

If you do not pay off the loan before it is due, the total finance charge will not change. But, if you repay the loan early, you will be charged more in finance charges in the early months than in a simple interest installment loan.

Suppose you have a \$500, one-year, 18% loan with a total finance charge of \$90. If you repay it at the end of one month, you will pay \$13.85 in finance charges if the lender uses the Rule of 78. If the loan is a simple interest installment loan, you will pay only \$7.50 in finance charges for that one month.

$$\$500 \times 0.18 \times \frac{12}{78} = \$13.85 \quad \text{one-month Rule of 78 finance charge}$$

$$\$500 \times 0.18 \times \frac{1}{12} = \$7.50 \quad \text{one-month simple interest charge}$$

To repay the Rule of 78 loan, you need \$513.85. To repay the simple interest installment loan, you need \$507.50.

EARNED AND UNEARNED FINANCE CHARGES The total finance charge for the Rule of 78 loan is $\$500 \times 0.18$, or \$90. Of that amount, the first month's finance charge, \$13.85, was *earned* by the lender because you borrowed the money for one month. The rest, or \$76.15, of the finance charge is *unearned* because you paid back the loan early.

The unearned finance charge is called the *finance charge refund* by lenders. When lenders lend you money, they expect that the loan will last its full term. So, they expect to receive the full amount of the finance charge. If you repay the loan early, they consider that the unearned interest has been lost to them, and treat it as a refund.

To calculate the earned and unearned finance charges when a Rule of 78 loan is repaid early, use the finance charge refund schedule shown. This schedule shows the percent of the total finance charge that is *unearned* by the lender.

BUSINESS TIP

The finance charge refund schedule does not show how much the borrower will get back if a loan is paid early. It shows the amount of the total finance charge that has been lost to the lender because the loan was repaid early.

Finance Charge Refund Schedule for Early Repayment of Installment Loan								
Percent of Finance Charges Refunded								
Elapsed Time of Loan in Months	Original Term of Loan in Months							
	3	6	9	12	15	18	21	24
1	50.00%	71.43%	80.00%	84.62%	87.50%	89.47%	90.91%	92.00%
2	16.67%	47.62%	62.22%	70.51%	75.83%	79.53%	82.25%	84.33%
3	0.00%	28.57%	46.67%	57.69%	65.00%	70.18%	74.03%	77.00%
4		14.29%	33.33%	46.15%	55.00%	61.40%	66.23%	70.00%
5		4.76%	22.22%	35.90%	45.83%	53.22%	58.87%	63.33%
6		0.00%	13.33%	26.92%	37.50%	45.61%	51.95%	57.00%
7			6.67%	19.23%	30.00%	38.60%	45.45%	51.00%
8			2.22%	12.82%	23.33%	32.16%	39.39%	45.33%
9			0.00%	7.69%	17.50%	26.32%	33.77%	40.00%
10				3.85%	12.50%	21.05%	28.57%	35.00%
11				1.28%	8.33%	16.37%	23.81%	30.33%
12				0.00%	5.00%	12.28%	19.48%	26.00%
15					0.00%	3.51%	9.09%	15.00%
18						0.00%	2.60%	7.00%
21							0.00%	2.00%

EXAMPLE 1

Vera Goode had a \$500, 12-month Rule of 78 loan. The total finance charge was \$80. Vera repaid the loan on the day the third monthly payment was due. Find the amount of finance charge the lender earned and the amount that was unearned.

SOLUTION

Find the unearned finance charge refund percentage in the finance charge refund schedule and change it to a decimal.

$$57.69\% = 0.5769 \quad \text{finance charge refund decimal rate} \\ \text{(3 months of loan have elapsed)}$$

Multiply the loan's total finance charge by the refund decimal rate.

$$\$80 \times 0.5769 = \$46.152, \text{ or } \$46.15 \quad \text{unearned finance charge}$$

Subtract the unearned finance charge from the total finance charge.

$$\$80 - \$46.15 = \$33.85 \quad \text{earned finance charge}$$

■ CHECK YOUR UNDERSTANDING

- A. Mario Mineto bought a large TV and paid for it with a 6-month Rule of 78 installment loan. The total finance charge for the loan was \$130. He decided to repay the loan at the end of the third month. What was the amount of his earned and unearned finance charges?
- B. Emily Polinski repaid a 9-month, \$2,000 Rule of 78 installment loan at the end of 6 months. The total finance charge for the loan was \$225. How much were the lender's earned and unearned finance charges?

■ Early Loan Repayments

To find the amount to repay a Rule of 78 loan early, find the total of the remaining payments. Subtract from that amount the unearned finance charge on the loan.

EXAMPLE 2

Sy Bauer has a 12-month, \$3,600 Rule of 78 installment loan. The total finance charge for the loan is \$420. The monthly installment payments are \$335. Sy has made 5 monthly installment payments and wants to repay the balance when the sixth payment is due. How much will Sy owe?

SOLUTION

Multiply the monthly payment amount by the number of monthly payments yet to be made to find the total of the remaining payments.

$$\$335 \times 7 = \$2,345 \quad \text{total amount of remaining payments}$$

Find the finance charge refund percentage and change it to a decimal.

$$26.92\% = 0.2692$$

Multiply to find the unearned finance charge.

$$\$420 \times 0.2692 = \$113.06 \quad \text{unearned finance charge}$$

Subtract to find the amount needed to repay the loan.

$$\$2,345 - \$113.06 = \$2,231.94 \quad \text{amount needed to repay loan}$$

Notice that 7 payments were remaining. Sy had only made 5 payments so far. Instead of making the sixth payment, he paid off the loan. However, the bank had lent him the money for a full 6 months. So, he owed finance charges on all 6 months.



■ CHECK YOUR UNDERSTANDING

- C. Lee Ivor repays a 9-month, \$5,300 Rule of 78 installment loan on the day that the 6th monthly payment is due. The total finance charge for the loan is \$336.70. The monthly installment payments are \$626.30. How much will Lee owe?
- D. Rob Jewel has a 12-month, \$8,400 Rule of 78 installment loan with a total finance charge of \$841.32. The monthly installment payments are \$770.11. He repays the loan on the day that the 6th monthly payment is due. How much will Rob pay to the bank?

Wrap Up

Look at Eva Lewis's loan in the Start Up section of the lesson. If the bank used the Rule of 78 to find the finance charge on the loan, she would need \$512.86 to repay it. The principal plus finance charges at 18% for the full term of the loan are \$545. The finance charge refund rate is 0.7143, and the unearned finance charge is \$32.14. The finance charge for the first month is $\$45 - \$32.14 = \$12.86$. A simple interest installment loan would have charged \$7.50 interest for one month.

Algebra Connection

Suppose you had a 12-month Rule of 78 installment loan with a finance charge of \$300. You want to pay off the loan at the end of 3 months. Instead of the Refund Schedule, you can use the following formula to find the refund decimal.

$$\frac{U(U+1)}{T(T+1)} = \text{Rule of 78 refund decimal}$$

U = The unelapsed term of the loan. In a 12-month loan paid off in 3 months, it is 9 months.

T = The term of the loan. In the example loan, it is 12 months.

F = The finance charge. In the example loan, it is \$300.

Substitute the values from the example loan into the formula.

$$\frac{9(9+1)}{12(12+1)} = 0.576923, \text{ or } 0.5769 \quad \text{Round to the nearest ten thousandth.}$$

Compare the decimal found by the formula to the one in the Refund Schedule.

EXERCISES

Rewrite as a decimal.

1. 245.6%.

2. 0.35%.

Perform the indicated operation.

3. $\$4,208 - \$3,489$

4. $\$63,418.36 - 48,897.57$

5. $\$189.87 \times 12$

6. $\$45.63 \times 3.148$

7. Rachel Carr has a 9-month, Rule of 78 installment loan. The total finance charge for the loan is \$450. She decides to repay the loan on the day that the third monthly payment is due. What is the unearned finance charge?
8. Levi Stein's 24-month, Rule of 78 installment loan has a total finance charge of \$675. He decides to repay the loan on the day that the twelfth payment is due. How much is the unearned finance charge?

Terry O'Doole has an 18-month installment loan for \$1,700 at 12% annual interest. Interest is computed using the Rule of 78. Terry decides to pay back the loan at the end of one month.

9. What is the total finance charge on the loan?
10. What is the unearned finance charge?
11. What is needed to repay the loan?

BEST BUY Lucy Smola can borrow \$2,500 at 18% for 12 months from lender A who uses the Rule of 78. She can also borrow \$2,500 at 18% from a lender B, who uses the simple interest method.

12. Which loan has the largest finance charge?
13. Which loan has the highest finance charge for the first month and by how much?
14. How much would it cost Lucy to pay back the loan from lender B at the end of the first month?
15. **CRITICAL THINKING** Why do you think Rule of 78 loans are so widely used for short-term loans? Why are Rule of 78 loans illegal in some states? Is the refund shown in the Refund Schedule really a refund?



MIXED REVIEW

16. Round 29,458 to the nearest hundred.
17. Estimate the product of $\$34.56 \times 24.8$.
18. What is the average of \$35.12, \$36.20, \$32.98, and \$33.56?
19. The cash price of a large color TV was \$2,400. Dee Hart bought it for \$240 down and 12 monthly payments of \$196. By what percent did the installment price exceed the cash price?

4.6

Annual Percentage Rates

GOAL

- Calculate the APR on a loan

Start Up

An auto dealer gives you a verbal quote for an interest rate on an installment loan for a car that you think is just terrific. Your friend thinks you are probably paying a higher interest rate. Who is probably right, you or your friend?



Math Skill Builder

Review these math skills and solve the exercises that follow.

- 1 Round** dollar amounts to the nearest cent.
Round $\$0.2978$ to the nearest cent. $\$0.2978 = \0.30
1a. $\$3.985$ 1b. $\$2.0793$ 1c. $\$8.0049$ 1d. $\$0.455$
- 2 Multiply** dollar amounts by 10, 100, and 1,000.
Find the product. $\$489.43 \times 100 = \$48,943$
2a. $\$189 \times 100$ 2b. $\$208.97 \times 1,000$ 2c. $\$790.72 \times 10$
- 3 Divide** dollar amounts by dollar amounts to the nearest thousandth.
Find the quotient to the nearest thousandth. $\$500 \div \$3,000 = 0.1666$, or 0.167
3a. $\$420 \div \$1,600$ 3b. $\$1,241 \div \$8,560$ 3c. $\$2,445 \div \$16,308$

■ Annual Percentage Rate (APR)

To find the rate of interest on a single-payment loan for one year, you divide the interest paid in a year by the principal. Finding the rate of finance charges on an installment loan is not as easy. The cost of borrowing money may include more than interest. It may also include service charges. Also, since you make payments on the loan each month, you are not borrowing the whole principal for the full time of the loan.

So the *Truth in Lending Act* makes the lender tell the borrower what annual rate is charged on the loan. The rate is called the **annual percentage rate (APR)**. It is usually higher than the interest rate of your loan. The amount you are actually borrowing is called the *amount financed*. This might be the sale price of a car less the down payment.

The easiest way to find the annual percentage rate is to use tables like the ones shown on the next page. To use the tables, you need to know the number of monthly payments for the loan and the finance charge per \$100 of the amount financed.

To find the *finance charge per \$100* of the amount financed, divide the finance charge by the amount financed. Then multiply the quotient by 100.

$$\text{Finance Charge per } \$100 \text{ of Amount Financed} = \frac{\text{Finance Charge}}{\text{Amount Financed}} \times \$100$$

After you have found the finance charge per \$100, you can use the tables to find the annual percentage rate

Number of Payments	Annual Percentage Rate										
	12 ³ / ₄ %	13%	13 ¹ / ₄ %	13 ¹ / ₂ %	13 ³ / ₄ %	14%	14 ¹ / ₄ %	14 ¹ / ₂ %	14 ³ / ₄ %	15%	15 ¹ / ₄ %
Finance Charge per \$100 of Amount Financed											
3	2.13	2.17	2.22	2.26	2.30	2.34	2.38	2.43	2.47	2.51	2.55
6	3.75	3.83	3.90	3.97	4.05	4.12	4.20	4.27	4.35	4.42	4.49
9	5.39	5.49	5.60	5.71	5.82	5.92	6.03	6.14	6.25	6.35	6.46
12	7.04	7.18	7.32	7.46	7.60	7.74	7.89	8.03	8.17	8.31	8.45
15	8.71	8.88	9.06	9.23	9.41	9.59	9.76	9.94	10.11	10.29	10.47

Number of Payments	Annual Percentage Rate										
	15 ¹ / ₂ %	15 ³ / ₄ %	16%	16 ¹ / ₄ %	16 ¹ / ₂ %	16 ³ / ₄ %	17%	17 ¹ / ₄ %	17 ¹ / ₂ %	17 ³ / ₄ %	18%
Finance Charge per \$100 of Amount Financed											
6	4.57	4.64	4.72	4.79	4.87	4.94	5.02	5.09	5.17	5.24	5.32
12	8.59	8.74	8.88	9.02	9.16	9.30	9.45	9.59	9.73	9.87	10.02

Number of Payments	Annual Percentage Rate										
	18 ¹ / ₄ %	18 ¹ / ₂ %	18 ³ / ₄ %	19%	19 ¹ / ₄ %	19 ¹ / ₂ %	19 ³ / ₄ %	20%	20 ¹ / ₄ %	20 ¹ / ₂ %	20 ³ / ₄ %
Finance Charge per \$100 of Amount Financed											
6	5.39	5.46	5.54	5.61	5.69	5.76	5.84	5.91	5.99	6.06	6.14
12	10.16	10.30	10.44	10.59	10.73	10.87	11.02	11.16	11.31	11.45	11.59

EXAMPLE 1

The finance charge for a 6-month, \$1,200 installment loan is \$72. Find the annual percentage rate on the loan.

SOLUTION

Divide the finance charge by the amount financed.

$$\$72 \div \$1,200 = 0.06$$

$$0.06 \times \$100 = \$6 \quad \text{Multiply the result by } \$100.$$

The finance charge per \$100 of amount financed is \$6.

Use the Annual Percentage Rate Tables. Read across the rows for 6 payments until you come to the amount closest to \$6. Since \$5.99 is the closest amount, use the rate, 20¹/₄%.

The annual percentage rate is 20¹/₄%.

MATH TIP

When necessary, round the finance charge to the nearest cent.

■ CHECK YOUR UNDERSTANDING

- Melina Cavaletti borrowed \$800 on a loan with a finance charge of \$78. Find the finance charge per \$100 of the amount financed.
- Chris Mathers borrowed \$250 on a 12-month loan that had a finance charge of \$20. Find the finance charge per \$100 of the amount financed and the annual percentage rate.

Wrap Up

The auto dealer is probably quoting you an annual interest rate based on the original amount you financed. Since you are paying back part of the amount financed each month, your interest rate is higher. You need to find the annual percentage rate on the car loan and compare that with the quote from the auto dealer.

Algebra Connection

An *approximate* annual percentage rate can be calculated for monthly level-payment loans without the use of tables using this formula:

$$\text{APR} = \frac{24F}{P(N + 1)}$$

F = Finance charge amount

P = Original principal

N = Number of payments per year

Revisit this Algebra Connection after completing the exercises. Enter the information from Exercise 19, find the APR to the nearest tenth percent, and compare your results to the answer to Exercise 19 you got using the table. Which gave a higher percent, the table or the formula? What was the amount of the difference? Would you rely on the formula to find the APR?

EXERCISES

Round to the nearest cent.

1. \$4.5627
2. \$105.3978

Find the product.

3. $\$98,208 \times 100$
4. $\$389.74 \times 1,000$
5. $\$1,078.43 \times 10$
6. $\$38.95 \times 100$

Find the quotient to the nearest thousandth.

7. $\$186 \div \828
8. $\$482 \div \$4,298.18$

9. Find the finance charge per \$100 on a loan of \$3,200 with a finance charge of \$480.

10. Find the finance charge per \$100 on a loan of \$12,600 with a finance charge of \$1,638.

Find the annual percentage rate for the following loans.

11. Finance charge of \$5.75 per \$100 for 9 payments.

12. Finance charge of \$2.50 per \$100 for 3 payments.

13. Finance charge of \$8.85 per \$100 for 15 payments.

14. **CRITICAL THINKING** What specific pieces of information should you look for in any loan contract? Why is this information important?

You borrow \$2,600 and repay the loan in 12 monthly installments of \$232.

15. What was the finance charge on your loan?
16. What was the finance charge per \$100 of the amount financed?
17. What was the annual percentage rate?

Olga Pozinski borrowed \$1,300 and repaid it in 12 monthly payments of \$116.

18. What was the finance charge on the loan?
19. What was the finance charge per \$100 of amount financed?
20. What was the annual percentage rate?
21. Charles repaid a loan of \$1,600 in 15 monthly installments of \$116.40 each. Find the APR on his loan.

INTEGRATING YOUR KNOWLEDGE Violeta Ramos borrowed \$4,000 and agreed to repay the loan in 12 equal monthly payments. To find the total finance charge, the lender calculated simple interest of 8% on the \$4,000.

22. What is the simple interest on \$4,000 for 1 year at 8%?
23. Find the total amount Violeta will pay over the life of the loan.
24. How much will Violeta's monthly payments be?
25. What is the finance charge per \$100 of amount financed?
26. What is the annual percentage rate?

MIXED REVIEW

27. $\$788 \times 100 =$
28. $\$9.75 \times 1,000 =$
29. $\$10.78 \times 10 =$
30. $\$3.97 \times 100 =$
31. Write $\frac{1}{4}$ as a ratio.
32. Rewrite $12\frac{5}{8}$ as a percent.
33. On January 31, Edith Nagel's bank statement balance was \$516.24. Her check register showed that a deposit for \$382.10 was outstanding. The following checks were also outstanding: #108, \$45.93; #109, \$108.12; #111, \$6.87. What was the corrected bank statement balance?
34. Rhonda Peterson pays a city tax of 1.5% on her taxable income. She also pays a state tax of 3.25% on her taxable income. What total city and state income taxes does she pay if her taxable income is \$38,109?
35. Hector Vadillo is a waiter and had these total food and beverage checks for the week: Tues., \$245.19; Wed., \$299.74; Thurs., \$349.73; Fri., \$612.50; Sat., \$575.33. If Hector received an average of 15% for tips during the week, how much money did he receive in tips?
36. Find the exact interest on \$1,050 at 14% for 126 days.

