN ENDED** Draw three congruent segments, and illustrate the Transitive Property using these segments.
 - Describe** how to determine whether a point B is between points A and C .

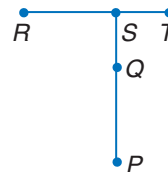
Guided Practice Justify each statement with a property of equality or a property of congruence.

- $\overline{XY} \cong \overline{XY}$
- If $\overline{GH} \cong \overline{MN}$, then $\overline{MN} \cong \overline{GH}$.
- If $AB = AC + CB$, then $AB - AC = CB$.
- Copy and complete the proof.

Given: $\overline{PQ} \cong \overline{RS}, \overline{QS} \cong \overline{ST}$

Prove: $\overline{PS} \cong \overline{RT}$

Proof:



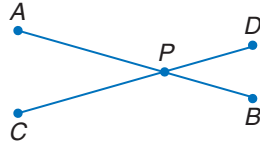
Statements	Reasons
a. $\underline{\quad ? \quad}, \underline{\quad ? \quad}$	a. Given
b. $PQ = RS, QS = ST$	b. $\underline{\quad ? \quad}$
c. $PS = PQ + QS, RT = RS + ST$	c. $\underline{\quad ? \quad}$
d. $\underline{\quad ? \quad}$	d. Addition Property
e. $\underline{\quad ? \quad}$	e. Substitution
f. $\overline{PS} \cong \overline{RT}$	f. $\underline{\quad ? \quad}$



PROOF For Exercises 8–10, write a two-column proof.

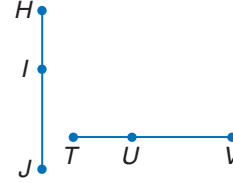
8. Given: $\overline{AP} \cong \overline{CP}$
 $\overline{BP} \cong \overline{DP}$

Prove: $\overline{AB} \cong \overline{CD}$



9. Given: $\overline{HI} \cong \overline{TU}$
 $\overline{HJ} \cong \overline{TV}$

Prove: $\overline{IJ} \cong \overline{UV}$



10. Symmetric Property of Congruence (Theorem 2.2)

Application

11. **GEOGRAPHY** Aberdeen in South Dakota and Helena, Miles City, and Missoula, all in Montana, are connected in a straight line by interstate highways. Missoula is 499 miles from Miles City and 972 miles from Aberdeen. Aberdeen is 473 miles from Miles City and 860 miles from Helena. Between which cities does Helena lie?

Practice and Apply

Homework Help

For Exercises	See Examples
14, 16, 17	1
12, 13, 15, 18–24	2

Extra Practice
See page 758.

Justify each statement with a property of equality or a property of congruence.

12. If $\overline{JK} \cong \overline{LM}$, then $\overline{LM} \cong \overline{JK}$.
13. If $AB = 14$ and $CD = 14$, then $AB = CD$.
14. If $W, X,$ and Y are collinear, in that order, then $WY = WX + XY$.
15. If $\overline{MN} \cong \overline{PQ}$ and $\overline{PQ} \cong \overline{RS}$, then $\overline{MN} \cong \overline{RS}$.
16. If $EF = TU$ and $GH = VW$, then $EF + GH = TU + VW$.
17. If $JK + MN = JK + QR$, then $MN = QR$.

18. Copy and complete the proof.

Given: $\overline{AD} \cong \overline{CE}, \overline{DB} \cong \overline{EB}$

Prove: $\overline{AB} \cong \overline{CB}$

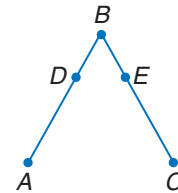
Proof:

Statements

- a. $\underline{\quad ? \quad}$
- b. $AD = CE, DB = EB$
- c. $AD + DB = CE + EB$
- d. $\underline{\quad ? \quad}$
- e. $AB = CB$
- f. $\overline{AB} \cong \overline{CB}$

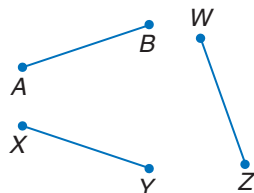
Reasons

- a. Given
- b. $\underline{\quad ? \quad}$
- c. $\underline{\quad ? \quad}$
- d. Segment Addition Postulate
- e. $\underline{\quad ? \quad}$
- f. $\underline{\quad ? \quad}$

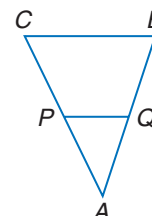


PROOF Write a two-column proof.

19. If $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$, then $\overline{XY} \cong \overline{AB}$.



20. If $\overline{AB} \cong \overline{AC}$ and $\overline{PC} \cong \overline{QB}$, then $\overline{AP} \cong \overline{AQ}$.



21. Copy and complete the proof.

Given: $\overline{WY} \cong \overline{ZX}$
 A is the midpoint of \overline{WY} .
 A is the midpoint of \overline{ZX} .

Prove: $\overline{WA} \cong \overline{ZA}$

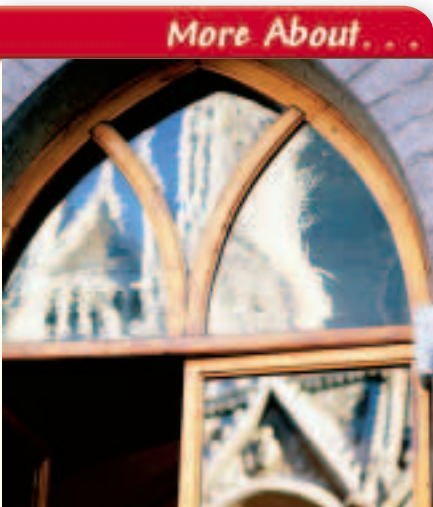
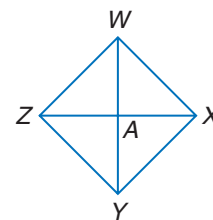
Proof:

Statements

- $\overline{WY} \cong \overline{ZX}$
 A is the midpoint of \overline{WY} .
 A is the midpoint of \overline{ZX} .
- $WY = ZX$
- ?
- $WY = WA + AY$, $ZX = ZA + AX$
- $WA + AY = ZA + AX$
- $WA + WA = ZA + ZA$
- $2WA = 2ZA$
- ?
- $\overline{WA} \cong \overline{ZA}$

Reasons

- ?
- ?
- Definition of midpoint
- ?
- ?
- ?
- ?
- Division Property
- ?



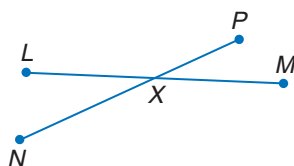
Design

Windows come in many different shapes and sizes. Some commonly used shapes for windows are circles, rectangles, squares, triangles, pentagons, and octagons.

Source: www.pella.com

PROOF For Exercises 22–24, write a two-column proof.

22. If $\overline{LM} \cong \overline{PN}$ and $\overline{XM} \cong \overline{XN}$, then $\overline{LX} \cong \overline{PX}$.

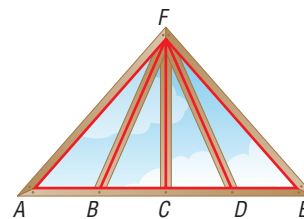


23. If $AB = BC$, then $AC = 2BC$.

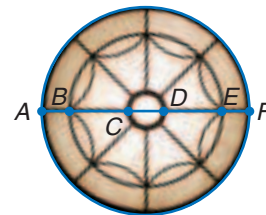


24. Reflexive Property of Congruence (Theorem 2.2)

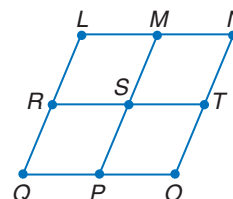
25. **DESIGN** The front of a building has a triangular window. If $\overline{AB} \cong \overline{DE}$ and C is the midpoint of \overline{BD} , prove that $\overline{AC} \cong \overline{CE}$.



26. **LIGHTING** The light fixture in Gerrard Hall of the University of North Carolina is shown at the right. If $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{DE}$, prove that $\overline{AC} \cong \overline{DF}$.



27. **CRITICAL THINKING** Given that $\overline{LN} \cong \overline{RT}$, $\overline{RT} \cong \overline{QO}$, $\overline{LQ} \cong \overline{NO}$, $\overline{MP} \cong \overline{NO}$, S is the midpoint of \overline{RT} , M is the midpoint of \overline{LN} , and P is the midpoint of \overline{QO} , list three statements that you could prove using the postulates, theorems, and definitions that you have learned.



28. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can segment relationships be used for travel?

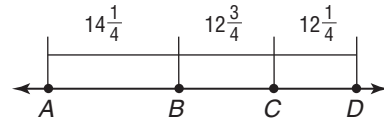
Include the following in your answer:

- an explanation of how a passenger can use the distances the pilot announced to find the total distance from San Diego to Dallas, and
- an explanation of why the Segment Addition Postulate may or may not be useful when traveling.



29. If P is the midpoint of \overline{BC} and Q is the midpoint of \overline{AD} , what is PQ ?

- (A) $\frac{1}{2}$ (B) 1
(C) 2 (D) $2\frac{1}{2}$



30. **GRID IN** A refreshment stand sells a large tub of popcorn for twice the price of a box of popcorn. If 60 tubs were sold for a total of \$150 and the total popcorn sales were \$275, how many boxes of popcorn were sold?

Maintain Your Skills

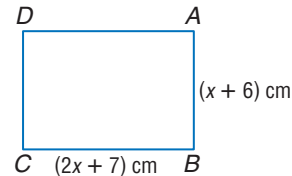
Mixed Review State the property that justifies each statement. (Lesson 2-6)

31. If $m\angle P + m\angle Q = 110$ and $m\angle R = 110$, then $m\angle P + m\angle Q = m\angle R$.
 32. If $x(y + z) = a$, then $xy + xz = a$.
 33. If $n - 17 = 39$, then $n = 56$.
 34. If $cv = md$ and $md = 15$, then $cv = 15$.

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain. (Lesson 2-5)

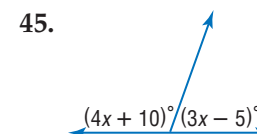
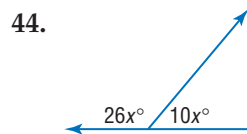
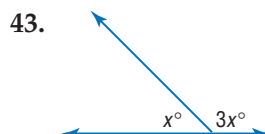
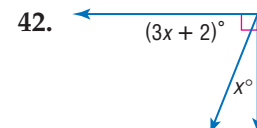
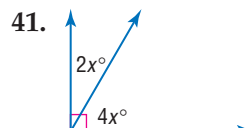
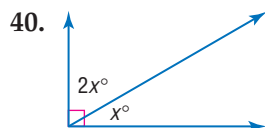
35. A midpoint divides a segment into two noncongruent segments.
 36. Three lines intersect at a single point.
 37. The intersection of two planes forms a line.
 38. Three single points determine three lines.

39. If the perimeter of rectangle $ABCD$ is 44 centimeters, find x and the dimensions of the rectangle. (Lesson 1-6)



Getting Ready for the Next Lesson

PREREQUISITE SKILL Find x . (To review **complementary and supplementary angles**, see Lesson 1-5.)



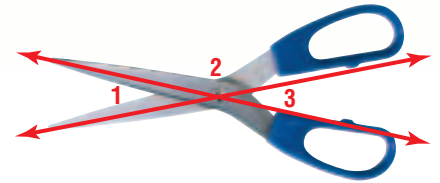
Proving Angle Relationships

What You'll Learn

- Write proofs involving supplementary and complementary angles.
- Write proofs involving congruent and right angles.

How do scissors illustrate supplementary angles?

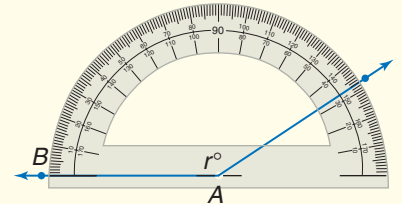
Notice that when a pair of scissors is opened, the angle formed by the two blades, $\angle 1$, and the angle formed by a blade and a handle, $\angle 2$, are a linear pair. Likewise, the angle formed by a blade and a handle, $\angle 2$, and the angle formed by the two handles, $\angle 3$, also forms a linear pair.



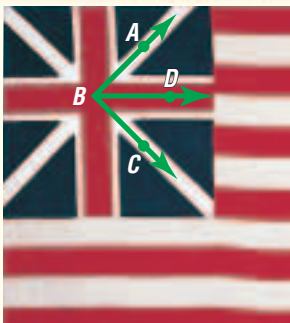
SUPPLEMENTARY AND COMPLEMENTARY ANGLES Recall that when you measure angles with a protractor, you position the protractor so that one of the rays aligns with zero degrees and then determine the position of the second ray. This illustrates the Protractor Postulate.

Postulate 2.10

Protractor Postulate Given \overline{AB} and a number r between 0 and 180, there is exactly one ray with endpoint A , extending on either side of \overline{AB} , such that the measure of the angle formed is r .



More About . . .



History

The Grand Union flag was the first flag used by the colonial United States that resembles the current flag. It was made up of thirteen stripes with the flag of Great Britain in the corner.

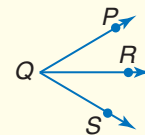
Source: www.usflag.org

In Lesson 2-7, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

Postulate 2.11

Angle Addition Postulate If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.



Example 1 Angle Addition

HISTORY The Grand Union Flag at the left contains several angles.

If $m\angle ABD = 44$ and $m\angle ABC = 88$, find $m\angle DBC$.

$$\begin{aligned} m\angle ABD + m\angle DBC &= m\angle ABC && \text{Angle Addition Postulate} \\ 44 + m\angle DBC &= 88 && m\angle ABD = 44, m\angle ABC = 88 \\ m\angle DBC &= 44 && \text{Subtraction Property} \end{aligned}$$

The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

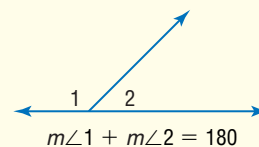
Study Tip

Look Back

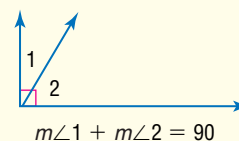
To review **supplementary** and **complementary** angles, see Lesson 1-5.

Theorems

2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.



You will prove Theorems 2.3 and 2.4 in Exercises 10 and 11.

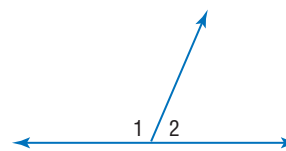
Example 2 Supplementary Angles

If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 67$, find $m\angle 1$.

$$m\angle 1 + m\angle 2 = 180 \quad \text{Supplement Theorem}$$

$$m\angle 1 + 67 = 180 \quad m\angle 2 = 67$$

$$m\angle 1 = 113 \quad \text{Subtraction Property}$$



CONGRUENT AND RIGHT ANGLES The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

Theorem 2.5

Congruence of angles is reflexive, symmetric, and transitive.

Reflexive Property $\angle 1 \cong \angle 1$

Symmetric Property If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property If $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

You will prove the Reflexive and Transitive Properties of Angle Congruence in Exercises 26 and 27.

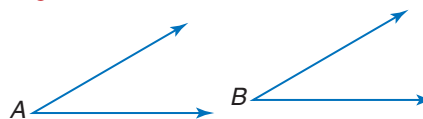
Proof Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$

Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.



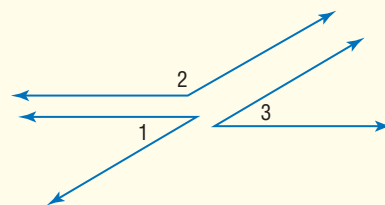
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

Theorems

2.6 Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation: \sphericalangle s suppl. to same \sphericalangle or $\cong \sphericalangle$ s are \cong .

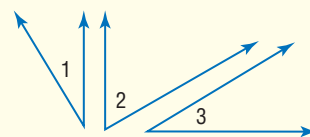
Example: If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation: \sphericalangle s compl. to same \sphericalangle or $\cong \sphericalangle$ s are \cong .

Example: If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 + m\angle 3 = 90$, then $\angle 1 \cong \angle 3$.



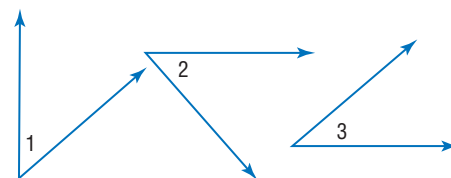
You will prove Theorem 2.6 in Exercise 6.

Proof Theorem 2.7

Given: $\angle 1$ and $\angle 3$ are complementary.
 $\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 2$

Proof:



Statements

- $\angle 1$ and $\angle 3$ are complementary.
 $\angle 2$ and $\angle 3$ are complementary.
- $m\angle 1 + m\angle 3 = 90$
 $m\angle 2 + m\angle 3 = 90$
- $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$
- $m\angle 3 = m\angle 3$
- $m\angle 1 = m\angle 2$
- $\angle 1 \cong \angle 2$

Reasons

- Given
- Definition of complementary angles
- Substitution
- Reflective Property
- Subtraction Property
- Definition of congruent angles

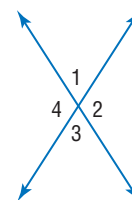
Example 3 Use Supplementary Angles

In the figure, $\angle 1$ and $\angle 2$ form a linear pair and $\angle 2$ and $\angle 3$ form a linear pair. Prove that $\angle 1$ and $\angle 3$ are congruent.

Given: $\angle 1$ and $\angle 2$ form a linear pair.
 $\angle 2$ and $\angle 3$ form a linear pair.

Prove: $\angle 1 \cong \angle 3$

Proof:



Statements

- $\angle 1$ and $\angle 2$ form a linear pair.
 $\angle 2$ and $\angle 3$ form a linear pair.
- $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 2$ and $\angle 3$ are supplementary.
- $\angle 1 \cong \angle 3$

Reasons

- Given
- Supplement Theorem
- \sphericalangle s suppl. to same \sphericalangle or $\cong \sphericalangle$ s are \cong .



Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example is a proof for the following theorem.

Study Tip

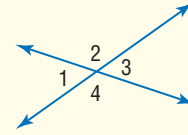
Look Back

To review **vertical angles**, see Lesson 1-5.

Theorem 2.8

Vertical Angles Theorem If two angles are vertical angles, then they are congruent.

Abbreviation: Vert. \angle s are \cong .



$$\angle 1 \cong \angle 3 \text{ and } \angle 2 \cong \angle 4$$

Example 4 Vertical Angles

If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = x$ and $m\angle 2 = 228 - 3x$, find $m\angle 1$ and $m\angle 2$.

$$\angle 1 \cong \angle 2 \quad \text{Vertical Angles Theorem}$$

$$m\angle 1 = m\angle 2 \quad \text{Definition of congruent angles}$$

$$x = 228 - 3x \quad \text{Substitution}$$

$$4x = 228 \quad \text{Add } 3x \text{ to each side.}$$

$$x = 57 \quad \text{Divide each side by 4.}$$

$$\begin{array}{l} m\angle 1 = x \\ \quad = 57 \end{array} \qquad \begin{array}{l} m\angle 2 = m\angle 1 \\ \quad = 57 \end{array}$$

The theorems you have learned can be applied to right angles. You can create right angles and investigate congruent angles by paper folding.



Geometry Activity

Right Angles

Make a Model

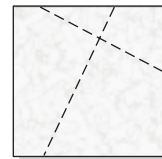
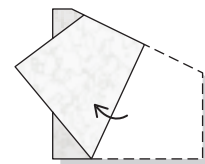
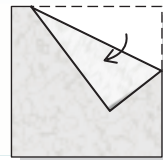
- Fold the paper so that one corner is folded downward.
- Fold along the crease so that the top edge meets the side edge.
- Unfold the paper and measure each of the angles formed.
- Repeat the activity three more times.

Analyze the Model

1. What do you notice about the lines formed?
2. What do you notice about each pair of adjacent angles?
3. What are the measures of the angles formed?

Make a Conjecture

4. What is true about perpendicular lines?
5. What is true about all right angles?



The following theorems support the conjectures you made in the Geometry Activity.

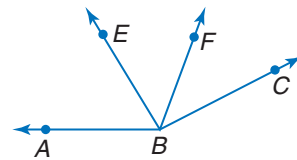
- 2.9** Perpendicular lines intersect to form four right angles.
- 2.10** All right angles are congruent.
- 2.11** Perpendicular lines form congruent adjacent angles.
- 2.12** If two angles are congruent and supplementary, then each angle is a right angle.
- 2.13** If two congruent angles form a linear pair, then they are right angles.

Check for Understanding

Concept Check 1. **FIND THE ERROR** Tomas and Jacob wrote equations involving the angle measures shown.

Tomas
 $m\angle ABE + m\angle EBC = m\angle ABC$

Jacob
 $m\angle ABE + m\angle FBC = m\angle ABC$

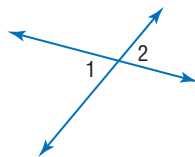


Who is correct? Explain your reasoning.

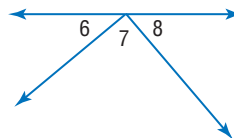
2. **OPEN ENDED** Draw three congruent angles. Use these angles to illustrate the Transitive Property for angle congruence.

Guided Practice Find the measure of each numbered angle.

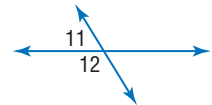
3. $m\angle 1 = 65$



4. $\angle 6$ and $\angle 8$ are complementary.
 $m\angle 8 = 47$



5. $m\angle 11 = x - 4$,
 $m\angle 12 = 2x - 5$



6. **PROOF** Copy and complete the proof of Theorem 2.6.

Given: $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$



Proof:

Statements

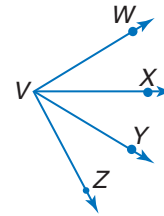
Reasons

a. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	a. <u>?</u>
b. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 4 = 180$	b. <u>?</u>
c. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	c. <u>?</u>
d. $m\angle 1 = m\angle 4$	d. <u>?</u>
e. $m\angle 2 = m\angle 3$	e. <u>?</u>
f. $\angle 2 \cong \angle 3$	f. <u>?</u>

7. **PROOF** Write a two-column proof.

Given: \overline{VX} bisects $\angle WVY$.
 \overline{VY} bisects $\angle XVZ$.

Prove: $\angle WVX \cong \angle YVZ$



Determine whether the following statements are *always*, *sometimes*, or *never* true.

8. Two angles that are nonadjacent are ? vertical.
 9. Two angles that are congruent are ? complementary to the same angle.

PROOF Write a proof for each theorem.

10. Supplement Theorem

11. Complement Theorem

Application **ALGEBRA** For Exercises 12–15, use the following information.

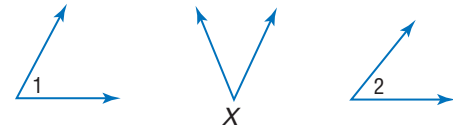
$\angle 1$ and $\angle X$ are complementary,
 $\angle 2$ and $\angle X$ are complementary,
 $m\angle 1 = 2n + 2$, and $m\angle 2 = n + 32$.

12. Find n .

14. What is $m\angle 2$?

13. Find $m\angle 1$.

15. Find $m\angle X$.



Practice and Apply

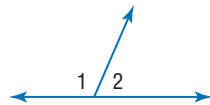
Homework Help

For Exercises	See Examples
16–18	1, 2
19–24	4
25–39	3

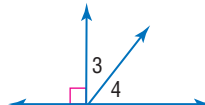
Extra Practice
See page 758.

Find the measure of each numbered angle.

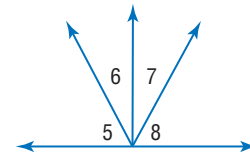
16. $m\angle 2 = 67$



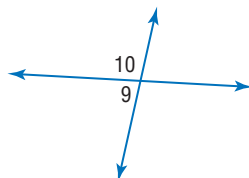
17. $m\angle 3 = 38$



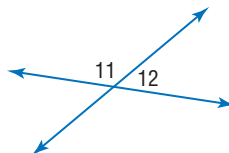
18. $\angle 7$ and $\angle 8$ are complementary. $\angle 5 \cong \angle 8$ and $m\angle 6 = 29$.



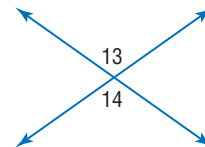
19. $m\angle 9 = 2x - 4$,
 $m\angle 10 = 2x + 4$



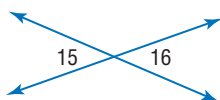
20. $m\angle 11 = 4x$,
 $m\angle 12 = 2x - 6$



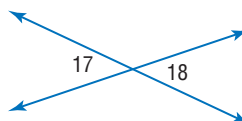
21. $m\angle 13 = 2x + 94$,
 $m\angle 14 = 7x + 49$



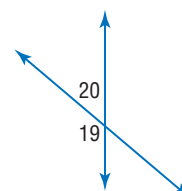
22. $m\angle 15 = x$,
 $m\angle 16 = 6x - 290$



23. $m\angle 17 = 2x + 7$,
 $m\angle 18 = x + 30$



24. $m\angle 19 = 100 + 20x$,
 $m\angle 20 = 20x$



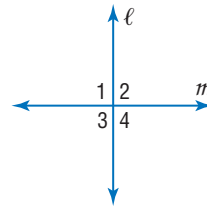
25. Prove that congruence of angles is reflexive.
26. Write a proof of the Transitive Property of Angle Congruence.

Determine whether the following statements are *always*, *sometimes*, or *never* true.

27. Two angles that are complementary ? form a right angle.
28. Two angles that are vertical are ? nonadjacent.
29. Two angles that form a right angle are ? complementary.
30. Two angles that form a linear pair are ? congruent.
31. Two angles that are supplementary are ? congruent.
32. Two angles that form a linear pair are ? supplementary.

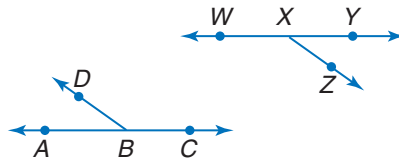
PROOF Use the figure to write a proof of each theorem.

33. Theorem 2.9
34. Theorem 2.10
35. Theorem 2.11
36. Theorem 2.12
37. Theorem 2.13

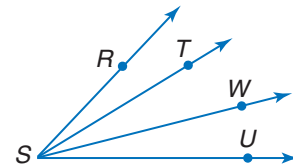


PROOF Write a two-column proof.

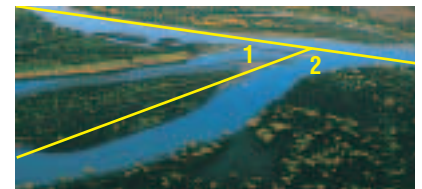
38. **Given:** $\angle ABD \cong \angle YXZ$
Prove: $\angle CBD \cong \angle WXZ$



39. **Given:** $m\angle RSW = m\angle TSU$
Prove: $m\angle RST = m\angle WSU$



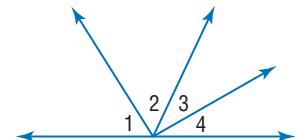
40. **RIVERS** Tributaries of rivers sometimes form a linear pair of angles when they meet the main river. The Yellowstone River forms the linear pair $\angle 1$ and $\angle 2$ with the Missouri River. If $m\angle 1$ is 28, find $m\angle 2$.



41. **HIGHWAYS** Near the city of Hopewell, Virginia, Route 10 runs perpendicular to Interstate 95 and Interstate 295. Show that the angles at the intersections of Route 10 with Interstate 95 and Interstate 295 are congruent.



42. **CRITICAL THINKING** What conclusion can you make about the sum of $m\angle 1$ and $m\angle 4$ if $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$? Explain.



Highways

Interstate highways that run from north to south are odd-numbered with the lowest numbers in the west. East-west interstates are even-numbered, and begin in the south.

Source: www.infoplease.com



43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do scissors illustrate supplementary angles?

Include the following in your answer:

- a description of the relationship among $\angle 1$, $\angle 2$, and $\angle 3$,
- an example of another way that you can tell the relationship between $\angle 1$ and $\angle 3$, and
- an explanation of whether this relationship is the same for two angles complementary to the same angle.



44. The measures of two complementary angles are in the ratio 4:1. What is the measure of the smaller angle?

(A) 15 (B) 18 (C) 24 (D) 36

45. **ALGEBRA** T is the set of all positive numbers n such that $n < 50$ and \sqrt{n} is an integer. What is the median of the members of set T ?

(A) 4 (B) 16 (C) 20 (D) 25

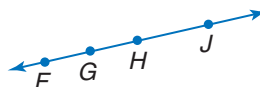
Maintain Your Skills

Mixed Review

PROOF Write a two-column proof. (Lesson 2-7)

46. **Given:** G is between F and H .
 H is between G and J .

Prove: $FG + GJ = FH + HJ$

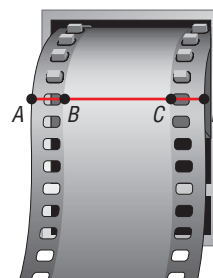


47. **Given:** X is the midpoint of \overline{WY} .

Prove: $WX + YZ = XZ$



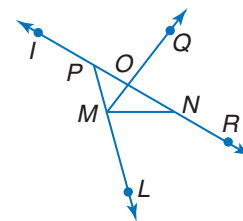
48. **PHOTOGRAPHY** Film is fed through a camera by gears that catch the perforation in the film. The distance from the left edge of the film, A , to the right edge of the image, C , is the same as the distance from the left edge of the image, B , to the right edge of the film, D . Show that the two perforated strips are the same width. (Lesson 2-6)



For Exercises 49–55, refer to the figure at the right.

(Lesson 1-4)

49. Name two angles that have N as a vertex.
50. If \overline{MQ} bisects $\angle PMN$, name two congruent angles.
51. Name a point in the interior of $\angle LMQ$.
52. List all the angles that have O as the vertex.
53. Does $\angle QML$ appear to be acute, obtuse, right, or straight?
54. Name a pair of opposite rays.
55. List all the angles that have \overline{MN} as a side.



Vocabulary and Concept Check

axiom (p. 89)	converse (p. 77)	inductive reasoning (p. 62)	postulate (p. 89)
biconditional (p. 81)	counterexample (p. 63)	informal proof (p. 90)	proof (p. 90)
compound statement (p. 67)	deductive argument (p. 94)	inverse (p. 77)	related conditionals (p. 77)
conclusion (p. 75)	deductive reasoning (p. 82)	Law of Detachment (p. 82)	statement (p. 67)
conditional statement (p. 75)	disjunction (p. 68)	Law of Syllogism (p. 83)	theorem (p. 90)
conjecture (p. 62)	formal proof (p. 95)	logically equivalent (p. 77)	truth table (p. 70)
conjunction (p. 68)	hypothesis (p. 75)	negation (p. 67)	truth value (p. 67)
contrapositive (p. 77)	if-then statement (p. 75)	paragraph proof (p. 90)	two-column proof (p. 95)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises Choose the correct term to complete each sentence.

1. A (*counterexample, conjecture*) is an educated guess based on known information.
2. The truth or falsity of a statement is called its (*conclusion, truth value*).
3. Two or more statements can be joined to form a (*conditional, compound*) statement.
4. A conjunction is a compound statement formed by joining two or more statements using (*or, and*).
5. The phrase immediately following the word *if* in a conditional statement is called the (*hypothesis, conclusion*).
6. The (*inverse, converse*) is formed by exchanging the hypothesis and the conclusion.
7. (*Theorems, Postulates*) are accepted as true without proof.
8. A paragraph proof is a (an) (*informal proof, formal proof*).

Lesson-by-Lesson Review

2-1 Inductive Reasoning and Conjecture

See pages
62–66.

Concept Summary

- Conjectures are based on observations and patterns.
- Counterexamples can be used to show that a conjecture is false.

Example

Given that points P , Q , and R are collinear, determine whether the conjecture that Q is between P and R is *true* or *false*. If the conjecture is false, give a counterexample.

In the figure, R is between P and Q . Since we can find a counterexample, the conjecture is false.



Exercises Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. See Example 2 on page 63.

9. $\angle A$ and $\angle B$ are supplementary.
10. X , Y , and Z are collinear and $XY = YZ$.
11. In quadrilateral $LMNO$, $LM = LO = MN = NO$, and $m\angle L = 90$.



2-2 Logic

See pages 67–74.

Concept Summary

- The negation of a statement has the opposite truth value of the original statement.
- Venn diagrams and truth tables can be used to determine the truth values of statements.

Example

Use the following statements to write a compound statement for each conjunction. Then find its truth value.

p : $\sqrt{15} = 5$ q : The measure of a right angle equals 90.

a. p and q

$\sqrt{15} = 5$, and the measure of a right angle equals 90.

p and q is false because p is false and q is true.

b. $p \vee q$

$\sqrt{15} = 5$, or the measure of a right angle equals 90.

$p \vee q$ is true because q is true. It does not matter that p is false.

Exercises Use the following statements to write a compound statement for each conjunction. Then find its truth value. See Examples 1 and 2 on pages 68 and 69.

p : $-1 > 0$ q : In a right triangle with right angle C , $a^2 + b^2 = c^2$.

r : The sum of the measures of two supplementary angles is 180.

12. p and q

13. q or r

14. $r \wedge p$

15. $p \wedge (q \vee r)$

16. $q \vee (p \vee r)$

17. $(q \wedge r) \wedge p$

2-3 Conditional Statements

See pages 75–80.

Concept Summary

- Conditional statements are written in if-then form.
- Form the converse, inverse, and contrapositive of an if-then statement by using negations and by exchanging the hypothesis and conclusion.

Example

Identify the hypothesis and conclusion of the statement *The intersection of two planes is a line*. Then write the statement in if-then form.

Hypothesis: two planes intersect

Conclusion: their intersection is a line

If two planes intersect, then their intersection is a line.

Exercises Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample. See Example 4 on page 77.

18. If an angle measure equals 120, then the angle is obtuse.

19. If the month is March, then it has 31 days.

20. If an ordered pair for a point has 0 for its x -coordinate, then the point lies on the y -axis.

Determine the truth value of the following statement for each set of conditions.

If the temperature is at most 0°C , then water freezes. See Example 3 on page 76.

21. The temperature is -10°C , and water freezes.
22. The temperature is 15°C , and water freezes.
23. The temperature is -2°C , and water does not freeze.
24. The temperature is 30°C , and water does not freeze.

2-4 Deductive Reasoning

See pages
82–87.

Concept Summary

- The Law of Detachment and the Law of Syllogism can be used to determine the truth value of a compound statement.

Example

Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following statements.

- (1) If a body in our solar system is the Sun, then it is a star.
- (2) Stars are in constant motion.

p : a body in our solar system is the sun

q : it is a star

r : stars are in constant motion

Statement (1): $p \rightarrow q$ Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If a body in our solar system is the Sun, then it is in constant motion.*

Exercises Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning. See Example 1 on page 82.
If two angles are adjacent, then they have a common vertex.

25. **Given:** $\angle 1$ and $\angle 2$ are adjacent angles.
Conclusion: $\angle 1$ and $\angle 2$ have a common vertex.
26. **Given:** $\angle 3$ and $\angle 4$ have a common vertex.
Conclusion: $\angle 3$ and $\angle 4$ are adjacent angles.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not follow, write *invalid*. See Example 3 on page 83.

27. (1) If a student attends North High School, then the student has an ID number.
(2) Josh Michael attends North High School.
(3) Josh Michael has an ID number.
28. (1) If a rectangle has four congruent sides, then it is a square.
(2) A square has diagonals that are perpendicular.
(3) A rectangle has diagonals that are perpendicular.
29. (1) If you like pizza with everything, then you'll like Cardo's Pizza.
(2) If you like Cardo's Pizza, then you are a pizza connoisseur.
(3) If you like pizza with everything, then you are a pizza connoisseur.

2-5 Postulates and Paragraph Proofs

See pages 89–93.

Concept Summary

- Use undefined terms, definitions, postulates, and theorems to prove that statements and conjectures are true.

Example

Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain. *Two points determine a line.*

According to a postulate relating to points and lines, two points determine a line. Thus, the statement is always true.

Exercises Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain. See Example 2 on page 90.

- The intersection of two lines can be a line.
- If P is the midpoint of \overline{XY} , then $XP = PY$.
- If $MX = MY$, then M is the midpoint of XY .
- Three points determine a line.
- Points Q and R lie in at least one plane.
- If two angles are right angles, they are adjacent.
- An angle is congruent to itself.
- Vertical angles are adjacent.

- PROOF** Write a paragraph proof to prove that if M is the midpoint of \overline{AB} and Q is the midpoint of \overline{AM} , then $AQ = \frac{1}{4}AB$.



2-6 Algebraic Proof

See pages 94–100.

Concept Summary

- The properties of equality used in algebra can be applied to the measures of segments and angles to verify and prove statements.

Example

Given: $2x + 6 = 3 + \frac{5}{3}x$

Prove: $x = -9$

Proof:

Statements

Reasons

1. $2x + 6 = 3 + \frac{5}{3}x$	1. Given
2. $3(2x + 6) = 3\left(3 + \frac{5}{3}x\right)$	2. Multiplication Property
3. $6x + 18 = 9 + 5x$	3. Distributive Property
4. $6x + 18 - 5x = 9 + 5x - 5x$	4. Subtraction Property
5. $x + 18 = 9$	5. Substitution
6. $x + 18 - 18 = 9 - 18$	6. Subtraction Property
7. $x = -9$	7. Substitution

Exercises State the property that justifies each statement. See Example 1 on page 94.

- 39. If $3(x + 2) = 6$, then $3x + 6 = 6$.
- 40. If $10x = 20$, then $x = 2$.
- 41. If $AB + 20 = 45$, then $AB = 25$.
- 42. If $3 = CD$ and $CD = XY$, then $3 = XY$.

PROOF Write a two-column proof. See Examples 2 and 4 on pages 95 and 96.

- 43. If $5 = 2 - \frac{1}{2}x$, then $x = -6$.
- 44. If $x - 1 = \frac{x - 10}{-2}$, then $x = 4$.
- 45. If $AC = AB$, $AC = 4x + 1$, and $AB = 6x - 13$, then $x = 7$.
- 46. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

2-7 Proving Segment Relationships

See pages
101–106.

Concept Summary

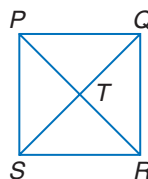
- Use properties of equality and congruence to write proofs involving segments.

Example

Write a two-column proof.

Given: $QT = RT$, $TS = TP$

Prove: $QS = RP$



Proof:

Statements	Reasons
1. $QT = RT$, $TS = TP$	1. Given
2. $QT + TS = RT + TS$	2. Addition Property
3. $QT + TS = RT + TP$	3. Substitution
4. $QT + TS = QS$, $RT + TP = RP$	4. Segment Addition Postulate
5. $QS = RP$	5. Substitution

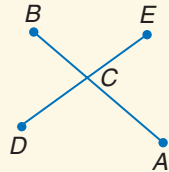
Exercises Justify each statement with a property of equality or a property of congruence. See Example 1 on page 102.

- 47. $PS = PS$
- 48. If $XY = OP$, then $OP = XY$.
- 49. If $AB - 8 = CD - 8$, then $AB = CD$.
- 50. If $EF = GH$ and $GH = LM$, then $EF = LM$.
- 51. If $2(XY) = AB$, then $XY = \frac{1}{2}(AB)$.
- 52. If $AB = CD$, then $AB + BC = CD + BC$.

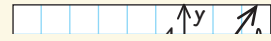
- Extra Practice, see pages 756–758.
- Mixed Problem Solving, see page 783.

PROOF Write a two-column proof. See Examples 1 and 2 on pages 102 and 103.

53. **Given:** $BC = EC, CA = CD$
Prove: $BA = DE$



54. **Given:** $AB = CD$
Prove: $AC = BD$



2-8 Proving Angle Relationships

See pages
107–114.

Concept Summary

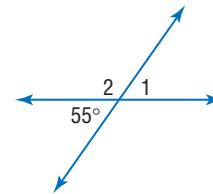
- The properties of equality and congruence can be applied to angle relationships.

Example

Find the measure of each numbered angle.

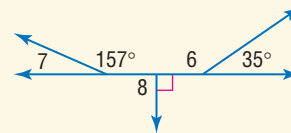
$m\angle 1 = 55$, since $\angle 1$ is a vertical angle to the 55° angle.
 $\angle 2$ and the 55° angle form a linear pair.

$$\begin{aligned} 55 + m\angle 2 &= 180 && \text{Def. of supplementary } \sphericalangle \\ m\angle 2 &= 125 && \text{Subtract 55 from each side.} \end{aligned}$$



Exercises Find the measure of each numbered angle. See Example 2 on page 108.

55. $m\angle 6$
 56. $m\angle 7$
 57. $m\angle 8$



58. **PROOF** Copy and complete the proof.

See Example 3 on page 109.

Given: $\angle 1$ and $\angle 2$ form a linear pair.
 $m\angle 2 = 2(m\angle 1)$

Prove: $m\angle 1 = 60$

Proof:

Statements

- a. $\angle 1$ and $\angle 2$ form a linear pair.
 b. $\angle 1$ and $\angle 2$ are supplementary.
 c. $\underline{\quad ? \quad}$
 d. $m\angle 2 = 2(m\angle 1)$
 e. $\underline{\quad ? \quad}$
 f. $\underline{\quad ? \quad}$
 g. $\frac{3(m\angle 1)}{3} = \frac{180}{3}$
 h. $\underline{\quad ? \quad}$

Reasons

- a. $\underline{\quad ? \quad}$
 b. $\underline{\quad ? \quad}$
 c. Definition of supplementary angles
 d. $\underline{\quad ? \quad}$
 e. Substitution
 f. Substitution
 g. $\underline{\quad ? \quad}$
 h. Substitution

Vocabulary and Concepts

1. Explain the difference between formal and informal proofs.
2. Explain how you can prove that a conjecture is false.
3. Describe the parts of a two-column proof.

Skills and Applications

Determine whether each conjecture is *true* or *false*. Explain your answer and give a counterexample for any false conjecture.

- | | | |
|--|--|--|
| 4. Given: $\angle A \cong \angle B$
Conjecture: $\angle B \cong \angle A$ | 5. Given: y is a real number
Conjecture: $-y > 0$ | 6. Given: $3a^2 = 48$
Conjecture: $a = 4$ |
|--|--|--|

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

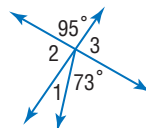
- p : $-3 > 2$ q : $3x = 12$ when $x = 4$. r : An equilateral triangle is also equiangular.
7. p and q
 8. p or q
 9. $p \vee (q \wedge r)$

Identify the hypothesis and conclusion of each statement and write each statement in if-then form. Then write the converse, inverse, and contrapositive of each conditional.

10. An apple a day keeps the doctor away.
11. A rolling stone gathers no moss.
12. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.
 - (1) Perpendicular lines intersect.
 - (2) Lines m and n are perpendicular.
 - (3) Lines m and n intersect.

Find the measure of each numbered angle.

13. $\angle 1$
14. $\angle 2$
15. $\angle 3$



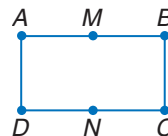
16. Write a two-column proof.

If $y = 4x + 9$ and $x = 2$, then $y = 17$.

17. Write a paragraph proof.

Given: $AM = CN$, $MB = ND$

Prove: $AB = CD$



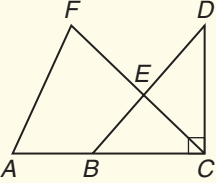
18. **ADVERTISING** Identify the hypothesis and conclusion of the following statement, then write it in if-then form. *Hard working people deserve a great vacation.*
19. **STANDARDIZED TEST PRACTICE** If two planes intersect, their intersection can be

I a line.	II three noncollinear points.	III two intersecting lines.	
(A) I only	(B) II only	(C) III only	(D) I and II only



Part 1 Multiple Choice

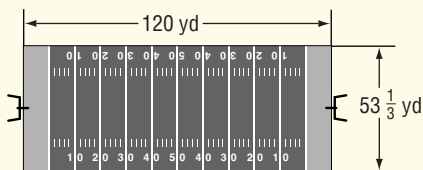
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- Arrange the numbers $|-7|$, $\frac{1}{7}$, $\sqrt{7}$, -7^2 in order from least to greatest. (Prerequisite Skill)
 - $|-7|$, $\sqrt{7}$, $\frac{1}{7}$, -7^2
 - -7^2 , $|-7|$, $\frac{1}{7}$, $\sqrt{7}$
 - $|-7|$, $\frac{1}{7}$, $\sqrt{7}$, -7^2
 - -7^2 , $\frac{1}{7}$, $\sqrt{7}$, $|-7|$
- Points A and B lie on the line $y = 2x - 3$. Which of the following are coordinates of a point noncollinear with A and B ? (Lesson 1-1)
 - $(7, 11)$
 - $(4, 5)$
 - $(-2, -10)$
 - $(-5, -13)$
- Dana is measuring distance on a map. Which of the following tools should Dana use to make the most accurate measurement? (Lesson 1-2)
 - centimeter ruler
 - protractor
 - yardstick
 - calculator
- Point E is the midpoint of \overline{DF} . If $DE = 8x - 3$ and $EF = 3x + 7$, what is x ? (Lesson 1-3)
 - 1
 - 2
 - 4
 - 13
- What is the relationship between $\angle ACF$ and $\angle DCF$? (Lesson 1-6)
 
 - complementary angles
 - congruent angles
 - supplementary angles
 - vertical angles
- Which of the following is an example of inductive reasoning? (Lesson 2-1)
 - Carlos learns that the measures of all acute angles are less than 90. He conjectures that if he sees an acute angle, its measure will be less than 90.
 - Carlos reads in his textbook that the measure of all right angles is 90. He conjectures that the measure of each right angle in a square equals 90.
 - Carlos measures the angles of several triangles and finds that their measures all add up to 180. He conjectures that the sum of the measures of the angles in any triangle is always 180.
 - Carlos knows that the sum of the measures of the angles in a square is always 360. He conjectures that if he draws a square, the sum of the measures of the angles will be 360.
- Which of the following is the contrapositive of the statement *If Rick buys hamburgers for lunch, then Denzel buys French fries and a large soda*? (Lesson 2-2)
 - If Denzel does not buy French fries and a large soda, then Rick does not buy hamburgers for lunch.
 - If Rick does not buy hamburgers for lunch, then Denzel does not buy French fries and a large soda.
 - If Denzel buys French fries and a large soda, then Rick buys hamburgers for lunch.
 - If Rick buys hamburgers for lunch, then Denzel does not buy French fries and a large soda.
- Which property could justify the first step in solving $3 \times \frac{14x + 6}{8} = 18$? (Lesson 2-5)
 - Division Property of Equality
 - Substitution Property of Equality
 - Addition Property of Equality
 - Transitive Property of Equality

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Two cheerleaders stand at opposite corners of a football field. What is the shortest distance between them, to the nearest yard? (Lesson 1-3)



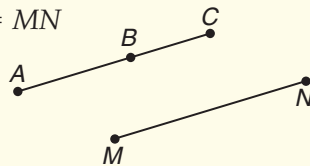
10. Consider the conditional *If I call in sick, then I will not get paid for the day*. Based on the original conditional, what is the name of the conditional *If I do not call in sick, then I will get paid for the day*? (Lesson 2-2)
11. Examine the following statements.
p: Martina drank a cup of soy milk.
q: A cup is 8 ounces.
r: Eight ounces of soy milk contains 300 milligrams of calcium.

Using the Law of Syllogism, how many milligrams of calcium did Martina get from drinking a cup of soy milk? (Lesson 2-4)

12. In the following proof, what property justifies statement *c*? (Lesson 2-7)

Given: $\overline{AC} \cong \overline{MN}$

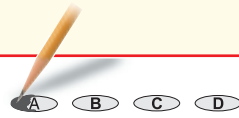
Prove: $AB + BC = MN$



Proof:

Statements	Reasons
a. $\overline{AC} \cong \overline{MN}$	a. Given
b. $AC = MN$	b. Definition of congruent segments
c. $AC = AB + BC$	c. ?
d. $AC + BC = MN$	d. Substitution

Test-Taking Tip



Question 6

When answering a multiple-choice question, always read every answer choice and eliminate those you decide are definitely wrong. This way, you may deduce the correct answer.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

13. In any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. From a single point in her yard, Marti measures and marks distances of 18 feet and 24 feet for two sides of her garden. Explain how Marti can ensure that the two sides of her garden form a right angle. (Lesson 1-3)
14. A farmer needs to make a 100-square-foot rectangular enclosure for her chickens. She wants to save money by purchasing the least amount of fencing possible to enclose the area. (Lesson 1-4)
- What whole-number dimensions, to the nearest yard, will require the least amount of fencing?
 - Explain your procedure for finding the dimensions that will require the least amount of fencing.
 - Explain how the amount of fencing required to enclose the area changes as the dimensions change.

15. **Given:** $\angle 1$ and $\angle 3$ are vertical angles.
 $m\angle 1 = 3x + 5$, $m\angle 3 = 2x + 8$

Prove: $m\angle 1 = 14$ (Lesson 2-8)

